ELEMENTS OF
PRACTICAL AERODYNAMICS

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BY

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PREFACE TO SECOND EDITION

Since the first presentation of this book, it has seemed desirable to include additional material which will be of value to a student and which properly merits place in an elementary textbook. For this reason in the present edition there has been added a description of N.A.C.A. airfoils, the standard taper of a monoplane wing, a graphical method of developing a streamline contour, a mathematical treatment of range and take-off distance, and an abridgement of Oswald's method of rapid prediction of performance. In various places throughout the book, especially in the chapter on longitudinal balance, the treatment of the subject has been enlarged in an attempt to clarify it for the reader.

In revising the original edition, the author wishes to acknowledge the valuable criticisms and suggestions of Professors A. Lavrow of the Detroit Institute of Technology, W. A. Bevan of the Iowa State College, V. W. Young of the Oklahoma A. and M. College, and T. E. Butterfield of Lehigh University. The author is especially grateful to his colleague at the University of Cincinnati, Mr. Mortimer Powell, for his valuable editorial assistance.

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PREFACE TO FIRST EDITION

The existence of several excellent works on the subject of aerodynamics demands that an explanation be offered for the appearance of a new one.

My experience in teaching has convinced me that a book intended primarily for the classroom must be, in general, different in character and scope from books intended for reference by practicing engineers. It is attempted in this textbook to present the subject matter as simply as possible. In two isolated cases of summation of forces, it is necessary to introduce the calculus to prove the correctness of certain statements. If these statements are accepted as being true, the proofs may be dispensed with, and no mathematical training higher than elementary algebra is needed for complete understanding of the text.

Although aerodynamics deals with air in motion and aerostatics with air at rest, a short chapter on aerostatics has been included because it is believed that students of aerodynamics should be acquainted with the elements of the sister subject.

Use has been made of data from various publications of the National Advisory Committee for Aeronautics and the Army and Navy Designers Handbook. I desire to acknowledge useful criticisms and suggestions from Professor Harold W. Sibert of this University.

BRADLEY JONES

UNIVERSITY OF CINCINNATI
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ELEMENTS OF PRACTICAL AERODYNAMICS

CHAPTER I

PHYSICAL PROPERTIES OF AIR

Aerodynamics is the study of the motion of air and of the forces on solids in motion relative to air.

In a fluid the particles cohere so slightly that they may be easily made to change their relative positions by the application of small forces; i.e., the modulus of shear is small. The amount of internal friction of a fluid may be very small, and one may conceive of a perfect fluid as a fluid wherein there is no internal friction, so that between two particles the action of any force must be normal to the contact surfaces, and cannot have any tangential component.

Air is often referred to as a perfect fluid. Air is a gas. It is a physical mixture, not a chemical compound.

The earth's atmosphere at sea-level has the following percentages by volume of these gases:

Nitrogen	78.08
Oxygen	20.94
Argon	0.94
Hydrogen	0.01
Neon	0.0012
Helium	0.0004
Carbon dioxide	0.03

Water vapor also is always present, the amount varying with the temperature and other factors, but averaging about 1.2 per cent at the earth's surface.

Up to altitudes encountered by aircraft (40,000 ft.) there are always winds and vertical air currents to keep the various constituents commingled in approximately the same proportion, as listed. At high altitudes, undoubtedly the different gases composing the atmosphere separate according to their respective densities, hydrogen forming the outermost layer, helium the next, and so on.

Because air is a gas, its density varies with temperature, pressure, etc. The standard conditions most commonly used (N.A.C.A.) are a barometric pressure of 29.92 in. (or 760 mm.) of mercury, and a temperature of 59° F. (or 15° C.).

Under these conditions, the mass density (ρ) of dry air is 0.002378 slug per cubic foot.

Using for standard acceleration of gravity (g) equal to 32.1740 ft. per sec. per sec., the specific weight of " standard " dry air is 0.07651 lb. per cu. ft.

Up till 1926 the National Advisory Committee for Aeronautics used for standard conditions a barometric pressure of 29.92 in. of mercury and a temperature of 60° F. (15.6° C.). The mass density of dry air under these conditions is 0.002372 slug and the weight density (g = 32.172 ft. per sec.^{2}) is 0.07635 lb. per cu. ft. In using data from N.A.C.A. reports or other sources of information, care should be taken to note the standard conditions of the test.

Air conforms to Boyle's law that at constant temperatures and up to certain limits the volume varies inversely as the pressure and to Charles's law that at constant pressure the volume varies directly as the absolute temperature. These two laws are usually expressed as the formula:

$$\frac{P_0 V_0}{T_0} = \frac{P_1 V_1}{T_1}$$

where P_0, V_0, and T_0 are pressure, volume, and absolute temperature under standard conditions, and P_1, V_1, and T_1 are under other than standard conditions.

As weight density is the weight divided by the volume, the density of a gas is increased by an increase in pressure, a decrease in volume, or a decrease in temperature. In a given volume, the density, as well as specific weight, varies directly as the pressure and inversely with absolute temperature.

$$\frac{\rho}{\rho_0} = \frac{P}{P_0}\,\frac{T_0}{T}$$

Example. What is the density of dry air if the pressure is 25.93 in. of mercury and the temperature is 45° F.?

The absolute temperature (T) = 45 + 459.4 = 504.4.

$$\rho = \frac{P}{P_0}\frac{T_0}{T}\rho_0 = \frac{25.93}{29.92} \times \frac{518.4}{504.4} \times 0.002378 = 0.00212$$

Example. What is the specific weight of dry air if the pressure is 16.38 in. of mercury and the temperature is −10° F.?

First method:

$$\rho = \frac{P}{P_0}\frac{T_0}{T}\rho_0 = \frac{16.38}{29.92} \times \frac{518.4}{449.4} \times 0.002378 = 0.001502$$

Specific weight $\rho \times g$ = 0.001502 × 32.1740 = 0.04832 lb. per cu. ft.

Second method:

Specific weight $= 0.07651 \times \frac{16.38}{29.92} \times \frac{518.4}{449.4} = 0.04832$ lb. per cu. ft.

Problems

1. Find the density of dry air at 20 in. pressure and 10° F.

2. Find the density of dry air at 18.52 in. pressure and 0° F.

3. Find the specific weight of dry air at 24 in. pressure and 25° F.

TABLE I

ALTITUDE-PRESSURE-DENSITY RELATION

Based on N.A.C.A. No. 218

Altitude, ft.	Temperature, °F.	Pressure, in. Hg	$\frac{\rho}{\rho_0}$	$\sqrt{\frac{\rho_0}{\rho}}$	ρ
0	59.0	29.92	1.000	1.000	0.002 378
1 000	55.4	28.86	0.9710	1.0148	.002 309
2 000	51.8	27.82	.9428	1.0299	.002 242
3 000	48.4	26.81	.9151	1.0454	.002 176
4 000	44.8	25.84	.8881	1.0611	.002 112
5 000	41.2	24.89	.8616	1.0773	.002 049
6 000	37.6	23.98	.8358	1.0938	.001 987
7 000	34.0	23.09	.8106	1.1107	.001 928
8 000	30.6	22.22	.7859	1.1280	.001 869
9 000	27.0	21.38	.7619	1.1456	.001 812
10 000	23.4	20.58	.7384	1.1637	.001 756
11 000	19.8	19.79	.7154	1.1822	.001 701
12 000	16.2	19.03	.6931	1.2012	.001 648
13 000	12.6	18.29	.6712	1.2206	.001 596
14 000	9.2	17.57	.6499	1.2404	.001 545
15 000	5.5	16.88	.6291	1.2608	.001 496
20 000	−12.3	13.75	.5327	1.3701	.001 267
25 000	−30.1	11.10	.4480	1.4940	.001 065
30 000	−48.1	8.88	.3740	1.6352	.000 889
35 000	−65.8	7.04	.3098	1.7961	.000 736
40 000	−67.0	5.54	.2447	2.0215	.000 582
45 000	−67.0	4.36	.1926	2.2786	.000 459
50 000	−67.0	3.44	.1517	2.5674	.000 361

Atmosphere. The air in the atmosphere close to the earth is compressed by the weight of the air above it. At higher altitudes, the air is under less pressure because there is less air above to cause pressure. The evidence from observations of meteors appears to indicate that our atmosphere extends upward at least 500 miles. In the upper limits the air is greatly rarified so that there is no exact demarcation at the upper edge.

Heat is radiated from the sun, and this radiation passes through our atmosphere without any appreciable heating effect. The sun's radiation heats the earth. The layer of air resting immediately on the earth is heated by conduction. This air in turn warms air superadjacent to it. Also the warm air will rise, and, in ascending to a region where the pressure is less, it will expand. If no heat is added or subtracted, which will be true if the upward movement is fairly rapid, when the air expands, its temperature drops. With increase in altitude above the earth, the temperature decreases. This decrease in temperature with altitude continues until at high altitudes the temperature has decreased to $-72°$ F. (at latitude 45°); then no further decrease in temperature takes place with increased altitude.

The lower region of the earth's atmosphere is called the troposphere. In the troposphere, the temperature decreases with altitude, winds may blow from any direction, and there is moisture in the air so that there may be clouds. The upper region of the earth's atmosphere is called the stratosphere. In the stratosphere, the temperature is constant, not varying with altitude; any winds that may blow are from a westerly direction and there is no moisture, consequently no clouds.

The dividing surface between the troposphere and the stratosphere is called the tropopause. The altitude of the tropopause is 56,000 ft. in the tropics, about 38,000 ft. over the United States, and about 28,000 ft. in the polar regions.

For convenience in aeronautics, a "standard" atmosphere has been adopted; that is, it has been agreed to assume that the temperature, pressure, and, consequently, density are fixed and constant for any altitude. This hypothetical "standard" atmosphere assumes that there is no moisture present in the air. This standard atmosphere represents average conditions at 40° latitude, but at any one time there may actually be considerable divergence from this standard, especially at low altitudes. The standard atmosphere has been approved by the International Commission for Air Navigation and is therefore frequently referred to as the I.C.A.N. atmosphere. The standard atmosphere is given in Table I.

The standard atmosphere assumes that at sea-level the barometric pressure is 29.92 in. and the temperature is 59° F. (15° C.). It assumes further that the temperature decreases 1° F. for every 280 ft. increase in altitude up to a height of 35,332 ft., where the temperature is −67° F. (−55° C.). From this altitude upward the temperature is assumed to be constant. It is assumed that the air is dry, that the change in acceleration due to gravity is negligible, and that air conforms to Boyle's and Charles' laws.

The temperature at any altitude is found by subtracting the temperature drop with altitude from the standard sea-level temperature.

$$T = 518.4 - aZ$$

where T = absolute temperature at altitude of Z feet
a = standard temperature gradient, 0.003566° F./ft.

The difference in barometric pressure at two different altitudes is due merely to the weight of the column of air of unit cross-section between the two heights. In order to ascertain this weight, it is necessary first to know the average temperature of the column. Because the density varies with the altitude, the harmonic mean temperature is used. For altitudes below 35,332 ft., the mean temperature is found by the following formula.

$$T_m = \frac{aZ}{\log_e \dfrac{T_0}{T_0 - aZ}}$$

where T_m = mean temperature (abs.)
T_0 = standard temperature (abs.)
Z = altitude in feet

If the mean temperature (T_m) is known, the barometric pressure (p) at any altitude (Z) may be found by the formula, due to Laplace;

$$\log_e \frac{p_0}{p} = \frac{\rho_0 g T_0}{p_0 T_m} Z$$

where p_0 = standard pressure
ρ_0 = standard density
T_0 = standard temperature
g = standard acceleration of gravity

By substituting the values of standard conditions and using 70.726 lb., the weight of one square foot of mercury, one inch high, to transform pressures from pounds per square foot to inches of mercury, the above formula is modified to

$$2.3026 \log_{10} \frac{29.921}{p} = \frac{0.002378 \times 32.174 \times 518.4 \times Z}{29.921 \times 70.73 \times T_m}$$

$$\log_{10} p = \log_{10} 29.921 - \frac{0.002378 \times 32.174 \times 518.4 \times Z}{2.3026 \times 29.921 \times 70.73 \times T_m}$$

$$= 1.47597 - 0.0081398 \frac{Z}{T_m}$$

Example. Find the temperature, pressure, and density at 18,000 ft. altitude in standard atmosphere.

Solution.

$$\begin{aligned}
T &= 518.4 - aZ \\
&= 518.4 - 0.003566 \times 18{,}000 \\
&= 518.4 - 64.2 \\
&= 454.2° \text{ F. absolute} \\
&= -5.2° \text{ F.}
\end{aligned}$$

$$\begin{aligned}
T_m &= \frac{aZ}{\log_e \dfrac{T_0}{T_0 - aZ}} \\
&= \frac{64.2}{2.3026 \log_{10} \dfrac{518.4}{454.2}} \\
&= 485.6° \text{ F. absolute}
\end{aligned}$$

$$\begin{aligned}
\log_{10} p &= 1.47597 - 0.0081398 \frac{Z}{T_m} \\
&= 1.47597 - 0.0081398 \frac{18{,}000}{485.6} \\
&= 1.17423 \\
p &= 14.936 \text{ in. of Hg.} \\
\rho &= \rho_0 \left(\frac{p}{p_0}\right)\left(\frac{T_0}{T}\right) \\
&= 0.002378 \times \frac{14.94}{29.92} \times \frac{518.4}{454.2} \\
&= 0.001355 \text{ slug per cu. ft.}
\end{aligned}$$

Problems

1. Find the density of air in the standard atmosphere at 21,000 ft. altitude.

2. Find the density of air in the standard atmosphere at 27,000 ft. altitude.

3. Find the density of air in the standard atmosphere at 32,500 ft. altitude.

For altitudes above 35,332 ft., the formula for mean temperature becomes

$$T_m = \frac{392.4\,Z}{Z - 4704.9}$$

Example. Find the temperature, pressure, and density at 40,000 ft. in standard atmosphere.

Solution.

$$\begin{aligned}
T &= 392.4° \text{ F. absolute (isothermal temperature)} \\
&= -67.0° \text{ F.} \\
T_m &= \frac{392.4\,Z}{Z - 4704.9} \\
&= \frac{392.4 \times 40{,}000}{40{,}000 - 4704.9} \\
&= 444.7° \text{ F. absolute} \\
\log_{10} P &= 1.47597 - 0.0081398\,\frac{Z}{T_m} \\
&= 1.47597 - 0.0081398\,\frac{40{,}000}{444.7} \\
P &= 5.54 \text{ in. of Hg.} \\
\rho &= \rho_0\left(\frac{P}{P_0}\right)\left(\frac{T_0}{T}\right) \\
&= 0.002378 \times \frac{5.54}{29.92} \times \frac{518.4}{392.4} \\
&= 0.000582 \text{ slug per cu. ft.}
\end{aligned}$$

Problems

1. Find the density of air in the standard atmosphere at 38,200 ft. altitude.

2. Find the density of air in the standard atmosphere at 44,400 ft. altitude.

3. Find the density of air in the standard atmosphere at 49,200 ft. altitude.

Viscosity. The viscosity of air is much less than for water or other liquids. It is not zero, nor in some calculations is it even negligible. In liquids, viscosity is due to the internal friction of the molecules. In gases, viscosity is due not so much to internal friction as to molecular vibration. Consider a rapidly moving stratum of gas immediately over another stratum of gas moving more slowly in the same direction. Some molecules from the upper layer, owing to their vibratory motion, will wander into the lower layer and will accelerate the motion of the lower strata. Molecules passing from the lower to the upper stratum will retard the latter. Any mingling of the molecules between the two strata results in the two velocities becoming more nearly equal.

In liquids, an increase in temperature causes a decrease in viscosity, because the intermolecular friction is less. In gases, an increase in temperature causes an increase in viscosity, because there is an increase in molecular vibration and therefore an increase in molecular interchange.

The coefficient of viscosity (μ) of air varies approximately as the $\frac{3}{4}$ power of the absolute temperature.

TABLE II

Coefficient of Viscosity

Temperature, °C.	C.g.s. units, poises	Slug-feet-second units
−30	$1\,554 \times 10^{-7}$	3.25×10^{-7}
−20	$1\,605 \times 10^{-7}$	3.35×10^{-7}
−10	$1\,657 \times 10^{-7}$	3.46×10^{-7}
0	$1\,709 \times 10^{-7}$	3.57×10^{-7}
10	$1\,759 \times 10^{-7}$	3.67×10^{-7}
20	$1\,808 \times 10^{-7}$	3.78×10^{-7}
30	$1\,856 \times 10^{-7}$	3.88×10^{-7}
40	$1\,904 \times 10^{-7}$	3.98×10^{-7}
50	$1\,951 \times 10^{-7}$	4.08×10^{-7}
60	$1\,997 \times 10^{-7}$	4.17×10^{-7}
70	$2\,043 \times 10^{-7}$	4.27×10^{-7}
80	$2\,088 \times 10^{-7}$	4.36×10^{-7}
90	$2\,132 \times 10^{-7}$	4.45×10^{-7}
100	$2\,175 \times 10^{-7}$	4.54×10^{-7}

The N.A.C.A. uses a value of 3.73×10^{-7} for coefficient of viscosity at 15° C.

Except for very high or very low pressure, the coefficient of viscosity, μ, is independent of pressure.

Kinematic Viscosity. The coefficient of kinematic viscosity (ν) is the ratio of the coefficient of viscosity (μ) to the density.

$$\nu = \frac{\mu}{\rho}$$

Because the density of air is affected by both pressure and temperature, whereas the coefficient of viscosity is affected only by temperature, the coefficient of kinematic viscosity of air is less at high altitudes than at the ground. For example, at 15° C., the coefficient of kinematic viscosity (ν) is 1.50×10^{-4} square feet per second; whereas, at $-30°$ C., the coefficient of kinematic viscosity (ν) is 1.21×10^{-4} square feet per second.

The kinematic viscosity is a factor in Reynolds' number, which will be discussed in a later chapter.

Modulus of Elasticity. For solids, the modulus of elasticity (E) is simply the ratio of stress to strain. In the case of gases under tension or compression, the temperature changes during stress should properly be considered. With rapid changes in the stress, such as would occur in compression waves, because of the low thermal conductivity of air, no appreciable heat is lost or gained so that the adiabatic law is assumed to be correct, i.e., that

$PV^n =$ a constant where $n =$ ratio of specific heats; for air, $n = 1.405$

Since density varies inversely as volume, the adiabatic law may be written as

$$P = k\rho^n$$

By differentiating with respect to ρ

$$\frac{dP}{d\rho} = nk\rho^{n-1}$$

The modulus of elasticity (E) being the ratio of stress to strain, if an increase of pressure from P to $P + \Delta P$ causes a change in volume from V to $V - \Delta V$, so that the strain is $-\Delta V/V$.

$$E = \frac{\Delta P}{-\frac{\Delta V}{V}}$$

but

$$-\frac{\Delta V}{V} = \frac{\Delta\rho}{\rho}$$

Therefore

$$E = \rho\frac{\Delta P}{\Delta\rho}$$

Expressed as a derivative

$$E = \rho\frac{dP}{d\rho}$$

but from the adiabatic law

$$
\begin{aligned}
\frac{dP}{d\rho} &= nk\rho^{n-1} \\
E &= \rho \times nk\rho^{n-1} \\
&= nk\rho^{n} \\
&= nP
\end{aligned}
$$

Velocity of Sound. Sound is propagated through air by means of compression waves. Newton proved that the velocity of sound through a fluid varies directly as the square root of the modulus of elasticity and inversely as the square root of the density.

$$
\begin{aligned}
\text{Velocity of sound} &= \sqrt{\frac{E}{\rho}} \\
&= \sqrt{\frac{nP}{\rho}}
\end{aligned}
$$

For gases, however,

$PV = \dfrac{P}{\rho} = RT$ where R = a constant for each gas (for air $R = 3075$)
T = absolute temperature Centigrade

If this is substituted in the previous equation,

$$
\begin{aligned}
\text{velocity of sound} &= \sqrt{nRT} \\
&= \sqrt{1.405 \times 3075 \times T} \\
&= 65.9\sqrt{T}
\end{aligned}
$$

For 15° C. ($T = 288°$), the velocity of sound is therefore 1,118 ft. per sec.

In designing propellers it is not considered good practice to have the tips rotating at a speed close to the velocity of sound. If the tip-speed corresponds to the velocity of sound, violent compression waves are initiated and there is great loss of power. At altitudes, the velocity of sound is greatly reduced and this must be reckoned with in designing propellers for sub-stratospheric flight. For example, at 40,000 ft. altitude (temperature, −55° C.) the velocity of sound is only 973 ft. per sec.

It is to be noted that changes in the barometric pressure have no effect on the velocity of sound.

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CHAPTER II

EFFECTS OF DEFLECTING AIR STREAMS

Isaac Newton formulated three well-known laws of motion.

1. Every body persists in its state of rest or uniform motion in a straight line, unless it be compelled by some force to change that state.

2. The rate of change of momentum of a body is proportional to the force acting on the body and is in the direction of the force.

3. Action and reaction are equal and opposite.

Air, though of less density than fluids or solids, does possess mass. It conforms to the laws of motion. If a mass of air is at rest, a force is required to put it in motion. If the mass of air is in steady rectilinear motion, a force is required to slow it, stop it, or change its direction. Air in motion has momentum.

When a quantity of air is put in motion work has been done on the air and it has acquired momentum. Momentum is the product of the mass and the velocity.

A cubic foot of air moving at a speed of V feet per second will have a momentum of ρV.

Kinetic Energy. A cubic foot of air at rest to be put in motion must have a force applied. Let F be the force applied to increase the velocity from zero to V feet per second. If this change in velocity takes place uniformly in t seconds, the acceleration is V/t feet per second per second. Since a force is measured by the mass times the acceleration, $F = \rho V/t$. In the t seconds during which the force is applied, the average velocity is $\frac{1}{2}V$ feet per second and the distance covered is this average velocity multiplied by the time in seconds, that is, $\frac{1}{2}Vt$. The work done on the cubic foot of air in imparting to it a velocity of V feet per second is the product of the force and the distance through which the force acts.

$$\text{Work} = \frac{\rho V}{t} \times \frac{1}{2}Vt = \frac{1}{2}\rho V^2 \qquad \begin{array}{l} W \text{ in foot-pounds} \\ \rho \text{ in slugs per cubic foot} \\ V \text{ in feet per second} \end{array}$$

This moving mass of air is capable of doing work and is therefore said to possess energy of motion or kinetic energy. The kinetic energy is numerically equal to the work which was required to put the air in motion, that is, $\frac{1}{2}\rho V^2$ foot-pounds.

Momentum Pressure. If a horizontal stream of moving air were to strike perpendicularly against a wall so as to lose all its momentum, the amount of momentum lost per second is equal to the momentum of the quantity of air arriving at the wall per second, which, by Newton's second law, is numerically equal to the force exerted on the wall in pounds. If the air stream is A square feet in cross-section and has a velocity of V feet per second, the volume of air arriving at the wall each second is AV cubic feet. Since the momentum of each cubic foot is the mass density times the velocity, ρV, the total momentum of AV cubic feet is ρV multiplied by AV or ρAV^2. The force acting on the wall will be equal to this, or

$$F = \rho A V^2$$

F in pounds
ρ in slugs per cubic foot
A in square feet
V in feet per second

The pressure in pounds per square foot will be

$$P = \rho V^2$$

P in pounds per square foot
ρ in slugs per cubic foot
V in feet per second

If conditions are altered so that, instead of a small stream striking against a large flat surface, a large stream strikes against a small flat surface, the same reasoning applies only that the area in the formula will be the area of flat plate instead of the cross-sectional area of the air stream. Again the force on the flat plate is exactly the same, whether the small plate is held stationary against a large stream of air which moves against the plate with a velocity of V feet per second, or whether the air is still and the plate is moved through the air with a velocity of V feet per second in a direction perpendicular to its flat surface.

The foregoing presupposes that the air disappears or vanishes on arriving at the flat surfaces. Actually, when particles of air are stopped at the flat surfaces, they cannot get out of the way of the following particles of air; some of the air is pocketed or trapped in the central portion of the plate; eddy currents and whirlpools of air are created. The amount of eddying depends on the size and shape of the flat surface. Because of these eddies, etc., the actual force against the flat surface is not exactly the same as given by the formula. Numerous tests have been made in which have been measured exactly the forces on flat plates of various sizes and shapes when in air streams of various velocities. The results of these tests have been quite consistent, so that, if the size and shape of the flat surface are known, the amount by which the true force differs from the theoretical force can be predicted.

It is usual to multiply the theoretical force by a correction factor K to obtain the actual force on the flat surface. The magnitude of K varies slightly with size if the area is only a few square feet; it also varies with the shape, i.e., whether the flat surface is circular, square, or rectangular. Except for very precise work, in aeronautics, it is customary to neglect these variations of K and assume that it has a constant value of 0.64, so that the equation for force on a flat plate normal to an air stream becomes

$$F = 0.64\,\rho A V^2 \qquad \begin{array}{l} \rho \text{ in slugs per cubic foot} \\ A \text{ in square feet} \\ V \text{ in feet per second} \end{array}$$

For standard atmosphere this becomes

$$\begin{aligned} F &= 0.64 \times 0.002378\,A V^2 \\ &= 0.00152\,A V^2 \end{aligned}$$

Since it is customary to measure airspeed in miles per hour, it is sometimes convenient to have a formula wherein velocity is in miles per hour.

$$\begin{aligned} F &= 0.64 \times 0.002378 \times A \times (\tfrac{88}{60} V)^2 \\ &= 0.00327\,A V^2 \end{aligned} \qquad \begin{array}{l} F \text{ in pounds} \\ V \text{ in miles per hour} \\ A \text{ in square feet} \end{array}$$

Example. A 40-mile-per-hour wind is blowing against a signboard 8 ft. by 10 ft. in size. Atmosphere is normal density. What is the force acting against the signboard?

Solution:

$$\begin{aligned} F &= 0.00327\,A V^2 \\ &= 0.00327 \times 8 \times 10 \times (40)^2 \\ &= 419 \text{ lb.} \end{aligned}$$

The force on the flat plate varies as the square of the airspeed. If the force acting when the relative speed is 1 mile per hour is known, the force at any other speed can be found by multiplying by the square of the airspeed.

Example. The force against an automobile windshield is 0.012 lb. when the car is moving forward at 1 mile per hour. What is the force when the car is traveling at 35 miles per hour?

Solution: F (pounds) $= 0.012 \times 35 \times 35 = 14.7$ lb.

Work and Power. The accomplishment of motion against the action of force tending to resist it is defined as work. Work is expressed in units of force times distance, for example, foot-pounds or mile-pounds. No time is involved. In overcoming a force of x pounds through a distance of y feet, the same work of xy foot-pounds is done whether accomplished in a short or long time.

Power involves the element of time. To do the same work in half the time means that twice the power is required. Power is expressed in units of work divided by time. Work is force times distance. Velocity is expressed in units of distance divided by time. Then power, which is force times distance divided by time, is also force times velocity. It is usual to express power in terms of an arbitrary unit, a horsepower, which is 550 ft-lb. per sec.

$$
\begin{aligned}
1 \text{ hp.} &= 550 \text{ ft-lb. per sec.} \\
&= 33{,}000 \text{ ft-lb. per min.} \\
&= 1{,}980{,}000 \text{ ft. lb. per hr.} \\
&= 375 \text{ mile-lb. per hr.}
\end{aligned}
$$

That is, overcoming a force of 1 lb. at a speed of 375 miles per hour, or of 375 lb. at a speed of 1 mile per hour, or 15 lb. at a speed of 25 miles per hour, requires 1 hp.

Except for minor corrections, if the plate is smaller than 12 sq. ft. in area, the force on a flat plate due to meeting air perpendicular to its surface is $0.64\,\rho A V^2$. If this is multiplied by the velocity to get power, then power is $0.64\,\rho A V^3$.

$$
\begin{aligned}
\text{H.P.} &= \frac{1.28 \frac{\rho}{2} A V^3}{550} & &\text{H.P. in horsepower} \\
& & &\rho \text{ in slugs per cubic foot} \\
& & &A \text{ in square feet} \\
&= 0.00233 \frac{\rho}{2} A V^3 & &V \text{ in feet per second}
\end{aligned}
$$

for standard density

$$
\begin{aligned}
\text{H.P.} &= 0.00000276\, A V^3 & &V \text{ in feet per second} \\
\text{H.P.} &= 0.0000087\, A V^3 & &V \text{ in miles per hour}
\end{aligned}
$$

Example. A flat surface 5 ft. square is moved through the air at a speed of 30 miles per hour in a direction perpendicular to its surface; what horsepower is required to do this?

Solution. The force on the plate will be

$$\begin{aligned} F &= 0.00327 \times (5)^2 \times (30)^2 \\ &= 73.6 \text{ lb.} \end{aligned}$$

the horsepower required will be

$$\begin{aligned} \text{H.P.} &= \frac{73.6 \times 30}{375} \\ &= 5.88 \text{ hp.} \end{aligned}$$

or directly

$$\begin{aligned} \text{H.P.} &= 0.0000087 \times 25 \times (30)^3 \\ &= 5.88 \end{aligned}$$

Example. Driving at 1 mile per hour, the force on a certain automobile windshield is 0.1 lb.; what horsepower is used up by the windshield at 70 miles per hour?

$$\begin{aligned} F &= 0.1 \times 70 \times 70 \\ &= 490 \text{ lb.} \\ \text{H.P.} &= \frac{490 \times 70}{375} \\ &= 91.5 \text{ hp.} \end{aligned}$$

Problems

1. What is the total force of a 40-mile-per-hour wind on a hangar door 40 ft. by 20 ft.?

2. An auto windshield is 42 in. wide by 12 in. high. Neglect correction for size and shape. What horsepower is used up by the windshield at 50 miles per hour?

3. The radiator of an automobile is 18 in. wide by 24 in. high. With shutters closed, what horsepower is expended in overcoming its head resistance at 45 miles per hour?

4. What horsepower is required for radiator in problem 3 if speed is 60 ft. per sec.?

Inclined Flat Plates. If the stream of air does not strike the flat surface perpendicularly, as considered in the last few paragraphs, but at an angle so that the air on hitting the plate moves off parallel to the surface, the problem is not the same. Let α be the angle between the direction of the air and the plane surface. If A is the area of the flat surface, the cross-section of the air stream which comes in contact with the surface will have an area of

$A \sin \alpha$ square feet. If the air stream is moving at a speed of V feet per second, then $A V \sin \alpha$ cubic feet of air will approach the flat surface every second. This volume of $A V \sin \alpha$ cubic feet will have a mass of $\rho A V \sin \alpha$ slugs and a momentum of $\rho A V^2 \sin \alpha$ pounds. If all this momentum were given up and lost by the moving air, then the force on the flat surface would be equal to the momentum or $\rho A V^2 \sin \alpha$ pounds. Since the air on meeting the plate is diverted and slides off in a direction more or less parallel to the surface it still retains some momentum. Also there are bound to be some eddy currents formed when the air stream meets the flat surface. The amount of turbulence will depend on the angle at which the air meets the surface. At small angles the turbulence will be relatively little, gradually increasing as the angle increases to 90°.

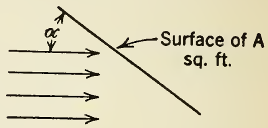


FIG. 1. Air striking an inclined plate.

Energy is required to put air in motion, and energy consumed in moving air about in whirlpools or eddy currents will subtract from the energy of the moving air.

The value of $\rho A V^2 \sin \alpha$ pounds can be considered as a maximum value of the force acting on the plate, provided that the effect of friction is neglected.

Necessarily when air flows along a surface there is bound to be friction. The moving air tends to drag the plate along in the direction parallel to the plane of the plate. If there were no friction, the only force on the flat plate due to the moving air would be perpendicular to the surface of the plate. With friction, there would be additional force parallel to the surface of the plate. The combination of these two forces is a resultant which is not perpendicular to the plate but is away from the perpendicular in the direction of flow of the air.

In aerodynamic work, the force resulting from air meeting a surface is not as important as the two components of this force perpendicular and parallel to the direction of the air stream. The component force perpendicular to the direction of the air stream is called the *lift*; the component parallel to the air stream is called the *drag*.

The components of lift and drag will vary with the density of the air, the area of the flat surface, and the square of the velocity.

The forces of lift and drag can therefore be expressed as a coefficient multiplied by $\frac{\rho}{2} A V^2$ (A in square feet, V in feet per second). The coefficient will be different for every different angle at which the flat plate is set with respect to the air stream. Figure 2 gives the value of the lift coefficient (C_L) and drag coefficient (C_D) plotted against angle of attack, the use of these coefficients being explained later in this text.

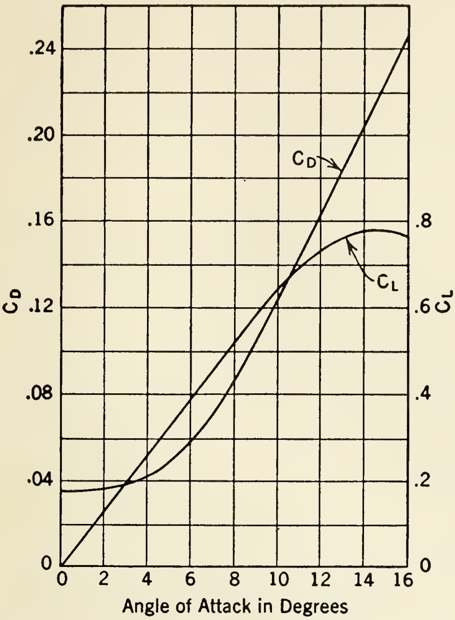


Fig. 2. Graph of C_L and C_D for flat plate.

The coefficients given in Fig. 2 were obtained by actual tests on a rectangular plane 90 cm. by 15 cm. in size, the plate being set so that the air stream first meets one of the long edges. These coefficients are correction factors to allow for losses or gains due to turbulence. The amount of turbulence would be different if the plate were turned so that a short side were first to meet the air; consequently the coefficients in this case would not be the same as in Fig. 2. If the ratio of long side to short side differs from six the coefficients will also be different.

Problems

1. A signboard is 18 ft. long by 3 ft. wide. A 30-mile-per-hour wind is blowing at an angle of 10° to the plane of the signboard. (*a*) What is the force in pounds on the signboard at right angles to the wind direction? (*b*) What is the force parallel to the wind direction? (*c*) What is the resultant of these two forces? (*d*) What is the component perpendicular to the face of the signboard?

2. A kite having 6 sq. ft. area is balanced by its tail so that it slants 15° to the horizontal. What is the lifting force in a 10-mile-per-hour wind?

3. A flat surface 6 ft. by 2 ft. is subject to a 25-mile-per-hour wind which comes in a direction 6° to the surface. (*a*) What is the force in pounds on the surface perpendicular to the wind? (*b*) What is the force parallel to the wind? (*c*) What angle does the resultant of these two forces make with the surface?

4. The side of a freight car is 60 ft. long and 10 ft. high. What is the force against the side of the car due to an 8-mile-per-hour wind from a direction perpendicular to the side (*a*) when freight car is stationary; (*b*) when car is moving forward at 30 miles per hour?

5. A flat plate is moving in a direction 15° from the plane at a speed of 250 ft. per sec. If plate has area of 50 sq. ft., what force applied in the direction of movement is required?

Curved Plates. A curved surface may be placed in a stream of air, so that the air meets the surface tangentially but is gradually deflected so that the air leaves the surface moving in a different direction from its original path (see Fig. 3). If the surface is smooth and there are no abrupt changes in direction, the air stream should suffer no diminishment in speed.

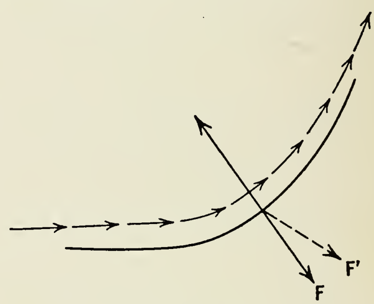


Fig. 3. Air striking curved plate.

Velocity has both speed and direction. In deflecting the air stream the velocity has been changed. By the first law of motion, any matter in motion in a straight line continues in that straight line unless acted upon by an (outside) force. A restatement of this is that if the direction of motion is changed there must exist a force which produced the change.

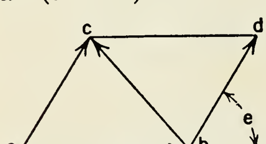


Fig. 4. Forces on a curved plate.

In Fig. 4, let *ab* represent in magnitude and direction the velocity (V) of the air stream before it meets the deflecting surface, *ac* the velocity as it leaves the surface, and ϵ the angle between. Constructing a parallelogram of velocity, *abcd*, the diagonal *bc* represents the only velocity which combined with velocity *ab* will give a resultant velocity of *ac*.

If the final velocity *ac* is equal in magnitude to entrance velocity *ab*, length *ac* equals length *ab*, and since $ab = bd = ac = cd$, triangles *abc* and *dbc* are equal, and angle *abc* equals angle *cbd*. Then the direction of the velocity *bc* bisects the angle *abd*, and since angle *abd* equals 180° − angle *bac*, angle *abc* equals $\frac{1}{2}(180° - \epsilon)$. By trigonometry, $\frac{1}{2}$ side *bc* = side *ab* × cos *abc*.

$$\text{Side } bc = 2\,V \cos \tfrac{1}{2}(180 - \epsilon)$$
$$= 2\,V\sqrt{\frac{1 + \cos\,(180 - \epsilon)}{2}}$$
$$= 2\,V\sqrt{\frac{1 - \cos \epsilon}{2}}$$
$$= V\sqrt{2(1 - \cos \epsilon)}$$

To change the direction of the air stream, a velocity of the magnitude of $V\sqrt{2(1 - \cos \epsilon)}$ must be impressed on the moving air. The force required to produce this change is equal to the change in momentum, which is equal to the mass times the change in velocity. If it requires t seconds for a particle of air to traverse the curved surface and undergo deflection, the mass of air deflected in t seconds will be $\rho A V t$. The change in velocity takes place during the time that the air is traversing the surface (t seconds); therefore the acceleration is $\frac{V\sqrt{2(1 - \cos \epsilon)}}{t}$, as was shown previously. The deflecting force is therefore $\rho A V^2\sqrt{2(1 - \cos \epsilon)}$.

Assuming that there is no burbling, and neglecting friction, the direction of the deflecting force will bisect the angle between the directions of the air coming towards and leaving the surfaces. For every action there is an equal and opposite reaction, so if the plate acts on the air stream as explained, the air stream will react on the plate in an equal manner but opposite in direction as shown by F in Fig. 3. If there is friction, the moving air will tend to drag the plate along with it, so that there will be a slight tangential force and the reaction will be more as shown by the dotted line F' in Fig. 3.

If the curved surface be the part of a circular cylinder, the air stream will follow a circular path while in contact with the surface. Let o be the center and R the radius of the arc which is the path of the particles in the center of the air stream, as shown in Fig. 5. Let a be the position of a particle on the circular path and b be its position a brief interval of time, t seconds later. Then, since the velocity is V, the length of arc ab is Vt. Let ac and bd be tangents to the circular path at a and b respectively, and call the intersection of these two lines point e. Draw lines from a, b, and e to o, the center of the circle. Then the central angle aob is equal to the exterior angle bec, and oe bisects angle aob. If angle bec is called θ, angle aoe is $\theta/2$. Draw ab intersecting oe at f. Provided angle aob is small, line ab is very nearly equal in length to arc ab. As ab is Vt feet in length, af will be $Vt/2$ feet long. The sine of angle aoe will be af divided by oa, or

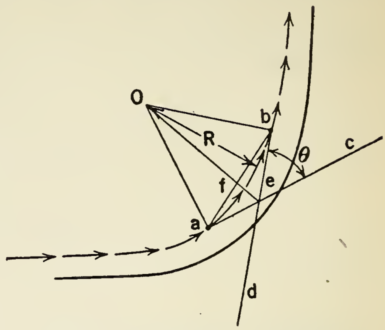


FIG. 5. Forces on a circular cylindrical plate.

$$\sin\frac{\theta}{2} = \frac{Vt/2}{R}$$

then

$$2\sin\frac{\theta}{2} = \frac{Vt}{R}$$

$$2\sqrt{\frac{1-\cos\theta}{2}} = \frac{Vt}{R}$$

$$\frac{V\sqrt{2(1-\cos\theta)}}{t} = \frac{V^2}{R}$$

In a previous paragraph, it has been shown that, if a curved surface deflects an air stream through a total angle of ϵ, the acceleration or change in velocity per second is $\dfrac{V\sqrt{2(1-\cos\epsilon)}}{t}$. If the surface is such that the air stream takes a circular path, and the radius is known, the acceleration can be expressed as V^2/R. This is the usual way of expressing the acceleration due to centrifugal force.

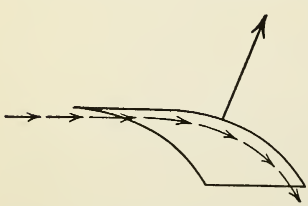


FIG. 6. Direction of reaction on circular cylindrical plate.

As an illustration of the application of this principle, let the following example be considered. A rectangular metal sheet, 36 ft. by 6 ft., is bent as shown in Fig. 6 so that a horizontal stream of air striking the sheet tangentially is gently deflected and leaves in a direction 10° below the horizontal. The air first meets the long edge of the deflecting surface. The air stream is 1 ft. thick; its velocity of 40 ft. per sec. is unchanged. What is the force of the air against the metal sheet, and in what direction does it act?

In 1 sec., a volume of air 36 ft. wide, 1 ft. deep, and 40 ft. long meets the surface and, in having its direction changed, reacts against the plate. Since, in passing from front to rear, the air traverses a distance of 6 ft., the time required will be 6/40 or 0.15 sec. In 0.15 sec., a volume of $0.15 \times 36 \times 1 \times 40$, or 216 cu. ft., will meet the surface. The mass of this volume of air will be 0.002378×216, or .514 slugs. The acceleration will be $\frac{40\sqrt{2(1 - \cos 10°)}}{0.15} = 46.5$ ft. per sec. per sec. The force will be 0.514×46.5, or 23.8 lb. Its direction, neglecting friction, will be upward and backward, 5° from the vertical.

Problems

1. A curved surface, 36 ft. by 6 ft., deflects an air stream through an angle of 5°. The air stream is 1 ft. thick and first meets a longer side of the surface. The airspeed is 40 ft. per sec. before and after deflection, and there is no turbulence. What is the force against the surface?

2. What is the force in problem 1 if the airspeed is 80 ft. per sec.?

3. What is the force in problem 1 if the angle of deflection is $2\frac{1}{2}$°?

CHAPTER III

AIR FLOW

If a body is moved through air, or if air flows around a body, the motion of the air may be either smooth or turbulent. Smooth flow is called continuous or streamline flow. Turbulent flow is discontinuous or sinuous.

A stream of air may be conceived of as consisting of a number of particles moving in the same direction. On meeting an obstruction, the path of some of these particles will be diverted. If the obstructing body is so shaped as to not cause a sudden and abrupt change in direction, the particles will move in new paths which are usually gentle curves. In this case, a particle, which was directly behind another particle in the original air stream, will follow exactly the new path taken by the leading particle in flowing around the obstruction. Succeeding particles will also follow this path. A continuous line describes this path, and the flow is called streamline. Figure 7 illustrates this flow pattern. Particle A follows the path $AA'A''$. Particle B, immediately to the rear of particle A, follows the same path as A, as does particle C and all other particles behind C.

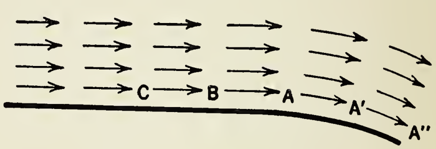


Fig. 7. Streamline flow.

In turbulent flow, the flow is discontinuous; that is, particles that follow each other in the original air stream do not follow in all cases the same path as preceding particles in flowing around obstructions. In Fig. 8, the air stream is shown meeting a flat plate. In Fig. 8a, particle A and a small number of succeeding particles will follow path $AA'A''$. Reaching the position A'', in order to continue in motion, particle A must break across path $AA'A''$, so that momentarily the flow is as shown in Fig. 8b. Particle A moves onward as shown by path $A''A'''A''''$. This particular path is followed only till most of the particles which moved into the vortex behind particle A have escaped onward, when the original path (Fig. 8*a*) is resumed. The flow is thus intermittent, particles first following one path then another.

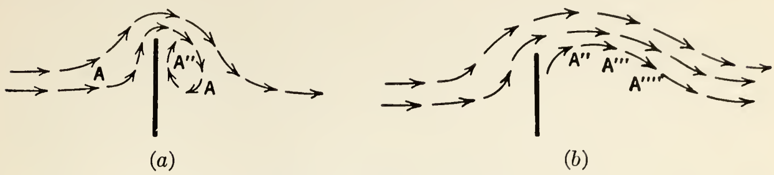


Fig. 8. Turbulent flow.

Skin Friction and Viscosity. Even in so-called streamline flow around an object, some turbulence is present. The layer of air immediately adjacent to the body is retarded in its forward movement by friction with the surface of the body. No matter how smooth the surface may be, there is bound to be some retardation. This friction between moving air and a stationary body will always be greater than between air particles themselves.

It is probably correct to assume that the layer of air in contact with the body has little, if any, forward motion. The adjoining layer of air is slowed up by friction with the motionless layer. Contiguous layers affect each other, in that layers nearer the body act to retard layers farther away. This interaction continues through successive layers, the retarding effect diminishing with increasing distance from the surface.

The viscosity and friction effects are very small, and it is only in a narrow zone clinging to the surface that this effect is noticeable. This region, in streamline flow, is only a few thousandths of an inch thick, and it is in this boundary layer that the velocity rises from zero at the immediate surface to its full value in the air stream. The velocity gradient, which is the ratio of velocity difference to distance between layers, is very high in the boundary layer, while very small in the main air stream.

The effect of the different speeds is to destroy the streamlines and break them up into tiny eddies as shown in Fig. 9, which is a greatly enlarged view of a portion of Fig. 7. Consider a particle in a streamline close to but not touching the body. Owing to the velocity gradient, on the outer side it is being rubbed by particles moving forward with greater velocity, on the inner side it is being rubbed by particles

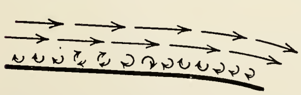


Fig. 9. Turbulence in Boundary Layer.

having a slower velocity. This tendency towards rotation of the individual particles serves to turn its path downward and causes minute vortices.

In the thin boundary layer, the viscosity of air plays the predominating part in determining the motion of the particles. Outside of the boundary layer, viscosity has little effect and air may be considered as a non-viscous fluid. With fluids of greater viscosity than air, the boundary layer will be thicker.

Reynolds' Number. The English physicist, Osborne Reynolds, made some interesting experiments with the flow of liquids in tubes. By using a glass tube, and injecting small streams of an insoluble colored fluid, he was able to study the form of flow.

Using water as a fluid, and starting with one size of tube, he found that at low velocity the flow of the colored liquid in the water was a continuous line; that is, the flow was streamline or laminar except for an infinitesimal layer touching the side walls of the tube, where there were slight eddies. As he increased the velocity the flow remained streamline, though the outside layer in which burbling was taking place became slightly thicker.

Increasing the rate of flow still more, he found that at a certain speed the main flow changed from streamline to turbulent. Above this critical speed the flow was turbulent; below, it was streamline. Increasing the diameter of the tube, he found that the critical speed, at which flow changed from streamline to turbulent, was lessened. Using fluid of different densities and different viscosities also varied the critical speed.

He evolved an expression which is called Reynolds' number and commonly abbreviated as R.N. Reynolds' number is dimensionless.

$$\text{R.N.} = \frac{\rho V R}{\mu}$$

V = average axial velocity (feet per second)
R = inner radius of tube (feet)
ρ = mass density of fluid (slugs per cubic foot)
μ = coefficient of viscosity of fluid (pound-second per square foot)

Reynolds found that for fluid flow through pipes or tubes if the R.N. was less than 1,100 the flow was laminar or streamline. Although later experimenters found a somewhat higher number as the critical condition, the value of 1,100 is the one commonly used as being perfectly safe. Below that value, the flow is certain to be laminar.

Reynolds' number is important to hydraulic engineers in deciding the proper size of piping. With a given size of pipe, the quantity of fluid that can be conducted through the pipe increases with the velocity until the critical Reynolds' number is reached. With turbulent flow, the quantity of fluid is decreased greatly.

It is to be noted that Reynolds' work was entirely with tubes and pipes. The critical value of Reynolds' number for flow inside of circular pipes is meaningless when the problem deals with flow of *unconfined* air *around* objects, such as airplanes. It is conjecturable, however, that even free air moving around an obstruction may have a critical factor, corresponding to the Reynolds' number for pipes, which will differentiate whether the flow will be streamline or turbulent.

Similar Flows. The important application of Reynolds' number for the aeronautic engineer is its use in comparing the flow of air at different speeds around objects of varying size. The manner of air flow around an object, as shown in Fig. 10*a*, would be

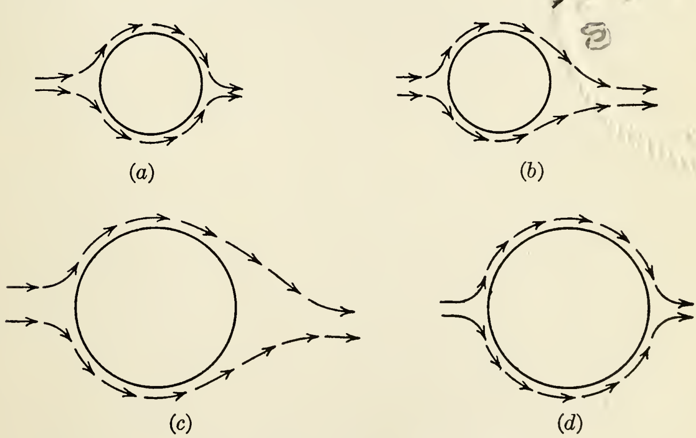


Fig. 10. Flow around similar figures.

changed to a flow more like that shown in Fig. 10*b* if the speed of the air were increased. If the speed of flow was the same as for the flow shown in Fig. 10*a*, but the size of the object were increased, the flow would be as shown in Fig. 10*c*. By decreasing the velocity in an inverse ratio to the increase in size, i.e., keeping

VL the same, where L is a representative dimension of the object, the flow would be as shown in Fig. 10*d*. With the same VL the flows are geometrically similar.

Again if the size of the object and speed of flow were the same as in Fig. 10*a* but the density of the air were increased, the flow would be changed to a flow, resembling that in Fig. 10*b*. A decrease in coefficient of viscosity would have the same effect on the shape of the air flow as an increase of density or velocity.

To summarize: If the Reynolds' number is the same, the flows are geometrically similar. With geometrically similar flows about two bodies of different sizes, at corresponding points in the two flows the direction of the streamlines will be the same and the magnitude of the forces will always have the same ratio to each other.

This relation is important in the design of airplanes. Models of wings or airplanes may be tested in wind tunnels, and the results of these tests may be used in the computation of the performance of the full-size airplane, provided that the Reynolds' number of the model is the same as the Reynolds' number of the airplane. Since the models are small in size in comparison with the actual wings, in order to have the Reynolds' number of the same magnitude as the Reynolds' number of the wing the velocity or the density of the air in the wind-tunnel test must be much greater than in the actual flight.

Where the Reynolds' number of the wind-tunnel test of a model wing is not the same as for the full-size wing in flight, the data obtained from the wind-tunnel tests cannot be expected to give exactly correct results when used in calculating forces on the full-size wings. At Langley Field, Virginia, the National Advisory Committee for Aeronautics has a variable-density wind tunnel. The tunnel is entirely enclosed in a steel shell. The air, after being drawn through the tunnel, past the model on test, flows around the outside of the tunnel proper to re-enter the throat once more. Since the air is at all times imprisoned inside the steel chamber it can be put under pressure. The Langley Field variable-density tunnel was designed for pressures as high as 21 atmospheres and air velocities past the model wings up to 75 ft. per sec.

In computing Reynolds' number, velocity (V) must be in feet per second; a linear dimension of the object (L) must be in feet.

For wings, the length of the chord is commonly used for this dimension (L). If the test is conducted under standard conditions (15° C. and 760 mm. pressure) the density of the air (ρ) is 0.002378 slug per cubic foot and the coefficient of viscosity (μ) is 0.000000373 slug per foot per second. If the temperature or pressure is not standard, corrections must be made to density (ρ) and coefficient of viscosity (μ).

Example. Find the Reynolds' number for a model wing of 3-in. chord, tests run at 100 miles per hour with standard air.

$$3 \text{ in.} = 0.25 \text{ ft.}$$
$$100 \text{ miles per hour} = 146.7 \text{ ft. per sec.}$$
$$\text{R.N.} = \frac{0.002378 \times 146.7 \times 0.25}{0.000000373} = 234{,}000$$

Example. Find the R.N. for model wing of 3-in. chord. Tests run at 100 miles per hour. Air at normal pressure but at 100° C. temperature.

$$\text{R.N.} = \frac{0.002378 \times \frac{288}{373} \times 146.7 \times 0.25}{0.000000454} = 148{,}000$$

Example. Find R.N. for model wing of 3-in. chord. Tests run at 100 miles per hour. Air at standard temperature but 21 atmospheres pressure. Note that μ is independent of pressure (except for extremely high or extremely low pressures).

$$\text{R.N.} = \frac{0.002378 \times 21 \times 146.7 \times .25}{0.000000373} = 4{,}910{,}000$$

Problems

1. Find R.N. for an airplane wing, 4-ft. chord moving at 120 miles per hour through standard atmosphere.

2. Find R.N. for an airplane wing with a 3-ft. 6-in. chord moving at 180 miles per hour through standard air.

3. Find R.N. for an airplane wing, 4-ft. chord moving at 150 miles per hour. Air is +40° C. and 21 in. barometer.

4. Find the velocity at which tests should be run in a wind tunnel on a model wing of 4-in. chord in order that the R.N. shall be the same as for a wing with a 4-ft. chord at 100 miles per hour. Air under standard conditions in both cases.

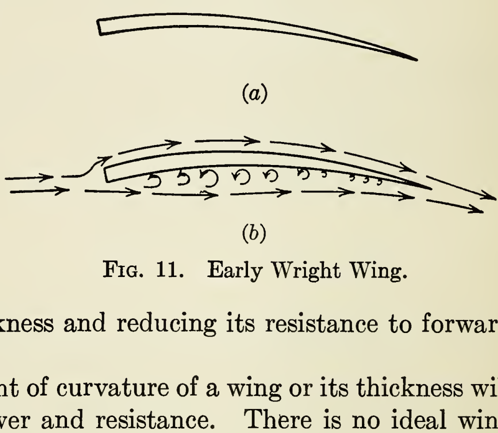
5. In a variable-density wind tunnel, under what pressure should tests be run on a model with a 3-in. chord, air velocity being 60 miles per hour, in order that the R.N. shall be the same as for a full-size wing of 4-ft. chord, moving at 100 miles per hour through the air? Air temperature is the same in each case.

CHAPTER IV

AIRFOILS

Flat plates are not suitable for airplane wings. To be structurally strong, there would have to be a considerable amount of bracing. This bracing, if located outside the plate, would cause much friction in moving through the air; if inside the plate, it would increase the thickness of the plate. By curving the plate even slightly it is possible to increase the weight-sustaining property greatly. By closing in the under side and making it more nearly streamline in shape, the resistance to forward movement through the air can be greatly decreased.

The wings of the first Wright glider were simply curved strips of wood covered only on the upper side with cloth (see Fig. 11*a*). The Wrights soon realized that when the wing was horizontal there would be a great amount of burbling on the under side (see Fig. 11*b*). They consequently covered up this space, giving the wing appreciable thickness and reducing its resistance to forward motion.



(*a*)

(*b*)

Fig. 11. Early Wright Wing.

Changing the amount of curvature of a wing or its thickness will change the lifting power and resistance. There is no ideal wing which is suitable for all types of airplanes. The proper shape must be chosen for each individual requirement.

Airfoils. Although the word *airfoil* may be used interchangeably with the word *wing,* common usage dictates that the word *wing* be used in referring to the actual wing of an airplane, and that the word *airfoil* be applied in describing the contour or shape of the vertical cross-section of a wing. Many differently shaped airfoils have been proposed and their properties investigated. Only a few of these airfoils will be discussed in this textbook.

For descriptions of other airfoils, the reader is referred to reports of the National Advisory Committee for Aeronautics and other aeronautical testing laboratories.

Chord and Camber. To describe the curvature of the upper and lower surfaces, a base line is used as a reference. Coordinates are given with respect to a point on this reference line to locate sufficient points on the airfoil to enable the curve to be drawn. This reference line is the chord of the airfoil.

If the airfoil has convex curvature on both upper and lower surfaces, as in Fig. 12*a*, the chord is the line joining the most forward point on the front edge (leading edge) to the rearmost point (trailing edge). If the lower surface of the airfoil is predominantly flat, as in Fig. 12*b*, the chord is the straight line that coincides for most of its length with the lower surface. With other shapes of airfoils the datum line is more or less arbitrary. This causes no confusion as the chord and the airfoil are never dissociated. Whenever an airfoil is described, the chord is known.

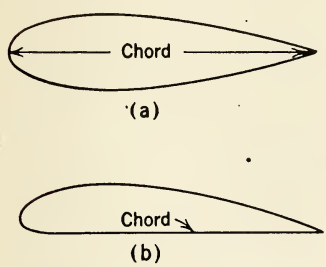


FIG. 12. Chords, (*a*) Double camber, (*b*) Single camber.

The chord length is the length of the projection of the airfoil section on its chord. Its symbol is c.

Camber is the length of the ordinate perpendicular to the chord. Top camber is the distance of a point in the upper surface of the airfoil from the chord line; bottom or lower camber is the distance of a point in the lower surface of the airfoil from the chord line. Because airfoil sections, which are geometrically similar but of different size, would be designated by the same name, amount of camber is always given as a percentage of the chord length. The abscissa, or length along the chord, of a point in the airfoil surface is also given in percentage of chord.

In Table III are given the dimensions of a Clark Y airfoil, a typical medium-thickness wing. When once the chord length is decided upon, the actual camber can be found from the above table by multiplying the chord length by the suitable percentages. The use of this table is shown in Fig. 13, the points laid out by means of these coordinates defining a curve.

TABLE III

SHAPE OF CLARK Y AIRFOIL

Percentage of Chord

Distance from Leading Edge	Upper Camber	Lower Camber
0	3.50	3.50
1.25	5.45	1.93
2.5	6.50	1.47
5	7.90	0.93
7.5	8.85	0.63
10	9.60	0.42
15	10.69	0.15
20	11.36	0.03
30	11.70	0
40	11.40	0
50	10.52	0
60	9.15	0
70	7.35	0
80	5.22	0
90	2.80	0
95	1.49	0
100	0.12	0

The maximum camber is an important dimension and refers to the greatest departure of the curve of the airfoil from the chord line. Mean camber is the camber of a point equidistant from the upper and lower surfaces.

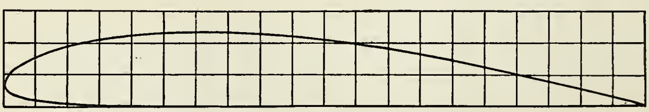


FIG. 13. Clark Y wing contour.

Span and Aspect Ratio. Span is the distance from wing tip to wing tip, inclusive of ailerons. It may be considered as the least width of hangar doorway through which the airplane can be pushed straight. The symbol for span is b.

The area of a wing is the area of the projection of the actual outline on the plane of the chord. The symbol for area is S.

The aspect ratio of a wing is the ratio of the span to the chord. This is for rectangular wings. For wings that are not rectangular in shape when viewed from above, the aspect ratio is the ratio of the square of the span to the area.

$$\text{A.R.} = \frac{b}{c} = \frac{b^2}{S}$$

It is customary in wind-tunnel tests to use model airfoils whose aspect ratio is 6. Force measurements from these tests need to be corrected when applying them to airfoils of differing aspect ratio.

Relative Wind. Relative wind is the motion of the air with reference to the wing. If the wing is moving horizontally forward, then the relative wind is horizontally backward. If the wing is moving both forward and downward, as when the plane is settling, the relative wind is upward as well as backward.

The angle of attack of a wing is the acute angle between the chord and the direction of the relative wind.

Forces on an Airfoil. Air flowing around an airfoil exerts a pressure on each little portion of the airfoil surface. This pressure is considered positive if it is greater than atmospheric, negative if it is less than atmospheric.

At small positive angles of attack the air flows smoothly over the upper surface. Each particle of air sweeping along a surface contributes to giving a small negative pressure as long as its motion is parallel to the surface. If its motion is not parallel but towards the surface it will contribute its component of positive impact pressure. The direction of flow is shown in Fig. 14*a*, and the pressures, to scale, are shown in Fig. 14*b*.

At larger angles of attack the shape of the stream lines and the magnitude of the pressures change. This is shown in Fig. 15.

As the angle of attack is increased still more, the air cannot follow the upper surface of the wing as that would entail too great a change in direction. The streamlines no longer conform to the contour of the airfoil (Fig. 16). Burbling starts at the trailing edge. If the angle of attack is made greater, the burbling will extend farther forward.

Wherever burbling takes place, that portion of the wing has lost its weight-sustaining property to a large extent. Burbling also increases the resistance of the wing to forward motion.

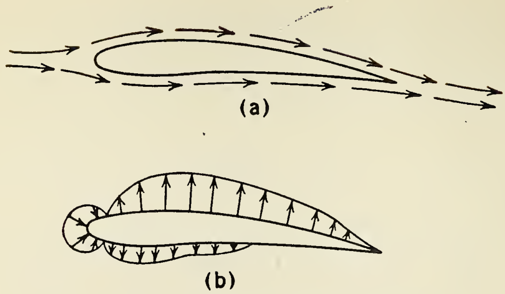


Fig. 14. Flow at low angle of attack.

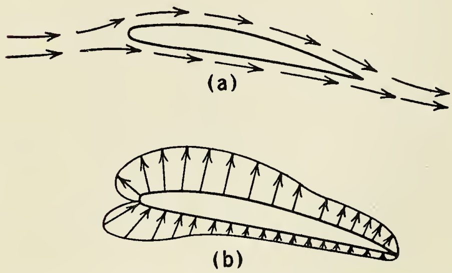


Fig. 15. Flow at medium angle of attack.

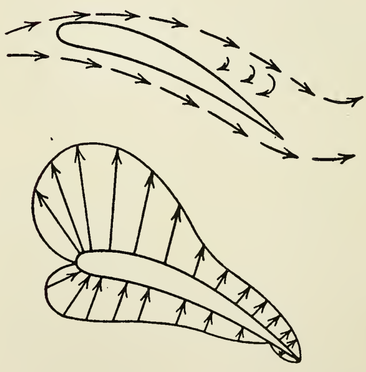


Fig. 16. Flow at high angle of attack.

It will be noted in Fig. 15*b* that the forces on the upper side of the airfoil are predominantly upward, and the magnitude of these forces is greater than that of the upward forces on the under side of the airfoil. Tests have shown that, for typical wings at 0° angle of attack, 100 per cent of the total upward force on a wing is derived from the upper surface; at 5° angle of attack, 74 per cent is due to forces on the upper surface; and at 10°, 68 per cent.

All the small forces acting on both upper and lower surfaces of the airfoil may be added together vectorially, i.e., taking into account both magnitude and direction, and this summation is called the resultant force. At small angle of attack, this resultant is small in magnitude and it acts near the trailing edge; as the angle of attack is increased, the resultant becomes larger and its point of action, called the center of pressure, moves forward.

Lift and Drag Components. The resultant can, of course, be described by giving its magnitude and direction. It can also be determined if its two components are given about known axes. It is customary and most useful to give a resultant in terms of lift and drag components.

Lift is the component of the resultant on a wing which is perpendicular to the relative wind.

Drag is the component of the resultant which is parallel to the relative wind.

The resultant force on a wing varies directly with the air density, the area of the wing, and the square of the velocity. It also depends on the angle of attack. The lift and drag components, also, vary in the same way. These relations could be expressed in a formula by stating that the component was a factor times air density times wing area times velocity squared, but it is more desirable to use a different form of the formula containing one-half the density instead of the density itself. The standard formulas are

$$\text{Lift} = C_L \frac{\rho}{2} S V^2$$

$$\text{Drag} = C_D \frac{\rho}{2} S V^2$$

Lift and drag are in pounds
ρ is in slugs per cubic foot
S is in square feet
V is in feet per second.

C_L is called the lift coefficient; C_D, the drag coefficient. They are " absolute " or dimensionless coefficients and therefore can be used with metric units of measurement. That is, the formulas for lift and drag are perfectly valid if air density is given in metric slugs, area in square meters, and velocity in meters per second. Lift and drag will then be in kilograms.

The equations may be written in the form

$$\text{Lift} = C_L q S \qquad q = \frac{\rho}{2} V^2$$

and

$$\text{Drag} = C_D q S$$

Characteristic Curves. The lift coefficient C_L, the drag coefficient C_D, and the center-of-pressure location, all for different angles of attack, are considered the characteristics of an airfoil. This information may be given in tables, but it is more usual to plot these data in the form of a curve. The characteristic curves for the Clark Y airfoil, for an aspect ratio of 6, are given in Fig. 17. Since lift coefficients are at most angles of attack much larger than drag coefficients, the drag coefficients are plotted on a larger scale in order to be legible.

Careful study should be given to Fig. 17. It should be noted that the lift coefficient (C_L) curve crosses the zero ordinate at $-5°$ angle of attack. This does not mean that there are zero forces acting on the wing at this angle of attack, but that the sum of the upward or positive lift forces equals the sum of the downward or negative lift forces. The angle of attack, where the upward and downward lift are equal, is called the angle of zero lift. For symmetrical sections, that is, airfoils that have the same camber on both upper and lower sides, the angle of zero lift is at 0° angle of attack.

From the angle of zero lift, the curve of lift coefficient is practically a straight line for a considerable distance. The slope is constant. The lift coefficient (C_L) is proportional to the angle of attack provided angle of attack is measured from the angle of zero lift.

At the larger angles of attack, the lift coefficient curve begins to deviate from a straight line. The lift increase is no longer proportional to the increase in angle of attack. At some angle of attack, for the Clark Y airfoil, it is $18\frac{1}{2}°$; the lift coefficient is a maximum. This angle of maximum lift is also called the critical angle, the burble point, or the stalling angle.

At angles above the angle of maximum lift, the lift coefficient decreases with increasing angle of attack till lift coefficient becomes zero at some angle slightly greater than 90°.

The drag coefficient curve resembles a parabola in part. At some small angle of attack, for the Clark Y it is $-3\frac{1}{2}°$, the drag coefficient has a minimum value. Whether the angle of attack is increased or decreased from this angle of minimum drag, the drag increases. For a few degrees above or below the angle of minimum drag, there is very little change in the value of the drag coefficient.

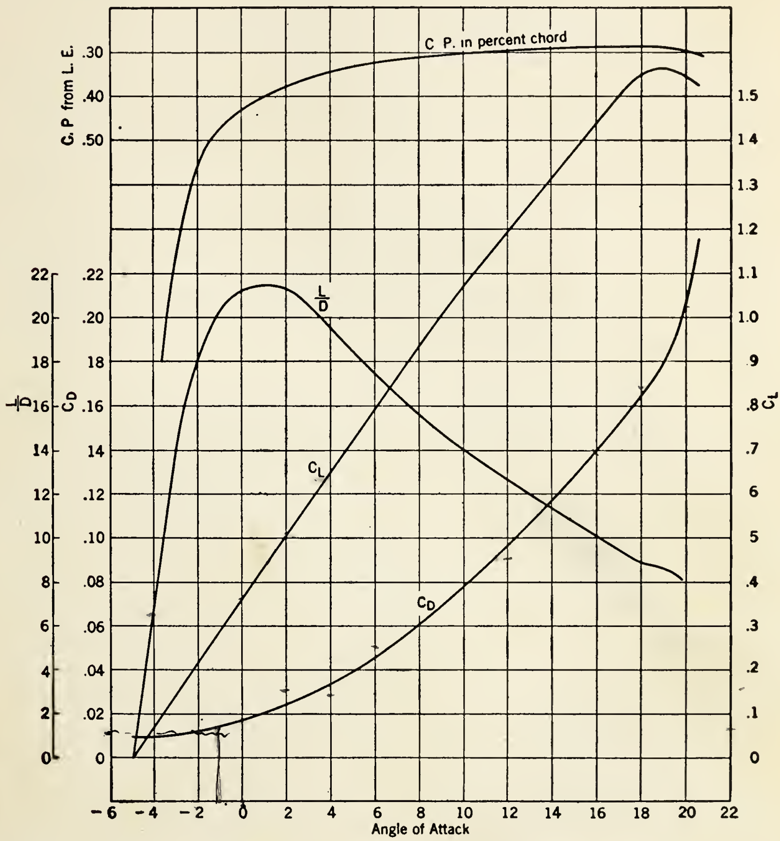


FIG. 17. Characteristics of Clark Y airfoil, aspect ratio 6.

With bigger increases in the angle of attack, the drag coefficient increases greatly.

With a symmetrical airfoil, the angle of minimum drag is at 0° angle of attack and the curve is symmetrical about a vertical axis through this point. With the more common airfoils, having more camber on the upper than the lower side, the angle of minimum drag is a small negative angle of attack and the part of the curve for more negative angles of attack has a steeper slope than the other side of the curve.

The center-of-pressure curve usually shows that at the angle of zero lift the center of pressure is at the trailing edge. With a slight increase in angle of attack, the center of pressure moves forward. It has its most forward position when the angle of attack is a few degrees below the angle of maximum lift. For symmetrical airfoils, there is practically no movement of the center of pressure; for this reason symmetrical airfoils are referred to as stable airfoils.

Lift Equation. For horizontal flight, except for a small vertical force on the tail and a small component of thrust which need not be considered at this time, the weight of the airplane equals the lift of the wing. If the lift is greater than the weight, the airplane will rise; if the weight is greater, the airplane will lose altitude. Then for horizontal flight

$$W = C_L \frac{\rho}{2} S V^2 \qquad \begin{array}{l} W \text{ in pounds} \\ S \text{ in square feet} \\ V \text{ in feet per second} \end{array}$$

The above equation may be used in other forms, viz.

$$C_L = \frac{W}{\frac{\rho}{2} S V^2}$$

$$S = \frac{W}{C_L \frac{\rho}{2} V^2}$$

$$V = \sqrt{\frac{W}{S}} \sqrt{\frac{1}{C_L \frac{\rho}{2}}}$$

The ratio of total weight to the area of the wing (W/S) is the wing loading, expressed in pounds per square foot. From the above formulas it will be seen that, for a given angle of attack, the proper velocity depends on the square root of the wing loading.

Example. What weight can an airplane have to fly level with a Clark Y wing 250 sq. ft. in area, at 4° angle of attack and airspeed of 100 miles per hour at sea-level?

From Fig. 17, C_L at 4° angle of attack = 0.649; 100 miles per hour = 146.7 ft. per sec.

$$W = 0.649 \times \frac{0.00237}{2} \times 250 \times \overline{146.7}^2 = 4{,}120 \text{ lb.}$$

Example. At what angle of attack should an airplane weighing 3,000 lb. be flown, if the wing is 300 sq. ft. in area, Clark Y section, airspeed 90 miles per hour?

$$C_L = \frac{3{,}000}{0.00118 \times 300 \times (90 \times 1.47)^2} = 0.483$$

Angle of attack (from Fig. 17) = +1.7°

Example. An airplane has a wing loading of 9 lb. per sq. ft.; the angle of attack of the Clark Y wing is 6°. What should be the airspeed?

$$V = \sqrt{9}\sqrt{\frac{1}{0.791 \times 0.00118}}$$
$$= 97.9 \text{ ft. per sec.}$$
$$= 66.8 \text{ miles per hour}$$

Problems

(Standard air density unless otherwise specified)

1. What is the lift on a Clark Y wing 300 sq. ft. in area at 8° angle of attack and airspeed of 80 miles per hour?

2. What is the lift on a Clark Y wing 450 sq. ft. in area at 6° angle of attack, airspeed 88 ft. per sec.?

3. A monoplane weighing 4,460 lb. has a rectangular Clark Y wing, 353 sq. ft. in area; at an airspeed of 100 miles per hour, what should be the angle of attack?

4. At what angle of attack should the airplane in problem 3 fly when airspeed is 85 miles per hour?

5. With a wing loading of 12 lb. per sq. ft., at what angle of attack should an airplane with a Clark Y wing fly, if airspeed is 70 miles per hour?

6. What should be the area of a Clark Y wing to support a total weight of 5,000 lb. when flying at 7° angle of attack and a velocity of 90 ft. per sec.?

7. What weight will be supported by a Clark Y wing 525 sq. ft. in area, at 5° angle of attack, and an airspeed of 120 ft. per sec. at sea-level?

8. What weight will be supported by the wing in problem 7, at the same angle of attack and same airspeed, but flying at 10,000 ft. altitude (air density at 10,000-ft. altitude is 0.00176 slug per cu. ft.; see Table I)?

9. At what airspeed should a 6,000-lb. airplane be flown, having an area of 700 sq. ft., Clark Y wing, at 10° angle of attack?

10. At what airspeed should the airplane described in problem 9 fly at an altitude of 10,000 ft.?

11. An airplane having a Clark Y wing at 6° angle of attack is to be flown at 150 miles per hour. What should be the wing loading?

12. What should be the wing loading of a plane with a Clark Y wing, if it is desired to fly at 6° angle of attack and 150 miles per hour airspeed at an altitude of 10,000 ft.?

Minimum Speed. Examination of formula $V = \sqrt{\frac{W}{S}}\sqrt{\frac{1}{C_L\left(\frac{\rho}{2}\right)}}$ shows that, with a fixed weight and a fixed wing area (fixed wing loading), the lift coefficient must vary inversely as the square of the velocity. It is axiomatic that small angles of attack mean high speed; large angles of attack mean slow speed. The smallest velocity will be when the lift coefficient is maximum. This slowest velocity is the stalling speed. Some airplanes (low-wing monoplanes), when gliding down to land, pocket air under their wing, which has a cushioning or buoyant effect, helping to support the airplane slightly. This added support enables the airplane to fly at slightly less speed than it could if the ground effect were not present. This effect is ordinarily negligible, and the minimum airspeed is considered to be the landing speed or take-off speed.

$$V_{\text{min}} = \sqrt{\frac{W}{S}}\sqrt{\frac{1}{C_{L\text{max.}} \times \frac{\rho}{2}}}$$

V in feet per second

Example. What is the landing speed of an airplane weighing 2,500 lb., having a Clark Y wing 300 sq. ft. in area?

$$\begin{aligned} V_{\text{min.}} &= \sqrt{\frac{2{,}500}{300}}\sqrt{\frac{1}{1.56 \times 0.00118}} \\ &= 67.0 \text{ ft. per sec.} \\ &= 45.7 \text{ miles per hour} \end{aligned}$$

Problems

1. What is the landing speed of an airplane weighing 4,500 lb., with a Clark Y wing 350 sq. ft. in area?

2. What is the landing speed of an airplane with a Clark Y wing and a wing loading of 12 lb. per sq. ft.?

3. What area should a Clark Y wing have in order that an airplane weighing 1,500 lb. shall not land faster than 30 miles per hour?

4. An airplane has a Clark Y wing. What would be the greatest wing loading in order that the landing speed should not exceed 35 miles per hour?

5. An airplane has a Clark Y wing 425 sq. ft. in area. What is the greatest weight this airplane can have with a landing speed not more than 40 miles per hour?

6. An airplane with a Clark Y wing has a wing loading of 14 lb. per sq. ft. What is the minimum speed at an altitude of 10,000 ft.?

7. It is desired that a pursuit airplane be able to fly at 50 miles per hour at an altitude of 10,000 ft. What should be the wing loading of its Clark Y wing?

8. An airplane weighing 5,000 lb. has a Clark Y wing 625 sq. ft. in area. What is its minimum speed (*a*) at sea-level, (*b*) at 10,000-ft. altitude?

Drag. Drag is the force, in pounds, with which a wing resists forward motion through the air. The drag multiplied by the velocity (in feet per second units) gives the power, in foot-pounds per second, required to move the wing forward. One horsepower is 550 ft-lb. per sec. Hence the drag times the velocity, divided by 550, gives the horsepower required to move the wing forward.

$$\text{H.P.}_{\text{req.}} = \frac{D \times V}{550} \qquad \begin{array}{l} D \text{ in pounds} \\ V \text{ in feet per second} \end{array}$$

But

$$D = C_D \frac{\rho}{2} S V^2$$

Therefore

$$\text{H.P.}_{\text{req.}} = \frac{C_D \frac{\rho}{2} S V^3}{550}$$

Example. A Clark Y wing 350 sq. ft. in area is moving through the air at 80 ft. per sec. at 6° angle of attack. What is the drag? What horsepower is required?

From Fig. 17, at $\alpha = 6°$, $C_D = 0.0452$

$$\text{Drag} = 0.0452 \times \frac{0.00237}{2} \times 350 \times (80)^2$$

$$= 120 \text{ lb.}$$

$$\text{H.P.}_{\text{req.}} = \frac{120 \times 80}{550}$$

$$= 17.5 \text{ hp.}$$

Problems

1. What is the drag of a Clark Y wing, 300 sq. ft. in area, at 8° angle of attack, at 120 ft. per sec. airspeed?

2. What is the drag of a Clark Y wing, 300 sq. ft. in area, at 8° angle of attack, at 120 miles per hour airspeed?

3. What is the horsepower required to move a Clark Y wing 430 sq. ft. in area at 2° angle of attack and airspeed of 150 miles per hour?

4. What is the horsepower required to move a Clark Y wing 270 sq. ft. in area at 10° angle of attack and an airspeed of 70 ft. per sec.?

5. At an altitude of 10,000 ft., what is the drag of a Clark Y wing, 480 sq. ft. in area, angle of attack 4°, airspeed 95 ft. per sec.? What horsepower is required?

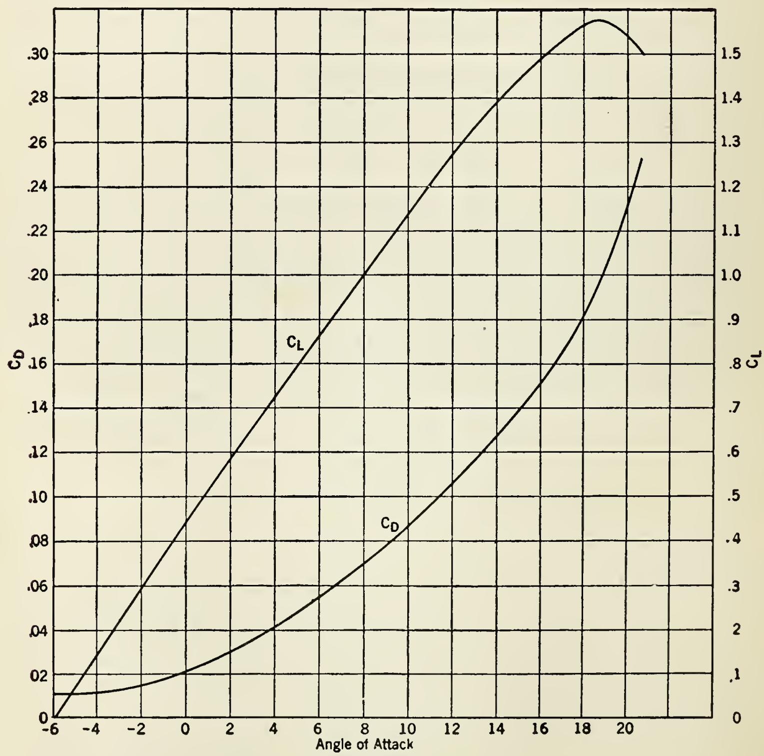


Fig. 18. Characteristics of Göttingen 398 airfoil, aspect ratio 6.

Difference in Airfoils. Many different shapes have been proposed and tested. Airfoils with contours radically different from those in general use do not ordinarily prove to be satisfactory.

This indicates that certain general rules must be followed in designing airfoils.

The leading edge should be slightly rounded. The camber of the upper surface should be such that the highest point or maximum ordinate is between one-quarter and one-third the chord length back from the leading edge.

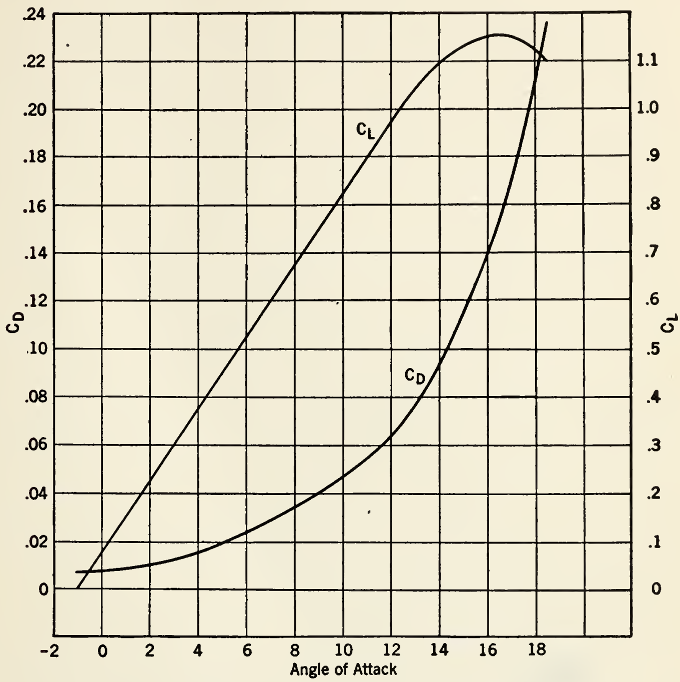


Fig. 19. Characteristics of C-80 airfoil, aspect ratio 6.

Increasing the camber increases the lift at any angle of attack but it also increases the drag. The maximum lift coefficient is increased, but the minimum drag coefficient is increased also. No wing should have an upper camber greater than one-quarter of the chord length.

An airfoil having a symmetrical section, the upper and lower surfaces of equal camber, is streamline in appearance and consequently will have less minimum drag than a non-symmetrical airfoil of the same thickness. Symmetrical airfoils have zero lift at zero angle of attack, and this is also the angle of minimum drag. Non-symmetrical airfoils have zero lift at a slightly negative angle of attack.

Concave lower camber increases the lift coefficient at any angle of attack but adds to the drag coefficient, especially at smaller angles of attack.

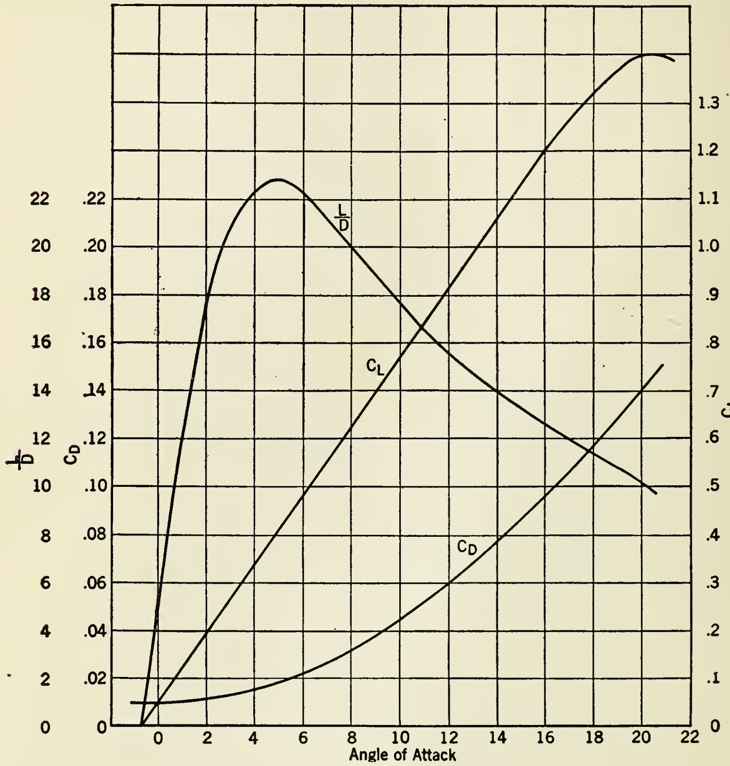


FIG. 20. Characteristics of M-6 airfoil, aspect ratio 6.

Figures 18, 19, 20, and 21 give characteristics of airfoils in common use. Thick wing sections are commonly called "high lift" sections because the maximum lift coefficients are large. With a predetermined landing speed and wing area, more load can be carried by a thick than a thin wing section.

Problems

1. (*a*) What is the lift on a Göttingen 398 wing 400 sq. ft. in area, at 2° angle of attack and airspeed of 90 ft. per sec.? (*b*) What is the drag?

2. What is the lift on a C-80 wing 400 sq. ft. in area at 2° angle of attack and airspeed of 90 ft. per sec.?

3. What is the lift on a C-80 wing 400 sq. ft. in area at 7° angle of attack and airspeed of 90 ft. per sec.?

4. What is the landing speed of an airplane weighing 5,000 lb. with a Göttingen 398 wing 400 sq. ft. in area?

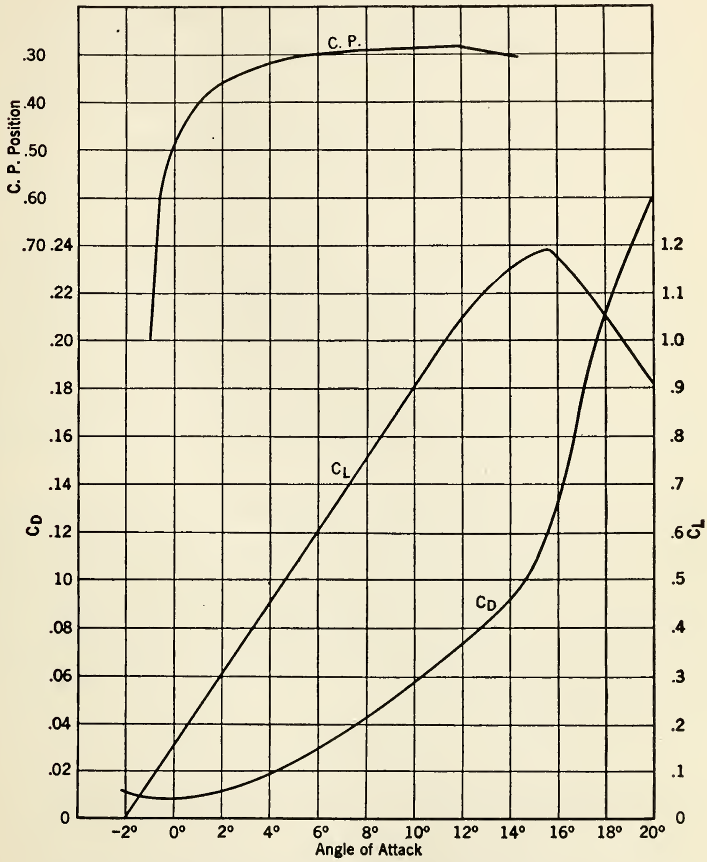


Fig. 21. Characteristics of RAF-15 airfoil, aspect ratio 6.

5. What is the landing speed of an airplane weighing 5,000 lb., with a C-80 wing 400 sq. ft. in area?

6. What total weight can a Göttingen 398 wing 400 sq. ft. in area sustain if landing speed must not exceed 45 miles per hour?

7. What total weight can a C-80 wing 400 sq. ft. in area sustain if landing speed must not exceed 45 miles per hour?

N.A.C.A. Four-Digit Airfoils. The National Advisory Committee for Aeronautics, in making a study of a large number of airfoils that had proved to be good in practice, discovered a certain relationship between dimensions of these airfoils. If a line is drawn equidistant between the upper and lower contours of an airfoil, this line is called the median line. Various airfoils may each have a different thickness; but, if the maximum thickness of each of several airfoils be made the same, the thickness at other points being reduced (or increased) in the same proportion as the maximum thickness was reduced (or increased), the various contours will coincide if the median line is straightened.

With the median line straight, the airfoil is symmetrical. The contour is a curve fitting the following equation

$$\pm y = 0.2969\sqrt{x} - 0.1260\, x - 0.3516\, x^2 + 0.2843\, x^3 - 0.1015\, x^4$$

where y and x are the vertical and horizontal coördinates in percentage of chord. The above equation gives a maximum thickness which is 20 per cent of the chord, and this maximum thickness is located 30 per cent of the chord back from the leading edge. To obtain a wing of any desired thickness t (expressed in fraction of chord), each value of y found by the above equation is multiplied by the constant $t/0.20$.

For non-symmetrical airfoils, the median line is made curved, the thickness at the various points along the chord being kept the same. The amount and shape of the curvature of the median line is defined by the distance of the maximum ordinate back of the leading edge (p), and the amount of the maximum ordinate. Forward of the maximum ordinate, the curvature of the median line fits the parabolic equation

$$y_c = \frac{m}{p^2}\,(2\,px - x^2)$$

Aft of the maximum ordinate the following equation applies

$$y_c = \frac{m}{(1 - p)^2}\,[1 - 2\,p + 2\,px - x^2]$$

The National Advisory Committee for Aeronautics has made an extensive and systematic study of airfoils conforming to this description. To identify each airfoil, a number consisting of four digits was assigned to each; the first digit representing the amount of camber of the median line in percentage of chord, the second digit representing the position of the point of maximum camber in tenths of chord from the leading edge, and the last two digits representing the maximum thickness in percentage of chord. For example, in airfoil N.A.C.A. 2421, the maximum ordinate of the median line is 2 per cent of the chord, this ordinate is 40 per cent of the chord back from the leading edge, and the greatest thickness of the airfoil is 21 per cent of the chord.

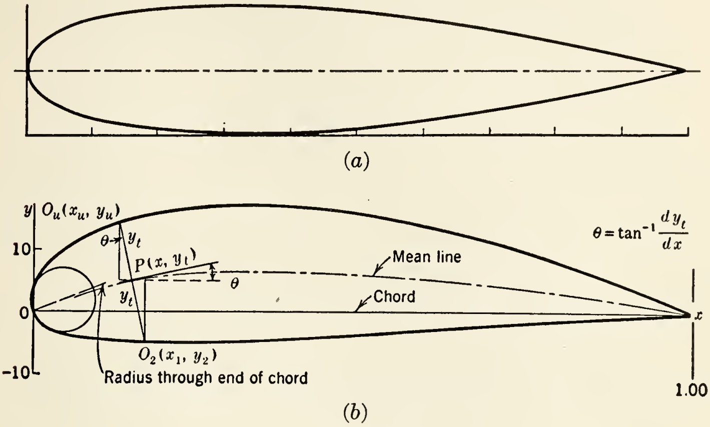


FIG. 22. N.A.C.A. airfoils.

N.A.C.A. Five-Digit Airfoils. Continuing their systematic study of airfoils, the N.A.C.A. wished to find the effect of changing the position of the point of maximum camber of the median line. They had found by investigation that moving the position of the maximum camber of the median line forward of 30 per cent of the chord had a pronounced effect on the maximum lift. Two groups have been investigated, one group having a simple curve for a median line, the other having a median line with reverse or reflex curvature. This reverse curvature was introduced to improve the pitching moment characteristics. The five-digit designation identifies the airfoils as follows. The first digit gives the amount of maximum camber of the median line in percentage of chord. The second digit gives the position of the point of maximum camber in twentieths of the chord. The third digit describes the median line; 0 if it is the simple curve; 1 if it is the reflexed curve. The last two digits are the thickness in percentage of chord. For example, in airfoil N.A.C.A. 24112, the maximum camber of the median line is 2 per cent of the chord; this ordinate is four-twentieths (20 per cent) of the chord back from the leading edge; the median line has a reverse curvature and the maximum thickness of the airfoil is 12 per cent of the chord.

Tapered Wings. The simplest wing shape is rectangular in plan. In tapering it is usual to leave the center section unchanged but, starting at the root (where the wing proper joins the center section), the chord is progressively decreased out to the tip. The decrease in chord length at any section is to the total decrease as

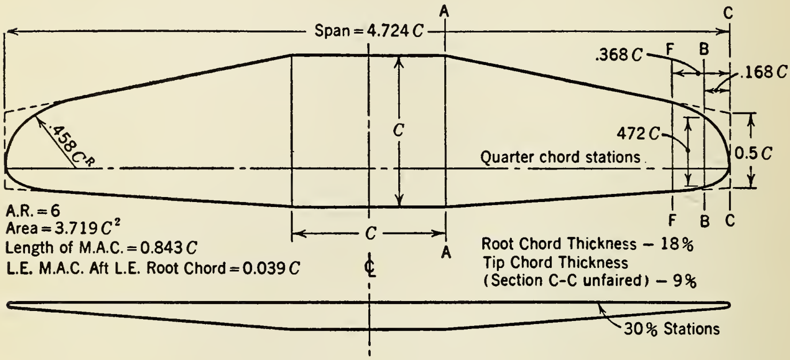


Fig. 23. Standard wing taper.

the distance from the root out to that section is to the total distance from root to tip. If the same airfoil is used throughout, because the chord is less towards the tip, the thickness is also less in the same ratio. It is usual to make the wing in such a manner that lines, joining the points of maximum upper camber in each section, will be horizontal; this will make the lower surfaces of a tapered wing slope upward to the tip.

It is becoming more customary to change arbitrarily the airfoil section from the root towards the tip. By doing this, it is possible to reduce the twisting action of the air. The standard taper adopted by the Army is illustrated in Fig. 23.

When a wing is tapered by making the chord progressively smaller from root to tip, while the same airfoil section is used throughout, it is termed " taper in planform only," even though the thickness does decrease from root to tip. When the thickness decreases at a faster rate than the chord, by using thinner airfoil sections towards the tip, the wing is said to " taper in planform and thickness."

Velocity versus Angle of Attack. An airplane, if it flies at a low angle of attack, must fly faster than if it flies at a high angle of attack. For each speed there is one angle of attack for level flight to be maintained. If the angle of attack is greater, the airplane will gain altitude; if the angle of attack is less, the airplane will descend.

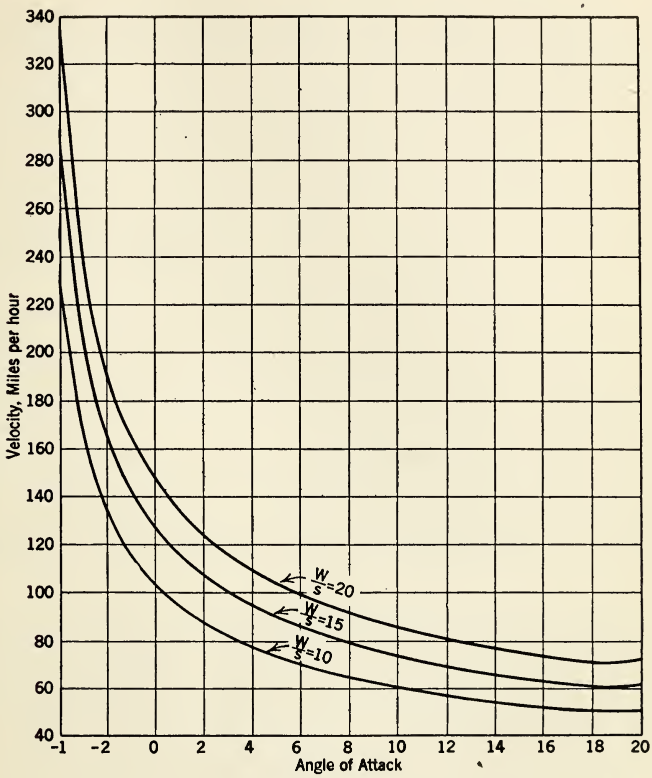


FIG. 24. Angle of attack versus velocity.

Conversely, for each angle of attack there is only one speed for level flight. This depends on the relation

$$C_L = \frac{W}{S} \frac{1}{\frac{\rho}{2} V^2}$$

If the wing loading (W/S) is changed, the relation between angle of attack and speed is changed. A partly loaded airplane, flying level, will either have to increase its speed or angle of attack if additional load is put on. Figure 24 shows the relation between airspeed and angle of attack for a Clark Y airfoil.

Problems

1. Plot airspeed versus angle of attack for a Göttingen 398 airfoil with a wing loading of 14 lb. per sq. ft.

2. Plot airspeed versus angle of attack for a R.A.F. 15 wing with a wing loading of 18 lb. per sq. ft.

3. For an M-6 airfoil, plot wing loading versus angle of attack for an airspeed of 100 miles per hour.

4. For a C-80 airfoil, plot wing loading versus airspeed, in feet per second, for an angle of attack of 4°.

Flying Level at Altitude. At altitudes above sea-level, the air density is less than its standard value. Since both lift and drag coefficients are multiplied by the air density, as well as wing area and the square of the velocity, to give lift and drag; at the same angle of attack or at the same airspeed, both lift and drag will be less at altitude. In order to fly level no matter what altitude, the lift must be equal to the weight.

As the wing area remains constant, when an airplane ascends to higher altitudes either lift coefficient or velocity, or both, must be increased to make up for the decrease in density. The case of flying at the same angle of attack as at the ground will be considered first.

The angle of attack being unchanged, C_L and C_D will be the same at all altitudes. Let ρ_0 and V_0 be the air density and velocity at zero altitude and ρ_x and V_x the density and velocity at any altitude, x feet.

$$W = C_L \frac{\rho_0}{2} S V_0^2$$

$$W = C_L \frac{\rho_x}{2} S V_x^2$$

Then $$\rho_0 V_0^2 = \rho_x V_x^2 \quad \text{and} \quad \frac{\rho_0}{\rho_x} V_0^2 = V_x^2$$

$$\text{Drag at zero altitude} = D_0 = C_D \frac{\rho_0}{2} S V_0^2$$

$$\text{Drag at } x \text{ feet altitude} = D_x = C_D \frac{\rho_x}{2} S V_x^2$$

But $$V_x^2 = \frac{\rho_0}{\rho_x} V_0^2$$

Therefore $$D_x = C_D \frac{\rho_x}{2} S \frac{\rho_0}{\rho_x} V_0^2$$

$$= C_D \frac{\rho_0}{2} S V_0^2$$

Therefore, whatever the altitude, the drag of the wing is the same, provided the angle of attack is the same. This may seem strange, but it should be borne in mind that with increased altitude the airspeed must be greater, and this exactly counterbalances the decrease in density.

Power required is the product of drag and velocity. Let H.P._0 be the horsepower at zero altitude and H.P._x the horsepower required at x-feet altitude.

$$\text{H.P.}_0 = \frac{D_0 \times V_0}{550}$$

$$\text{H.P.}_x = \frac{D_x \times V_x}{550}$$

But $$D_0 = D_x$$

and $$V_x = V_0 \sqrt{\frac{\rho_0}{\rho_x}}$$

Therefore $$\text{H.P.}_x = \frac{D_0 \times V_0}{550} \sqrt{\frac{\rho_0}{\rho_x}}$$

$$= \text{H.P.}_0 \sqrt{\frac{\rho_0}{\rho_x}}$$

The density becomes less as ascent is made in the atmosphere; that is, ρ_x is less than ρ_0, so that ρ_0/ρ_x is greater than 1. With the same angle of attack, the horsepower required to move a wing forward through the air will always be greater at altitude than at sea-level.

When the angle of attack is changed, keeping the same airspeed, a lightly loaded plane is affected differently from a heavily loaded plane. At ordinary flying angles, the lift coefficient varies approximately as angle of attack. The drag coefficient changes very little with angle of attack at small angles of attack corresponding to high speeds. At slow speeds, corresponding to high angles, there is a big change of drag coefficient with angle of attack changes.

When attempt is made to fly at the same speed at high altitude as at the ground, the angle of attack must be increased to obtain the greater lift coefficient to offset the decrease in density. The larger angle of attack will mean an increased drag coefficient. If at sea-level the airplane was being flown at low angle of attack, a moderate increase in altitude would mean a very small increase in drag coefficient. It is quite possible that the decrease in density could be greater in proportion than the increase in drag coefficient. In this case, the drag and consequently power required would be less at altitude than at sea-level. A plane flying slowly at the ground (high angle of attack) and rising to a moderate height, or an airplane flying fast at the ground but rising to extreme altitudes, will usually result in the drag coefficient increasing disproportionately more than the decreasing of density so that the drag and horsepower will be greater at the higher altitude.

Example. An airplane has a Clark Y wing 400 sq. ft. in area. It flies 100 ft. per sec. at 2° angle of attack. What are the lift, wing drag, and horsepower required at sea-level? What are the lift, wing drag, and horsepower required at 10,000-ft. altitude?

At sea-level,

$$L = 0.5 \times \frac{0.002378}{2} \times 400 \times \overline{100}^2$$

$$= 2{,}378 \text{ lb.}$$

$$D = 0.024 \times \frac{0.002378}{2} \times 400 \times \overline{100}^2$$

$$= 114 \text{ lb.}$$

$$\text{H.P.} = \frac{114 \times 100}{550}$$

$$= 20.7 \text{ hp. required}$$

At 10,000-ft. altitude

$$L = 0.5 \times \frac{0.001756}{2} \times 400 \times \overline{100}^2$$

$$= 1{,}756 \text{ lb.}$$

$$D = 0.024 \times \frac{0.001756}{2} \times 400 \times \overline{100}^2$$

$$= 84.3 \text{ lb.}$$

$$\text{H.P.} = \frac{84.3 \times 100}{550}$$

$$= 15.3 \text{ hp.}$$

Example. An airplane weighing 4,000 lb. has a Clark Y wing 350 sq. ft. in area and is flying at sea-level at 100 miles per hour. What are the wing drag and horsepower required for the wing? If the airplane is flying at an altitude of 10,000 ft., at 100 miles per hour, what are the wing drag and horsepower required?

At sea-level

$$C_L = \frac{4{,}000}{\frac{0.002378}{2} \times 350 \times (1.47 \times 100)^2}$$

$$= 0.445$$

Therefore $\alpha = 1\frac{1}{2}°$

and $C_D = 0.0217$

$$D = 0.0217 \times \frac{0.002378}{2} \times 350 \times \overline{1.47 \times 100}^2$$

$$= 195 \text{ lb.}$$

$$\text{H.P.} = \frac{195 \times 147}{550}$$

$$= 52.2 \text{ hp.}$$

At 10,000-ft. altitude

$$C_L = \frac{4{,}000}{\frac{0.001756}{2} \times 350 \times (1.47 \times 100)^2}$$

$$= 0.604$$

Therefore $\alpha = 3\frac{1}{2}°$

and $C_D = 0.0308$

$$D = 0.0308 \times \frac{0.001756}{2} \times 350 \times \overline{147}^2$$

$$= 204 \text{ lb.}$$

$$\text{H.P.} = \frac{204 \times 147}{550}$$

$$= 54.6 \text{ hp.}$$

Problems

1. An airplane weighing 3,000 lb. has a Clark Y wing 250 sq. ft. in area and is flying at sea-level at 150 miles per hour. (*a*) What are the wing drag and horsepower required for the wing? (*b*) What are the wing drag and horsepower if flying at 150 miles per hour at 15,000-ft. altitude? (*c*) What are the wing drag and horsepower if flying at 150 miles per hour at 30,000-ft. altitude?

2. An airplane weighing 6,000 lb. has a Clark Y wing 250 sq. ft. in area. It flies at 125 miles per hour. (*a*) What are the wing drag and horsepower required for the wing at sea-level; (*b*) at 15,000-ft. altitude; (*c*) at 25,000-ft. altitude?

3. An airplane weighing 5,000 lb. has a Clark Y wing 275 sq. ft. in area. It flies at 4° angle of attack. (*a*) What are the wing drag and horsepower required at sea-level; (*b*) at 15,000 ft.?

4. An airplane weighing 4,000 lb. has a Clark Y wing 350 sq. ft. in area. It flies at 150 miles per hour. (*a*) What is the horsepower required at sea-level; (*b*) at 20,000-ft. altitude?

5. An airplane whose wing loading is 12 lb. per sq. ft. has a Clark Y wing and is flying at 200 ft. per sec. at sea-level. (*a*) What is the angle of attack? (*b*) What is the angle of attack if flying at 20,000-ft. altitude at the same speed?

Lift-Drag Ratio. Air flowing around a wing causes forces to come into action, and the resultant of these forces is usually expressed in terms of its lift and drag components. In level flight the relative wind is horizontal, so that the lift component is vertical, the drag component horizontal. The lift component sustains the weight of the airplane. The drag component is the resistance to forward motion of the wing. In a complete airplane there are other parts, such as the fuselage, landing gear, and struts, which offer resistance to forward motion through the air. The resistance of these other parts of the airplane to forward motion is called the parasite drag. The sum of the wing drag and parasite drag constitute the total drag of the airplane. This total drag is the backward force that must be balanced by the forward thrust of the propeller in order to sustain forward movement of the airplane.

The sole purpose of a wing is to provide a sustaining force for the airplane. It is to be expected that a wing will offer resistance to movement through the air. The wing that offers the least resistance and at the same time furnishes the most lift would be the most desirable from this standpoint.

Wings must be capable of being made structurally strong. A very thin wing might have merits from an aerodynamic standpoint, but it might be so shallow that the spars usable in such a wing would be too light to have sufficient strength. The amount of movement of the center of pressure on a wing is also important in the securing of longitudinal balance. These matters will affect the selection of the wing section to be used, so that big lift with little drag will not be the sole consideration.

The term " efficiency " as used in engineering has a very exact meaning, namely the *power* output divided by the *power* input. In engineering, efficiency is always less than unity. The term *efficiency* cannot be correctly applied to ratio of the lift force to the drag force of a wing, since it is a ratio of forces, not of powers. The expression " efficacy of the wing " which has been suggested is rather cumbersome, and it is practically universal to employ the expression " lift-drag ratio " or " L over D."

The lift-drag ratio is the same as the ratio of lift coefficient to drag coefficient.

$$\frac{\text{Lift}}{\text{Drag}} = \frac{C_L \frac{\rho}{2} S V^2}{C_D \frac{\rho}{2} S V^2} = \frac{C_L}{C_D}$$

and is the tangent of the angle which the resultant force on the wing makes with the horizontal plane.

At small angles of attack, drag coefficient is small but lift coefficient is also small. At large angles of attack, lift coefficient is large but drag coefficient is also large. At the angle of attack of minimum drag coefficient, it will be found that an increase of a couple of degrees in the angle of attack will cause only a slight increase in drag coefficient but it will cause a considerable increase in the lift coefficient. It will therefore be at an angle of attack $1\frac{1}{2}°$ or $2°$ greater than the angle of minimum drag coefficient that L/D will have its greatest value.

The values of L/D or C_L/C_D for various angles of attack of the Clark Y airfoil are shown in Fig. 17. It will be noted that the angle of minimum drag coefficient is $-3\frac{1}{4}°$, while the angle of maximum L/D is slightly greater, i.e., $+\frac{3}{4}°$.

The angle of maximum L/D is important. For level flight, lift is considered as practically equal to the weight. With weight constant, lift is constant; the drag will be least when D/L is least, that is, when L/D is greatest.

In a preceding paragraph, it was shown that for level flight there is one velocity corresponding to each angle of attack. The velocity corresponding to the angle of maximum L/D will be the velocity at which the wing will have the least drag.

Since parasite drag coefficients vary only slightly with angle of attack, it is approximately correct to state that, if the angle of attack of the wing is that of maximum L/D, the whole airplane will have less total drag than in any other position. This position will require the least thrust *force* from the propeller.

It should be noted that power contains the element of speed or time, so that less *power* may be required from the engine if the airplane is flown at slightly less speed, i.e., slightly greater angle of attack. This will be treated later in this book. The gasoline consumption depends on horsepower-hours, which, in turn, depends on L/D total. In flying from one point to another, the least amount of fuel will be required if the airplane is flown entirely with the wing at the angle of attack of maximum L/D.

For best performance of the engine and propeller and for comfort it is desirable that the longitudinal axis of the airplane be horizontal. In order that in that position of the airplane the wing shall be at the angle of maximum L/D, the wing is usually set at a small positive angle to the axis of the airplane. This angle is called the angle of incidence.

In a race, the nose of an airplane would be depressed, causing the angle of attack to be less than that of maximum L/D. The drag coefficient will be less at this angle of attack than at the angle of maximum L/D, but in order that there shall be sufficient lift to sustain the plane in the air, velocity must be greater than at the larger angle. Because velocity is greater, the drag is greater at the lower angle even though the drag coefficient is less.

Example. In level flight, what is the least drag of the Clark Y wing of an airplane weighing 5,000 lb.?

Solution. From Fig. 17, $(L/D)_{\text{max.}}$ for Clark Y is 21.5

$$\frac{L}{D} = \frac{W}{D} = \frac{5{,}000}{D} = 21.5$$

$$D = \frac{5{,}000}{21.5} = 233 \text{ lb.}$$

Problems

1. (*a*) Plot L/D versus angle of attack for the Göttingen 398 airfoil. (*b*) What is the least drag of a Göttingen 398 wing for an airplane weighing 4,000 lb.? (*c*) What is the drag for this airplane when the wing is at 6° angle of attack? (*d*) What is the drag when the wing is at $-3°$ angle of attack?

2. (*a*) Plot L/D versus angle of attack for the M-6 airfoil. (*b*) What is the least drag of an M-6 wing for an airplane weighing 2,000 lb.? (*c*) What is the angle of attack for least drag?

3. (*a*) Plot L/D versus angle of attack for the R.A.F. 15 airfoil. (*b*) What is the least drag of an R.A.F. 15 wing for an airplane weighing 2,000 lb.? (*c*) What is the angle of attack for least drag? (*d*) What is the drag at zero degrees angle of attack?

Polars. Instead of plotting lift and drag coefficients against angle of attack, the information may be given in other ways. Quite frequently the lift coefficient is plotted against drag coefficient, as in Fig. 25. The curve represents the changes in C_D with

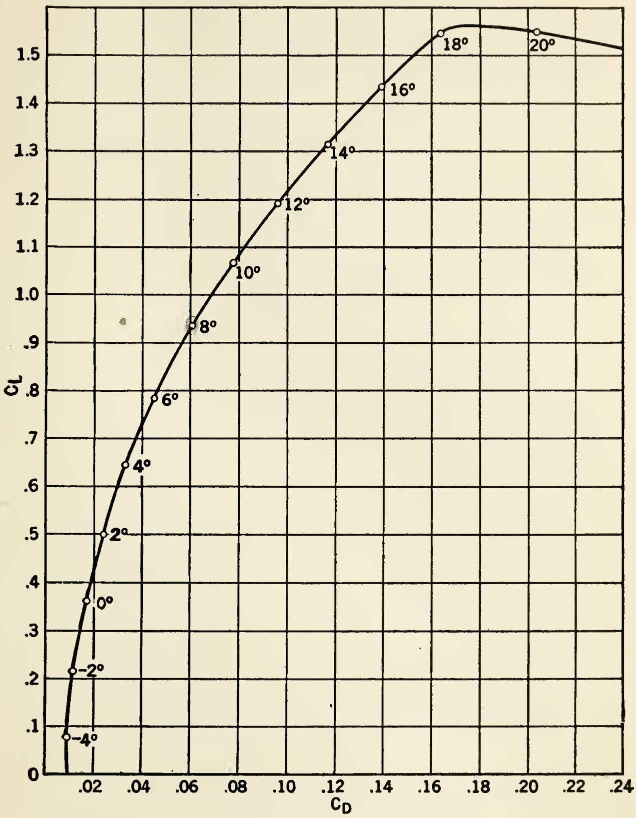


FIG. 25. Polar curve for Clark Y airfoil, aspect ratio 6.

changes in C_L, or vice versa. The angle of attack is designated on the curve itself. This form of plotting is called a *polar* curve.

Since C_D is small in comparison with C_L it is customary to use a much larger scale for plotting C_D than for C_L. If the same scale were used for plotting both C_L and C_D, a line drawn from the origin to any point on the curve would represent the resultant coefficient, both in direction and magnitude. Its length could be scaled off in the same units as the C_L and C_D scale. Multiplied by $(\rho/2)SV^2$, it would represent the force acting on the wing.

Using different scales for C_L and C_D, the length and direction of a line from the origin to a point on the polar curve are meaningless. However, irrespective of scales, the line which makes the largest angle with the horizontal base line, having the greatest tangent, will have the highest C_L/C_D ratio. Therefore a line drawn from the origin tangent to the polar curve will locate the angle of maximum L/D. By reading the coordinates of the point of tangency, the maximum value of L/D may be quickly found.

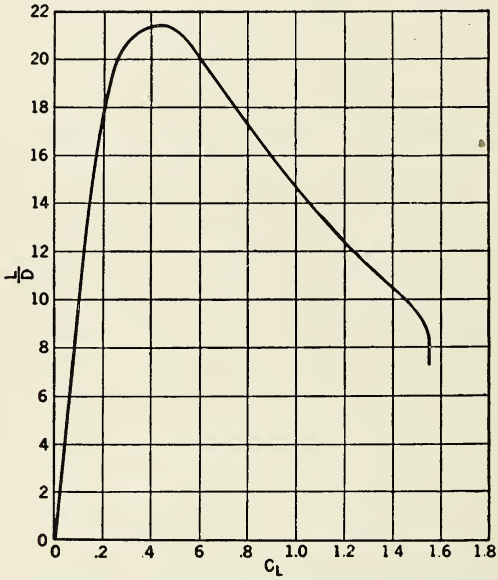


Fig. 26. L/D versus C_L for Clark Y airfoil.

Another form of graph is shown in Fig. 26, where L/D is plotted against C_L. Because lift coefficient varies almost directly with angle of attack up to near the burble point, this graph resembles very much the plot of L/D against angle of attack in Fig. 17.

In comparing one airfoil with another, the actual angles of attack are of little consequence. The important factors are C_L, C_D, and L/D. These are all given by one polar curve; the information would have to be obtained from three separate curves otherwise.

Example. An airplane weighing 3,000 lb. has a Clark Y wing 350 sq. ft. in area. What horsepower is required for the wing when flying at 90 ft. per sec.?

$$C_L = \frac{W}{\frac{\rho}{2} S V^2}$$

$$= \frac{3{,}000}{\frac{0.002379}{2} \times 350 \times (90)^2}$$

$$= 0.891$$

From Fig. 25, when $C_L = 0.891$, $C_D = 0.0561$

$$\text{H.P.} = \frac{C_D \frac{\rho}{2} S V^3}{550}$$

$$= \frac{0.0561 \times 0.001189 \times 350 \times (90)^3}{550}$$

$$= 31.0 \text{ hp.}$$

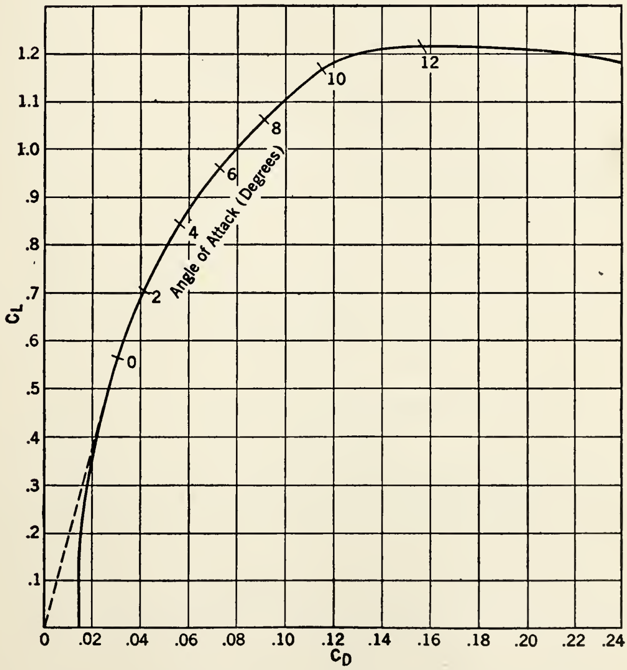


FIG. 27. Polar curve for U.S.A.-35A airfoil, aspect ratio 6.

Example. Compare maximum L/D of a Clark Y airfoil with that of a U.S.A.-35 airfoil.

In Fig. 25, the line from origin tangent to curve is tangent at point where $C_L = 0.43$ and $C_D = 0.020$.

$$\text{For Clark Y, Maximum } L/D = \frac{0.43}{0.020} = 21.5$$

In Fig. 27, the line from origin tangent to curve is tangent at point where $C_L = 0.55$ and $C_D = 0.03$.

$$\text{For U.S.A.-35 max. } L/D = \frac{0.55}{0.03} = 18.3$$

or directly from Fig. 26,

$$\text{For Clark Y, Maximum } L/D = 21.0$$

Example. An airplane weighing 2,000 lb. has a C-80 wing 180 sq. ft. in area. What horsepower is required for the wing when the airspeed is 120 miles per hour?

Solution.

$$C_L = \frac{W}{\frac{\rho}{2}SV^2}$$

$$= \frac{2,000}{0.001189 \times 180 \times (120 \times 1.47)^2}$$

$$= 0.301$$

From Fig. 28, when $C_L = 0.301$, $L/D = 23.2$
For horizontal flight, $W = L$

$$\frac{2,000}{D} = 23.2$$

$$D = 86.2 \text{ lb.}$$

$$\text{H.P.} = \frac{86.2 \times 120}{375}$$

$$= 27.6 \text{ hp.}$$

Problems

1. An airplane weighing 2,500 lb. has a C-80 wing 200 sq. ft. in area. What horsepower is required for the wing when airspeed is 200 ft. per sec.?

2. An airplane weighing 1,800 lb. has a C-80 wing. What area should the wing have in order that only 25 hp. will be required for the wing when the airspeed is 150 ft. per sec.?

3. An airplane weighing 4,000 lb. has a C-80 wing. It is flying at the angle of attack which has C_L of 0.6. What is the drag?

4. An airplane has a C-80 wing. When C_L is 0.7 and the drag is 300 lb., what is the lift?

5. An airplane weighing 2,400 lb. has a U.S.A.-35A wing 210 sq. ft. in area. What is the drag when flying at an airspeed of 90 ft. per sec.?

6. An airplane weighing 3,500 lb. has a U.S.A.-35A rectangular wing of 54-ft. span and 9-ft. chord. What horsepower is required by the wing when the airspeed is 150 ft. per sec.?

7. What horsepower is required by the wing in problem 6 when the airplane is flying at 100 ft. per sec.?

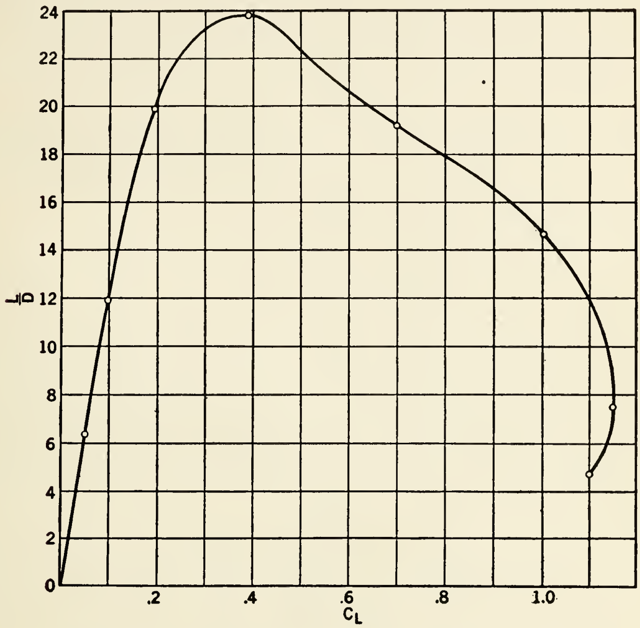


FIG. 28. L/D versus C_L for C-80 airfoil, aspect ratio 6.

8. An airplane with a U.S.A.-35A wing has a wing loading of 12 lb. per sq. ft. What is the L/D when airspeed is 100 miles per hour?

9. An airplane with a U.S.A.-35A wing 600 sq. ft. in area is flying at a 6° angle of attack with an airspeed of 90 miles per hour. What is the lift and what is the drag?

10. An airplane with a U.S.A.-35A wing has a wing loading of 14 lb. per sq. ft. It is flying at a 4° angle of attack. What should be the airspeed?

Absolute Coefficients with Metric Units. The absolute coefficients of lift and drag, C_L and C_D, are dimensionless, that is, they are pure numbers. Because of this, C_L and C_D are usable in the standard equations for lift and drag, provided the other factors are consistent. With the English units: lift and drag forces are in pounds, mass density is in slugs per cubic foot, area is in square feet, and velocity is in feet per second. With metric units: lift and drag forces are in kilograms, density is in metric slugs per cubic meter, area is in square meters, and velocity is in meters per second.

Under standard conditions, a cubic meter of air weighs 1.2255 kg. Since the standard acceleration of gravity is 9.807 meters per second per second, the standard mass density of air is 1.2255 ÷ 9.807 or 0.12497 metric slug per cubic meter.

Example. An airplane having a Göttingen 398 wing 35 square meters in area is flying at 4° angle of attack, with a velocity of 40 meters per second. What is the lift?

Solution. From Fig. 18, $C_L = 0.76$.

$$\begin{aligned} \text{Lift} &= C_L \frac{\rho}{2} S V^2 \\ &= 0.76 \times \frac{0.125}{2} \times 35 \times \overline{40}^2 \\ &= 2{,}660 \text{ kg.} \end{aligned}$$

Example. An airplane with a wing loading of 30 kg. per square meter is flying with its Göttingen 398 wing at a 2° angle of attack. What should be the airspeed?

Solution. From Fig. 18, $C_L = 0.585$.

$$\begin{aligned} V &= \sqrt{\frac{W}{S}} \times \sqrt{\frac{1}{C_L \frac{\rho}{2}}} \\ &= \sqrt{30} \times \sqrt{\frac{1}{0.585 \times 0.0625}} \\ &= 28.7 \text{ meters per second} \end{aligned}$$

Engineering Coefficients. In the United States, it was formerly universal custom to employ engineering coefficients, K_x and K_y, instead of the absolute coefficients, C_L and C_D. K_x and K_y included in themselves the standard density of air as well as a factor for changing miles per hour into feet per second. With engineering coefficients, the standard formulas for lift and drag become

$$L = K_y S V^2 \qquad S \text{ in square feet}$$
$$D = K_x S V^2 \qquad V \text{ in miles per hour}$$

K_y is the lift in pounds of a wing, 1 sq. ft. in area, traveling at a velocity of 1 mile per hour; K_x is the drag of that wing, the movement taking place through standard air.

The K_y and K_x coefficients given by characteristic curves for various airfoils are for use only when the airfoils are moving through air of standard density. They should be more properly designated K_{y_0} and K_{x_0} since their application is only at zero altitude. At altitudes where the air density differs from that at sea-level, the K_y and K_x shown on the graphs must be corrected for the new air density. The symbol σ (sigma) is used to express the ratio of the density at any altitude to standard density. The corrected K_y and K_x for any altitude is found by multiplying the K_{y_0} and K_{x_0} (taken from the graphs) by the σ for that altitude.

$$K_y = \sigma K_{y_0} = \frac{\rho}{\rho_0} K_{y_0}$$

$$K_x = \sigma K_{x_0} = \frac{\rho}{\rho_0} K_{x_0}$$

By comparing the equations for lift and drag containing absolute coefficients with the similar equations with engineering coefficients, a relation is found between the two sets of coefficients, as follows.

$$K_y = C_L \times \left(\frac{0.002378}{2}\right) \times \left(\frac{5{,}280}{3{,}600}\right)^2$$

$$= 0.00256\ C_L$$

and $$K_x = 0.00256\ C_D$$

or $$C_L = 390.7\ K_y$$

and $$C_D = 390.7\ K_x$$

C_L and C_D, being each multiplied by the same constant of transformation, 0.00256, to obtain K_y and K_x respectively, L/D is equal to the ratio of engineering coefficients, since

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{0.00256\ C_L}{0.00256\ C_D} = \frac{K_y}{K_x}$$

Engineering coefficients are much more convenient to use since in practice airspeed is usually measured in miles per hour rather than in feet per second. There is incongruity, however, in not using the same unit of length in calculating area as in computing velocity. Engineering coefficients cannot be used for metric units without transformation.

The characteristics of the Clark Y airfoil are shown in Fig. 29. It will be noted that the curves are identical with those in Fig. 17, but the scales are different.

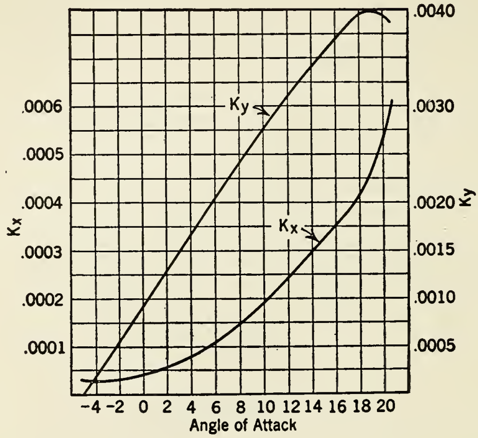


FIG. 29. Characteristics for Clark Y airfoil, aspect ratio 6, with engineering coefficients.

Example. A Clark Y wing having an area of 400 sq. ft. at 4° angle of attack is moving through the air at a speed of 125 miles per hour. What are the lift and drag at sea-level? At 10,000-ft. altitude?

Solution. From Fig. 29:

At 4° angle of attack: $K_y = 0.00169$

$$K_x = 0.000089$$

At zero altitude:

$$L = 0.00169 \times 400 \times \overline{125}^2$$
$$= 10{,}600 \text{ lb.}$$

$$D = 0.000089 \times 400 \times \overline{125}^2$$
$$= 556 \text{ lb.}$$

From Table I, σ at 10,000 ft. = 0.738.

At 10,000-ft. altitude:

$$L = 0.738 \times 0.00169 \times 400 \times \overline{125}^2$$
$$= 0.738 \times 10{,}600$$
$$= 7{,}820 \text{ lb.}$$

$$D = 0.738 \times 0.000089 \times 400 \times \overline{125}^2$$
$$= 0.738 \times 556$$
$$= 410 \text{ lb.}$$

Example. An airplane weighing 5,000 lb. has a Clark Y wing 400 sq. ft. in area; what should be the airspeed at 4° angle of attack at sea-level? At 10,000-ft. altitude? What is the drag under each condition?

Solution.

At zero altitude:

$$V = \sqrt{\frac{5{,}000}{400 \times 0.00169}}$$
$$= 86.0 \text{ miles per hour}$$
$$D = 0.000089 \times 400 \times \overline{86.0}^2$$
$$= 263 \text{ lb.}$$

At 10,000-ft. altitude:

$$V = \sqrt{\frac{5{,}000}{400 \times 0.738 \times 0.00169}}$$
$$V = \sqrt{\frac{1}{0.738}} \times 86.0$$
$$= 100 \text{ miles per hour}$$
$$D = 0.738 \times 0.000089 \times 400 \times \overline{100}^2$$
$$= 0.738 \times 0.000089 \times 400 \times \left(\sqrt{\frac{1}{0.738}} \times 86.0\right)^2$$
$$= 0.000089 \times 400 \times \overline{86.0}^2$$
$$= 263 \text{ lb.}$$

Note: With weight fixed, at a constant angle of attack, drag is independent of altitude.

Problems (Use Fig. 29).

1. A wing 150 sq. ft. in area has a Clark Y section. What is the maximum load that can be carried at 50 miles per hour?

2. For the wing in problem 1, what is the minimum speed to carry a load of 1,600 lb.?

3. For the wing in problem 1, what must be the angle of attack for the wing to lift 1,600 lb. when the airspeed is 80 miles per hour?

4. For a wing loading of 14 lb. per sq. ft., (*a*) what should be the angle of attack for a Clark Y wing at an airspeed of 90 miles per hour at sea-level; (*b*) at 10,000-ft. altitude?

5. An airplane weighing 2,000 lb. has a Clark Y wing 240 sq. ft. in area. (*a*) What is the drag at 120 miles per hour at sea-level; (*b*) at 10,000-ft. altitude?

6. For an airplane with a Clark Y wing, what should be the wing loading to fly 150 miles per hour at 0° angle of attack: (*a*) at sea-level; (*b*) at 10,000-ft. altitude?

7. An airplane, with a wing loading of 18.2 lb. per sq. ft., has a Clark Y wing whose angle of attack is 7°. For level flight what should be the velocity (*a*) at sea-level; (*b*) at 10,000-ft. altitude?

8. An airplane has a minimum speed of 40 miles per hour at sea-level, what is its minimum speed at 10,000-ft. altitude?

9. An airplane has a Clark Y wing 270 sq. ft. in area. With wing at 3° angle of attack, what should the airspeed be in order that the drag does not exceed 300 lb. (*a*) at sea-level; (*b*) at 10,000-ft. altitude?

10. An airplane weighing 1,800 lb. has a Clark Y wing. What should be the wing area, if the airspeed is 130 miles per hour when the wing is at 1° angle of attack?

Power with Engineering Coefficients. With engineering coefficients, velocity is in miles per hour units. Power required by the wing, being the product of drag and velocity, will be in mile-pounds per hour units, if velocity in miles per hour is multiplied by drag force in pounds. One horsepower is 550 ft-lb. per sec. or 375 mile-lb. per hour. Using this factor, the formulas for horsepower required by the wing become

$$\text{H.P.}_{\text{req.}} = \frac{DV}{375} = \frac{K_x S V^3}{375} \qquad \begin{array}{l} D \text{ in pounds} \\ S \text{ in square feet} \\ V \text{ in miles per hour} \end{array}$$

Example. What is the horsepower required by a Clark Y wing 225 sq. ft. in area at 2° angle of attack and 125 miles per hour (*a*) at sea-level; (*b*) at 15,000-ft. altitude?

Solution. From Fig. 29, at 2° angle of attack, K_x is 0.0000614.

At sea-level:

$$\text{H.P.}_{\text{req.}} = \frac{0.0000614 \times 225 \times \overline{125}^3}{375} = 72.0 \text{ hp.}$$

At 15,000-ft. altitude:

$$\text{H.P.}_{\text{req.}} = \frac{(0.629 \times 0.0000614) \times 225 \times \overline{125}^3}{375} = 0.629 \times 72.0 = 45.2 \text{ hp.}$$

Problems

1. What horsepower is required by a Clark Y wing 325 sq. ft. in area at 7° angle of attack and airspeed of 85 miles per hour?

2. What should be the airspeed in order that a Clark Y wing 200 sq. ft. in area should require only 45 hp. at 4° angle of attack?

3. What should be the area of a Clark Y wing so that 50 hp. is required to fly at 90 miles per hour at 5° angle of attack?

4. What horsepower is required by a Clark Y wing 260 sq. ft. in area, on an airplane weighing 1,750 lb., when flying at 3° angle of attack (*a*) at sea-level; (*b*) at 15,000-ft. altitude?

5. What horsepower is required by a Clark Y wing 260 sq. ft. in area, on an airplane weighing 1,750 lb., when flying at 100 miles per hour (*a*) at sea level; (*b*) at 15,000-ft. altitude?

Power in Terms of $C_D/C_L^{3/2}$. The power required to move a wing forward through the air may be expressed in a form which does not contain velocity (V).

$$\text{H.P.} = \frac{1}{550}\left(C_D\frac{\rho}{2}SV^2\right)V$$

V = vel.

but

$$V^2 = \frac{W}{C_L\frac{\rho}{2}S}$$

and

$$V = \sqrt{\frac{W}{C_L\frac{\rho}{2}S}}$$

Then

$$\text{H.P.} = \frac{1}{550}C_D\frac{\rho}{2}S\frac{W}{C_L\frac{\rho}{2}S}\sqrt{\frac{W}{C_L\frac{\rho}{2}S}}$$

$$= \frac{1}{550}\frac{1}{\sqrt{\frac{\rho}{2}}}W\sqrt{\frac{W}{S}}\frac{C_D}{C_L^{3/2}}$$

An airplane, with a given weight and wing area, will require the least power to move the wing forward at the particular angle of attack at which $C_D/C_L^{3/2}$ is the least. In other words, the angle of attack at which $C_L^{3/2}/C_D$ is the maximum is the angle of attack for which the least power is required.

The above expression also shows that at any one angle of attack, the power required varies directly as $W^{3/2}$, inversely as the square root of the wing area S, and inversely as the square root of the air density ρ.

Example. An airplane weighs 4,000 lb., the wing area is 300 sq. ft. The wing is flying at an angle of attack for which $C_L = 1.2$ and $C_D = 0.1$. What horsepower is required for the wing?

Solution.

$$\text{H.P.} = \frac{1}{550\sqrt{\frac{\rho}{2}}} \times W\sqrt{\frac{W}{S}} \times \frac{C_D}{C_L{}^{3/2}}$$

$$= \frac{1}{550\sqrt{\frac{0.002378}{2}}} \times 4{,}000\sqrt{\frac{4{,}000}{300}} \times \frac{0.1}{1.2^{3/2}}$$

$$= 58.6$$

Example. An airplane weighing 5,000 lb. requires 100 hp. to move the wing at a certain angle of attack. If 500 lb. is added to the weight of the airplane, what horsepower is required for the wing if flown at the same angle of attack as before?

Solution.

$$\frac{\text{H.P.}_1}{\text{H.P.}_2} = \left(\frac{W_1}{W_2}\right)^{3/2}$$

$$\frac{100}{\text{H.P.}_2} = \left(\frac{5{,}000}{5{,}500}\right)^{3/2}$$

$$\text{H.P.}_2 = 115.4$$

Problems

1. An airplane wing requires 90 hp. to be flown at a certain angle of attack. If the wing tips are clipped so as to reduce the wing area from 300 sq. ft. to 250 sq. ft., and the total weight is the same as before, what horsepower is needed to fly at the same angle of attack as before?

2. An airplane requires 40 hp. to fly at an angle of attack at which $C_L = 0.9$ and $C_D = 0.06$. What power is required for the wing when flying at an angle of attack at which $C_L = 0.6$ and $C_D = 0.03$?

3. An airplane weighs 6,000 lb., the wing area is 450 sq. ft., the C_D of the wing is 0.02, and C_L is 0.45. What power is required by the wing at sea-level?

4. An airplane weighing 5,000 lb. requires 30 hp. for the wing to fly at one angle of attack. What horsepower is needed for the wing at that same angle of attack but with 800 lb. less load?

5. Plot a curve of $C_L{}^{3/2}/C_D$ versus angle of attack for the Clark Y airfoil, aspect ratio of 6.

Moment Coefficient and Center of Pressure. In studying the problem of longitudinal balance and stability, it is necessary to know not only the magnitude and direction of the resultant of the forces on the wing, but also the position of this resultant. Lift and drag forces are perpendicular to each other and are components of the resultant force. The lift squared plus the drag squared equals the resultant squared. Like its two components, the resultant varies as the air density, the wing area, and the square of the velocity; therefore a resultant coefficient may be used in a similar manner to the lift and drag coefficients.

$$\text{Resultant} = R = \sqrt{L^2 + D^2} = C_R \frac{\rho}{2} S V^2$$

S is area in square feet

V is velocity in feet per second

$C_R = \sqrt{C_L{}^2 + C_D{}^2}$

The direction in which the resultant force acts is the angle $\cot^{-1} L/D$, backward from the direction in which the lift component acts.

The point on the chord through which the line of action of the resultant force passes is termed the center of pressure. This is abbreviated as C.P. Its location in percentage of chord length is given by a curve which is customarily included among the characteristic curves of an airfoil.

For all unsymmetrical airfoils, that is, those having greater camber on the upper than the lower surface, the curves of center-of-pressure location bear a close resemblance. At angles close to the angle of zero lift, the center of pressure is near the trailing edge of the wing. As the angle becomes more positive, the center of pressure moves forward. At some angle, usually a few degrees below the angle of maximum lift, the center of pressure is at its most forward position. The maximum forward position of the center of pressure is about 28 per cent of the chord length back from the leading edge.

As the angle of attack is decreased below that for maximum forward position, the center of pressure moves backward. The resultant is an upward and backward force. At the exact angle of zero lift, the resultant instead of being a single force becomes a couple tending to depress the leading edge and raise the trailing edge, plus the drag component. If the angle of attack becomes more negative than the angle of zero lift, the resultant reappears as a single force again but acting downward and rearward, with the center of pressure near the trailing edge. Depressing the leading edge still more, the resultant, as a larger downward and backward force, moves forward.

The above-described movement is termed " unstable " center-of-pressure movement. When the wing is balanced at one angle of attack, if this angle of attack is momentarily increased by a gust of wind or otherwise, the forward movement of the upward resultant force tends to tip the leading edge of the wing upward still more. This increase of attack angle moves the center of pressure still further forward, tending to increase the nosing-up still more, so that a stall would eventually result. Conversely, a decrease in the angle of attack from a previously balanced condition would cause a backward movement of the center of pressure which, by lifting on the trailing edge, tends to decrease the angle of attack more.

With symmetrical airfoils, that is, those having both surfaces convex and of the same camber, there is practically no center-of-pressure movement. These airfoils are termed " stable " airfoils. On the more common non-symmetrical airfoils, a reverse curvature, i.e., having the trailing edge curve upward, gives an airfoil with little or no center-of-pressure movement.

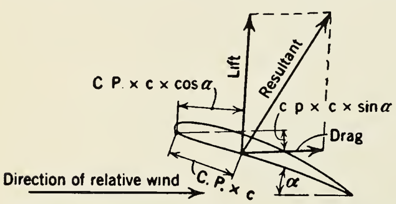


Fig. 30. Diagram of moments about leading edge.

The *moment* of the resultant force on the wing is the product of the force and the distance from the line of action of the force to the point about which the moment is taken. Moments which act in a manner tending to increase the angle of attack are called stalling moments and are designated by a positive sign. Moments which tend to decrease the angle of attack are called diving moments and are negative in sign.

When the moment is taken about the leading edge, the sum of the moments of the two components may be used; see Fig. 30.

$$\text{Moment about leading edge} = M_0 = \text{C.P.} \times c \times \cos\alpha \times L + \text{C.P.} \times c \times \sin\alpha \times D$$

moment is in foot-pounds.

C.P. is in percentage of chord length, c is chord length in feet, L and D are in pounds.

This may also be written as follows:

$$\text{Moment about leading edge} = M_0 = C_{M_0} c \frac{\rho}{2} S V^2$$

$C_{M_0} = (\text{C.P.})(C_L \cos\alpha + C_D \sin\alpha)$, V in feet per second

In engineering units:

$$\text{Moment about leading edge} = M_0 = K_{M_0} cSV^2$$

$$K_{M_0} = (\text{C.P.})(K_y \cos \alpha + K_x \sin \alpha), \ V \text{ in miles per hour}$$

Since α is always a small angle, cosine α is very close to unity, and only a slight error is introduced if the term C_L is used instead of $C_L \cos \alpha$. Also since C_D is usually small compared with C_L, $C_D \sin \alpha$ will be still smaller compared with C_L and only a slight additional error will be incurred if the term $C_D \sin \alpha$ is dropped. Then a sufficiently close approximation for most work is to let

$$C_{M_0} = -(\text{C.P.})(C_L) \qquad V \text{ in feet per second}$$

and $$K_{M_0} = -(\text{C.P.})(K_y) \qquad V \text{ in miles per hour}$$

minus signs being used to designate diving moment.

Equations $\left(M_0 = C_{M_0} c \frac{\rho}{2} SV^2\right)$ and $(M_0 = K_{M_0} cSV^2)$ may be transformed to read

$$C_{M_0} = \frac{M_0}{c\frac{\rho}{2}SV^2} \qquad V \text{ in feet per second}$$

and $$K_{M_0} = \frac{M_0}{cSV^2} \qquad V \text{ in miles per hour}$$

Example. Find moment about leading edge of a R.A.F. 15 airfoil of 42-ft. span and 7-ft. chord at a 2° angle of attack with an airspeed of 120 ft. per sec.

Solution.

From Fig. 21, when $\alpha = 2°$

$$C_L = 0.295$$
$$C_D = 0.0156$$
$$\text{C.P.} = 36 \text{ per cent}$$

Then:

Moment about leading edge =

$$0.36 \times (0.295 \cos 2° + 0.0156 \sin 2°) \times 7 \times \frac{0.002378}{2} \times 42 \times 7 \times \overline{120}^2$$
$$= -3740 \text{ ft-lb.}$$

Problems

1. Plot M_c versus angle of attack for an R.A.F. 15 airfoil.

2. What is the moment about the leading edge of an R.A.F. 15 wing of 8 ft. chord and 48-ft. span at a 7° angle of attack with an airspeed of 70 miles per hour?

3. What is the moment about the leading edge of an R.A.F. 15

wing of 45-ft. span and $7\frac{1}{2}$-ft. chord at a 4° angle of attack with an airspeed of 120 miles per hour?

4. What is the moment about the leading edge of an R.A.F. 15 wing of 36-ft. span and 6-ft. chord at a 3° angle of attack with an airspeed of 90 miles per hour?

5. What vertical force must be applied 5 ft. back from the leading edge to prevent rotation of a R.A.F. 15 wing of 54-ft. span and 9-ft. chord at an angle of attack of 0° at an airspeed of 115 miles per hour?

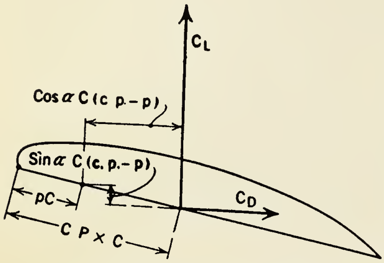


Fig. 31. Moments about point not at leading edge.

Center of Pressure. Instead of finding the moments about the leading edge of the wing, the moment may be found about a point located back of the leading edge. Let pc feet be the distance from the leading edge to this point, where p is in percentage of chord length; see Fig. 31. Then the moment about this point will be

$$-M = (C_L \cos \alpha + C_D \sin \alpha)\frac{\rho}{2}SV^2\,[(\overline{\text{C.P.}} \times c) - pc]$$

the minus sign being used because the force shown in Fig. 31 will cause a diving or negative moment. An approximate form of the above is

$$-M = C_L\frac{\rho}{2}SV^2(\overline{\text{C.P.}} \times c - pc)$$

dividing by $\frac{1}{2}\,\rho SV^2c$ gives

$$\frac{M}{\frac{\rho}{2}SV^2c} = \overline{\text{C.P.}} \times C_L - pC_L$$

letting $\dfrac{M}{\frac{\rho}{2}SV^2c} = C_M$, moment coefficient about point p,

$$-C_M = \overline{\text{C.P.}} \times C_L - pC_L$$

That is, for any angle of attack, the moment coefficient about any point along the chord is the lift coefficient for that angle multiplied by the difference between the position of the center of pressure and the position of the point measured in percentage of chord.

For most airfoils, if, from experimental data, $1/C_L$ is plotted against C.P., a curve resembling that in Fig. 32 results. It closely approximates a straight line up to values of C_L close to the burble point. Any deviations from the straight line may be accounted as due to experimental error. Assuming that it is a

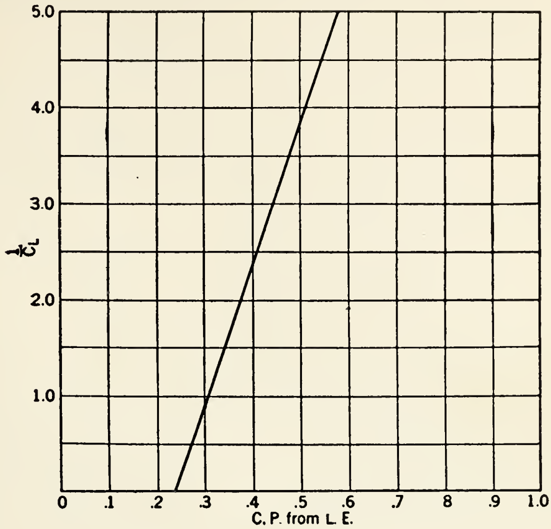


FIG. 32. $1/C_L$ versus C.P. for Clark Y airfoil.

straight line, it represents an equation of the form $x = ay + b$. The intersection of the straight line with the horizontal axis is approximately 0.25 for practically every airfoil investigated. That is, when $1/C_L$ equals zero, C.P. equals 0.25, and since the straight line has a constant slope, from the experimental data, it may be stated that

$$\overline{\text{C.P.}} = 0.25 + k\left(\frac{1}{C_L}\right)$$

or

$$k = C_L(\overline{\text{C.P.}} - 0.25)$$

From the previously derived equation that $-C_M = C_L(\overline{\text{C.P.}} - p)$, where C_M is the moment coefficient about a point that is p per cent of the chord back from the leading edge, the moment coefficient about a point that is a quarter-chord length back from the leading edge is

$$-C_{M0.25} = C_L(\text{C.P.} - 0.25)$$

The constant slope k of the graph of C.P. versus $1/C_L$ is therefore equal to the moment coefficient about the quarter-chord point, $C_{M0.25}$. Since the moment coefficient, and consequently the moment, about the quarter-chord point are constant, this point is called the aerodynamic center of the airfoil. The moment coefficient about the quarter-chord point for various airfoils is tabulated in Table V.

Since
$$-C_{M0.25} = C_L(\text{C.P.} - 0.25)$$
$$\text{C.P.} = 0.25 - \frac{C_{M0.25}}{C_L}$$
substituting this value for C.P. in the equation for moment coefficient about any point p per cent of the chord back from the leading edge gives
$$\begin{aligned} -C_M &= -pC_L + C_L\left(0.25 - \frac{C_{M0.25}}{C_L}\right) \\ &= -pC_L + 0.25\,C_L - C_{M0.25} \end{aligned}$$

An inspection of this relation shows that, when C_L equals zero, the moment coefficient about any point is equal to the moment coefficient about the quarter point. Therefore at the angle of zero lift the moment coefficient about the leading edge is equal to the moment coefficient about the quarter point, which is approximately constant for all angles.
$$C_{M(0.25)} = C_{M0(C_L=0)}$$

Example. An airplane whose wing loading is 8 lb. per sq. ft. is flying level at an airspeed of 100 miles per hour. If at angle of zero lift, $C_{M_0} = -0.067$, what is the center of pressure?

Solution. $V = 100$ miles per hour $= 146.7$ ft. per sec.
$$\begin{aligned} C_L &= \frac{W}{\frac{\rho}{2} S V^2} \\ &= \frac{8}{\frac{0.002378}{2} \times \overline{146.7}^2} \\ &= 0.313 \\ \text{C.P.} &= 0.25 - \frac{C_{M0.25}}{C_L} \\ &= 0.25 - \frac{-0.067}{0.313} \\ &= 0.25 + 0.214 \\ &= 0.46 \end{aligned}$$
or 46 per cent chord length back of leading edge.

Example. At 6° angle of attack, an airfoil has a $C_L = 0.84$, $C_D = 0.06$, and $C_{M_0} = -0.27$. Find position of center of pressure by both approximate and exact methods.

Solution.

$$\text{Approximate C.P.} = \frac{-C_{M_0}}{C_L} = -\left(\frac{-0.27}{0.84}\right) = 0.322$$

$$\text{Exact C.P.} = \frac{-C_{M_0}}{C_L \cos \alpha + C_D \sin \alpha} = \frac{-(-0.27)}{0.84 \times 0.995 + 0.06 \times 0.105} = \frac{+0.27}{0.836 + 0.006} = 0.321$$

Problems

1. An airplane with wing loading of 15 lb. per sq. ft. is flying at an airspeed of 125 miles per hour. At angle of zero lift $C_{M_0} = -0.06$. By approximate method, where is the center of pressure?

2. A monoplane weighing 2,592 lb. with wing span of 36 ft. and wing chord of 6 ft. is flying at a speed of 120 miles per hour. If the center of pressure is 35 per cent of chord length back of the leading edge: (*a*) what is the moment about the leading edge; (*b*) what is the moment coefficient about the leading edge; (*c*) what is the moment about the leading edge at angle of zero lift?

3. An airfoil at 3° angle of attack has a $C_L = 0.65$, $C_D = 0.036$, and $C_{M_0} = -0.24$. Find location of the center of pressure (*a*) by approximate method; (*b*) by exact method. (*c*) Find C_{M_0} at angle of zero lift.

4. At 0° angle of attack a certain airfoil has $C_L = 0.16$ and the center of pressure is 30 per cent of the chord length back from the leading edge. What is the moment coefficient about the leading edge when lift is zero?

5. For a certain airfoil, when lift is zero, the moment coefficient about the leading edge is 0.06; what is C_L when the center of pressure is 40 per cent of chord length back of leading edge?

6. What value of moment coefficient about the leading edge at angle of zero lift will give a center of pressure of 0.52 at a speed of 150 miles per hour for an airplane weighing 8,000 lb. and having a wing area of 750 sq. ft.?

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CHAPTER V

INDUCED DRAG OF MONOPLANES

In the previous chapter, the action of air in flowing over wings was described. All the data given, or used in problems, were, however, for wings having an aspect ratio of 6; i.e., the span was 6 times the chord length.

By chance, early measurements of the lift and drag of airfoils were made on small model wings having an aspect ratio of 6. Later when it was desired to know whether a change in the camber would improve the characteristics, the models of the newer airfoils were made geometrically similar to the older models, i.e., with the same aspect ratio, so that any difference in the characteristics could be attributed solely to the difference in contour of the airfoil. Practically all tests on airfoils are therefore made on models having an aspect ratio of 6, and unless a definite statement is made to the contrary, it may be safely assumed that data published about airfoils refer to airfoils having this standard aspect ratio.

The changes in characteristics due to changes in aspect ratio are now quite definitely known. For convenience, it is becoming more customary to furnish data for airfoils having an infinite aspect ratio. These data can then be corrected to be applicable to wings of any aspect ratio.

Variation of Lift across Span. The motion of the air in flowing around a wing is very complex. Air flows backward over the top side and under the lower side. This motion is complicated because of other incidental conditions.

The air pressure on the upper side of a wing in motion is slightly less than atmospheric pressure; that on the under side is slightly greater than atmospheric pressure. Fluids will always go from a region of high to one of low pressure. Therefore, in flight, air is spilling out from below the wing tip and up into the region of low pressure on top of the wing. Therefore, on the upper side of the wing near the wing tips, the pressure is not quite as low as over the rest of the wing because of the excess coming up from below partly filling this low-pressure area. On the under side of the wing near the tip, because of the air passing out and up, the positive pressure is not as great as under the inner portions of the wing.

Lift, being due to the difference in air pressure between the lower and upper sides of the wing, is not uniform over the span of even a rectangular wing. Tests have been repeatedly run which check with these deductions. Pressure measurements, made at various points across the span, show that difference in pressure is maximum at the center of the span, decreasing by small amounts towards the wing tips.

The distribution of lift along the span of a rectangular wing is shown in Fig. 33*a*. Close to the center of the span the decrease

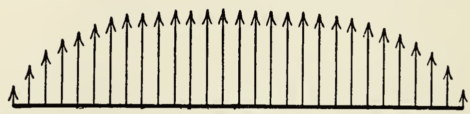


FIG. 33*a*. Lift distribution across span of a rectangular wing.

from maximum is slight. At a greater distance from the center the rate of decrease is more pronounced.

In calculating the necessary strength of the spars the distribution of the supporting forces of lift should be known. The Department of Commerce arbitrarily prescribes that, for internally braced wings, the lift shall be considered uniform from the center of the span to a point one chord length in from the tip, and from there it shall decrease to 80 per cent of the uniform lift (see Fig.

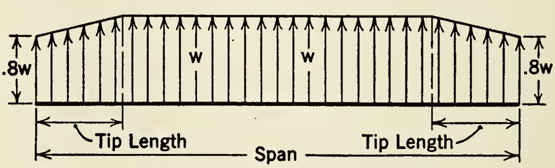


FIG. 33*b*. Department of Commerce lift distribution for internally braced wing.

33*b*). For non-rectangular or externally braced wings, slightly different assumptions are made. Although these lift distributions are not exactly correct, they do make some allowances for the decrease in lift at the wing tips and are sufficiently close for strength computations.

It is to be noted that, with a wing of large aspect ratio, the decrease in lift at the wing tip is less in proportion to the total than with a wing of small aspect ratio. An imaginary wing of infinite aspect ratio, having no wing-tip losses, would have uniform lift across the span.

Inward and Outward Flow. Because, on the under side of the wing, air near the tip is flowing out and upward, air nearer the center flows out to replace this air. The air on the under side of the wing has therefore not only a backward motion relative to the wing but also an outward motion as well. Near the center of the span this outward component is weaker than near the tip. The resultant motion is illustrated in Fig. 34*a*.

On the upper side of the wing, owing to air coming up and inward over the tips, there is an inward component, which is

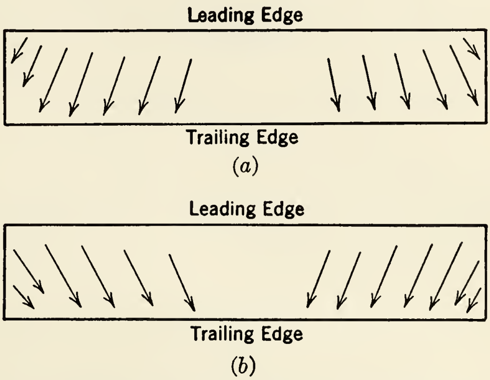


Fig. 34. Air motion (*a*) under side, (*b*) upper side of wing.

strongest at the tips. The resultant of this inward movement of air and the rearward velocity is shown in Fig. 34*b*.

The theoretical wing of infinite aspect ratio would have no inward and outward flow. The flow would be directly backward and downward.

Action at Trailing Edge. At any point on the trailing edge, coming over the upper side of the wing, is a streamline of air which has a direction backward, downward, and inward towards the center of the span, and coming under the lower side of the wing is a streamline of air which has a direction backward and outward. The juxtaposition of these two streamlines at the trailing edge initiates a tiny vortex. These vortices are formed at an infinite number of points along the trailing edge.

On the left wing trailing edge, these vortices are counter-clockwise, viewed from the front; on the right wing they are clockwise. These small vortices, immediately after their formation, merge into two main vortices, one at each wing tip. These vortices are powerful air movements, and in flying in close formation great care must be taken that the wing of a following airplane shall not protrude in a wing-tip vortex of a leading plane.

The effect of the vortex at each wing tip is to give an upward motion to air outside the wing tip and an inward and downward motion to air inside it. Since at each wing tip air is coming inward, the momentums neutralize each other and can be neglected. The downward components of motion are, however, of considerable importance.

On a wing of infinite aspect ratio, there are no wing-tip vortices. Air flowing over and under the wing is given a downward deflection

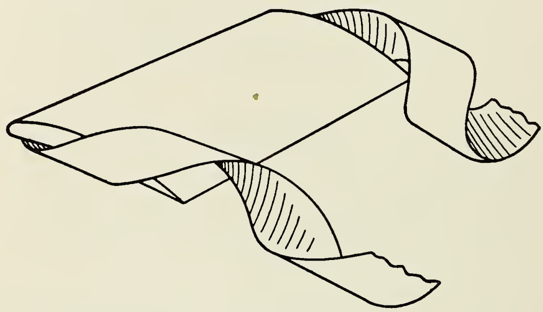


Fig. 35. Wing-tip vortices.

as explained in the last chapter. A streamline, under these conditions, is always in a vertical plane parallel to the vertical plane of symmetry of the airplane. This type of flow is two dimensional and is called profile flow.

With a finite aspect ratio, there are wing-tip vortices. The down flow due to these vortices is in addition to the down flow of the profile flow. With smaller aspect ratio the importance of this down flow due to the wing-tip vortices becomes greater. The downward velocity caused by the wing tips is called the induced velocity and is denoted by the symbol w.

Induced Angle of Attack. The geometric angle of attack of a wing, heretofore referred to simply as the angle of attack, has been defined as the angle between the relative wind and the wing **chord.** The relative wind is the direction from which the air comes in meeting the wing, this direction being the direction of the air stream before it has been disturbed by the approaching wing.

With a wing of finite aspect ratio, the air through which the wing is passing has a downward velocity due to the wing-tip vortices. Then the wing is not traveling through air which has a relative direction exactly opposite to the wing's forward direction, but, instead, through air which has a motion both backward and downward past the wing. The true velocity of the air relative to the wing is the resultant of the two velocities, the backward velocity V, equal in magnitude to the airspeed, and the induced downward velocity w; see Fig. 36a. This resultant may be considered a true relative wind as it is the actual direction of the air passing backward and downward past the wing.

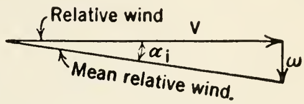


FIG. 36a. Relative wind and mean relative wind.

The downward velocity w is always small with respect to the airspeed V, so that the resultant true velocity is very little different in magnitude from the airspeed V. This slight difference is negligible, and the magnitude of the true velocity is considered to be V.

The angle between relative wind V and the true relative wind is called the induced angle of attack and, though small, is very important. It is represented by the symbol α_i.

The geometric angle of attack α may be considered as being made up of two parts, the induced angle of attack α_i and the effective angle of attack α_0, where

$$\alpha = \alpha_0 + \alpha_i$$

With an infinite aspect ratio, there being no wing-tip vortices, there is no downward motion of the air in which the wing is moving so that the direction of the relative wind measured forward of the wing in undisturbed air is the same as the direction of the relative wind right at the wing. The induced angle of attack is therefore zero, and the effective angle of attack is the same as the geometric angle of attack. For this reason the effective angle of attack is sometimes called the angle of attack for infinite aspect ratio.

A wing of finite aspect ratio in traveling forward horizontally is always moving in air that has a downward motion. The resultant relative wind is therefore the direction of the air which must be considered, and it is the effective angle of attack which determines the air flow and the forces acting on the wing. The lift and drag forces depend on the effective angle of attack rather than the geometric angle of attack.

Two wings, having the same airfoil section, the same area, and moving at the same forward speed but having different aspect ratios, will have the same forces acting over the surface of the wing and the same resultant force provided the effective angle of attack is the same for both wings. The wings having different aspect ratios, the strength of the wing-tip vortices will be different so that the induced angles of attack will be different. The wing with the greater aspect ratio will have the smaller induced angle of attack.

Profile and Induced Drag. In Fig. 36*b*, let C_R be the coefficient of the resultant of all the small forces acting over the upper and lower surfaces of the wing. Resolve C_R into two components, C_{L0} perpendicular to the true or resultant relative wind and C_{D0} parallel to the true relative wind. These would be the lift and drag coefficients for a wing of infinite aspect ratio. For practical wings, the true relative wind does not coincide with the free air direction, and these coefficients must therefore be transformed to a different set of axes.

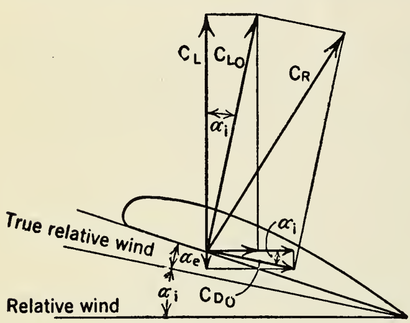


FIG. 36*b*. Induced drag coefficient.

Resolving these forces into components perpendicular and parallel to the undisturbed relative wind, it is found that

$$C_L = C_{L0} \cos \alpha_i - C_{D0} \sin \alpha_i$$

and

$$C_D = C_{L0} \sin \alpha_i + C_{D0} \cos \alpha_i$$

where

$$\alpha_i = \tan^{-1} \frac{w}{V}$$

The induced angle of attack, α_i, is always small, so that $\cos \alpha_i$ may be taken as unity. C_{D0} is small in comparison with C_{L0}, and $\sin \alpha_i$ is a small fraction; therefore $C_{D0} \sin \alpha_i$ is very small in comparison with C_{L0} and may be neglected. If angle α_i is in radians, for small angles the sine is equal to the angle itself; i.e., $\alpha_i = \sin \alpha_i$. The equations thus become:

$$C_L = C_{L0}$$

and

$$C_D = C_L\alpha_i + C_{D0}$$

or

$$C_D = C_{Di} + C_{D0} \qquad \text{where } C_{Di} = C_L\alpha_i$$

The drag coefficient is composed of two parts. One part, represented by the coefficient C_{Di}, is caused by the induced downward velocity of the wing-tip vortices and is called the induced drag coefficient. The other part, C_{D0}, is called the profile drag coefficient. In the same way, the drag itself is divided into two corresponding parts: the induced drag equal to $C_{Di}\,\frac{\rho}{2}SV^2$ and the profile drag equal to $C_{D0}\,\frac{\rho}{2}SV^2$.

When the wing has an infinite aspect ratio, there is no induced drag and all the drag is that due to the profile or two-dimensional flow. The drag is then due only to the skin friction of the air with the surface of the airfoil. The amount of this drag depends chiefly on the shape of the airfoil at angles below where excessive burbling takes place. It is because of the profile of the airfoil determining its magnitude that D_0 is called profile drag. Profile drag is independent of angle of attack, up to the burble point, and is independent of aspect ratio.

With finite aspect ratio, in addition to the profile drag, there is the induced drag. The coefficient of induced drag depends on the lift — which depends on the angle of attack — and on the induced angle of attack, which depends on the effect of the wing-tip vortices, which in turn depends on the aspect ratio.

The induced drag, being a component of lift, depends not only on the lift but also on the slant of the lift backward from the perpendicular to the relative wind. Dr. M. Munk has proved that this angle, α_i, is minimum when the downwash is constant along the span, and this constant downwash is obtained when the lift distribution along the span varies in such a manner that, when plotted with the lift as ordinates and distance along span as abscissas, the resulting curve is a semi-ellipse with the span as the axis.

When a wing moves forward the air which comes in immediate contact with the surface is affected and has motion imparted to it.

Farther away from the wing the air is affected less. At great distances above or below the wing, there is still an effect on the air but so slight as to be negligible. The actual situation is that air is affected by a varying amount from close to the wing to infinite distances away from the wing where the effect is zero, but for the purpose of calculation it is assumed that the effect is the same as if a definite area is affected uniformly. This fictitious area of uniform effect is termed the *swept area* and is denoted by S'. It is a cross-section of the air stream taken perpendicular to the direction of motion of the wing.

Then the mass of air affected by the wing in unit time is $\rho S'V$. The downward momentum imparted to this mass in unit time is $\rho S'V \times V\alpha_i$ where $V\alpha_i$ is considered equal to w, the downward velocity; it is measured in radians. It has been proved mathematically that the downward momentum in unit time is equal to one-half the lift, i.e.,

$$\rho S'V^2\alpha_i = \frac{L}{2}$$

Then

$$\alpha_i = \frac{L}{2\,\rho S'V^2}$$

Prandtl has shown that, with semi-elliptic lift distribution, the swept area is a circle whose diameter is the span. Then

$$S' = \frac{\pi b^2}{4}$$

$$\alpha_i = \frac{L}{2\,\rho\frac{\pi b^2}{4}V^2} = \frac{C_L\frac{\rho}{2}SV^2}{2\,\rho\frac{\pi b^2}{4}V^2} = \frac{C_LS}{\pi b^2}$$

But by definition aspect ratio is equal to b^2/S. Therefore

$$\alpha_i = \frac{C_L}{\pi\text{ A.R.}}$$

also

$$C_{Di} = C_L\alpha_i$$
$$= \frac{C_L^2S}{\pi b^2}$$
$$= \frac{C_L^2}{\pi\text{ A.R.}}$$

It must be emphasized that these two expressions are true only when the lift plotted against span is a semi-elliptic curve. This makes α_i the same all across the span. This particular lift distribution is obtained from wings that are elliptic in plan view. For wings of other shape, such as rectangular or tapered wings, these expressions are not exactly correct. The error in using them is very slight, however, and they are commonly employed without any corrections.*

The expression given above for α_i gives the induced angle of attack in radians. Since 1 radian is 57.3°

$$\alpha_i \text{ (degrees)} = \frac{C_L}{\pi \text{ A.R.}} \times 57.3$$
$$= \frac{18.24\, C_L}{\text{A.R.}}$$

The induced drag coefficient being known, the induced drag may be found, as follows

$$D_i = C_{Di}\frac{\rho}{2}SV^2$$
$$= \left[\frac{C_L{}^2S}{\pi b^2}\right]\frac{\rho}{2}SV^2$$
$$= \frac{C_L{}^2\left(\frac{\rho}{2}\right)^2 S^2V^4}{\pi\frac{\rho}{2}b^2V^2}$$

* Glauert has shown that, for rectangular wings, more nearly correct formulas are

$$\alpha_i = \frac{C_L}{\pi \text{ A.R.}}(1 + T)$$

and

$$C_{Di} = \frac{C_L{}^2}{\pi \text{ A.R.}}(1 + S)$$

where the correction factors T and S vary with aspect ratio as follows:

A.R.	T	S	A.R.	T	S
3	0.11	0.022	7	0.20	0.064
4	.14	.033	8	.22	.074
5	.16	.044	9	.23	.083
6	.18	.054			

The neglect of these correction factors never results in an error exceeding 5 per cent.

$$= \frac{L^2}{\pi \frac{\rho}{2} b^2 V^2} \qquad V \text{ in feet per second}$$

$$= \frac{0.148\, L^2}{\frac{\rho}{2} b^2 V^2} \qquad V \text{ in miles per hour}$$

For level flight, the lift is always substantially equal to the weight. Therefore for level flight

$$D_i = \frac{1}{\pi q}\left(\frac{W}{b}\right)^2 \qquad q = \frac{\rho}{2} V^2$$

The weight divided by the span is called the span loading. For level flight the induced drag varies directly with the square of the span loading and inversely as the dynamic pressure.

At sea-level (i.e., standard density)

$$D_i = \frac{1}{\pi \times 0.00119\, V^2}\left(\frac{W}{b}\right)^2$$

$$= \frac{268\left(\frac{W}{b}\right)^2}{V^2} \qquad V \text{ in feet per second}$$

$$= \frac{125\left(\frac{W}{b}\right)^2}{V^2} \qquad V \text{ in miles per hour}$$

The horsepower required to overcome induced drag is

$$\text{H.P.}_{Di} = \frac{D_i V}{550} \qquad V \text{ in feet per second}$$

$$= \left[\frac{W^2}{\pi \frac{\rho}{2} b^2 V^2}\right] \frac{V}{550}$$

$$= \frac{W^2}{1{,}730 \frac{\rho}{2} b^2 V}$$

$$= \frac{0.00116\, W^2}{\rho V b^2}$$

At standard density, this becomes

$$\text{H.P.}_{Di} = \frac{0.488\left(\frac{W}{b}\right)^2}{V} \qquad V \text{ in feet per second}$$

or

$$\text{H.P.}_{Di} = \frac{125\left(\frac{W}{b}\right)^2}{375\ V}$$

$$= \frac{\left(\frac{W}{b}\right)^2}{3\ V} \qquad V \text{ in miles per hour}$$

That is, the horsepower required at sea-level to overcome the induced drag is one-third the span loading squared divided by the velocity in miles per hour. At any altitude the horsepower required to overcome induced drag is the horsepower required at sea-level, at that speed, divided by the relative density at that altitude:

$$\text{H.P.}_{Di} = \frac{\rho_0}{\rho} \times \frac{\left(\frac{W}{b}\right)^2}{3\ V} \qquad V \text{ in miles per hour}$$

Example. A rectangular monoplane wing has a span of 39 ft. and a chord of 6 ft. What are the induced angle of attack and the induced drag coefficient, when the lift coefficient is 0.8?

Solution.

$$\text{A.R.} = \frac{39}{6} = 6.5$$

$$\alpha_i \text{ (degrees)} = \frac{18.24 \times 0.8}{6.5} = 2.22°$$

$$C_{Di} = \frac{0.8^2}{\pi \times 6.5} = 0.031$$

Example. A monoplane weighing 2,000 lb. has a span of 38 ft. What is the induced drag at 10,000-ft. altitude if the airspeed is 80 miles per hour? What horsepower is required to overcome the induced drag?

Solution.

$$
\begin{aligned}
80 \text{ miles per hour} &= 117 \text{ ft. per sec.} \\
\text{Span loading} &= \frac{W}{b} \\
&= \frac{2{,}000}{38} \\
&= 52.6 \text{ lb. per ft.} \\
D_i &= \frac{\overline{52.6}^2}{\pi \times \frac{1}{2} \times 0.00176 \times 117^2} \\
&= 73 \text{ lb.} \\
\text{H.P.}_{Di} &= \frac{0.00116 \times \overline{52.6}^2}{0.00176 \times 117} \\
&= 15.5 \text{ hp.}
\end{aligned}
$$

Problems

1. A rectangular monoplane wing has 42-ft. span and 5-ft. chord. When $C_L = 0.65$, what are the induced angle of attack and the induced drag coefficient?

2. A rectangular monoplane wing has 38-ft. span and 7-ft. chord. When $C_L = 0.72$, what are the induced angle of attack and the induced drag coefficient?

3. At an airspeed of 90 miles per hour at sea-level what is the induced drag of a monoplane weighing 4,700 lb. and having a wing span of 52 ft.?

4. The Northrop Delta weighs 7,000 lb., and its span is 48 ft. What horsepower is required to overcome induced drag of wing when flying at sea-level at 200 miles per hour?

5. The Taylor Cub weighs 925 lb., and its wing span is 35 ft. $2\frac{1}{2}$ in. What horsepower is required at sea-level to overcome induced wing drag at an airspeed of 60 miles per hour?

6. The Lockheed Vega weighs 4,750 lb.; its wing span is 41 ft. At sea-level, what is the induced drag at 210 ft. per sec.?

7. A Stinson Reliant weighs 3,125 lb.; its wing span is 43 ft. $3\frac{1}{8}$ in. At 5,000 ft., what is induced drag at 150 ft. per sec.?

8. A Fairchild monoplane weighs 1,600 lb.; its wing span is 32 ft. 10 in. At 10,000-ft. altitude what horsepower is required to overcome induced drag at 110 miles per hour?

9. A monoplane weighs 3,000 lb. At sea-level, flying at 150 ft. per sec., what is the induced drag (*a*) if span is 35 ft.; (*b*) if span is 30 ft.; (*c*) if span is 25 ft.?

10. A monoplane weighs 3,000 lb.; its span is 30 ft. What is the induced drag, at sea-level, (*a*) at airspeed of 100 ft. per sec.; (*b*) at 200 ft. per sec.?

If engineering coefficients are used, since $K_y = 0.00256\ C_L$, the expression for α_i becomes

$$\alpha_i \text{ (degrees)} = \frac{18.24\ K_y}{0.00256 \text{ A.R.}}$$
$$= \frac{7130\ K_y}{\text{A.R.}}$$

Also in engineering coefficients, the coefficient for induced drag is K_{xi}, where $K_{xi} = 0.00256\ C_{Di}$.

Since

$$C_{Di} = \frac{C_L^2}{\pi \text{ A.R.}}$$
$$0.00256\ C_{Di} = 0.00256 \left[\frac{\left(\frac{K_y}{0.00256}\right)^2}{\pi \text{ A.R.}} \right]$$
$$K_{xi} = \frac{K_y^2}{0.00256\ \pi \text{ A.R.}}$$
$$= \frac{125\ K_y^2}{\text{A.R.}}$$

Corrections for Aspect Ratio of Monoplane. As shown in a previous paragraph, the geometric angle of attack is made up of two parts, the effective angle of attack and the induced angle of

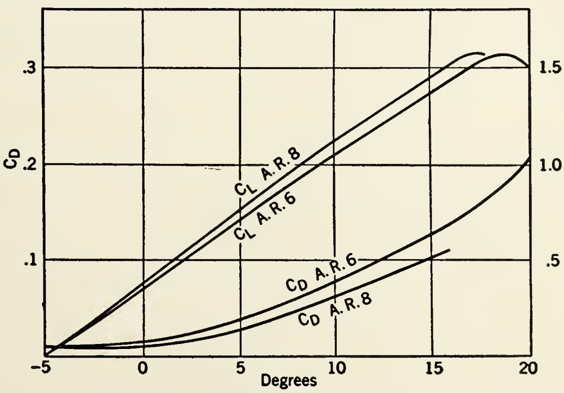


FIG. 37. Effect of aspect ratio on Clark Y characteristics.

attack. Two wings of the same area, same airfoil section, and same airspeed will have the same lift, provided the effective angle of attack is the same in both cases. If these two wings have a different aspect ratio, the one having the smaller aspect ratio will have a larger induced angle of attack and will consequently need to have a greater geometric or total angle of attack.

These two wings having the same effective angle of attack will have the same profile drag, but since the induced drag is greater on the wing of smaller aspect ratio, the total drag will be greater on that wing.

If the total angle of attack is known that gives a certain lift coefficient with a wing of one aspect ratio, the total or geometric angle of attack that will be needed for a wing of a different aspect ratio to give the same lift coefficient can be found as follows. The difference between the geometric angles will be the difference between the induced angles. Let A and B be the two aspect ratios, α_A being the total angle of attack for wing of aspect ratio A which gives the same lift as the total angle of attack α_B gives for the wing of aspect ratio B.

Then

$$\alpha_A - \alpha_B = \frac{18.24\ C_L}{A} - \frac{18.24\ C_L}{B}$$

$$= 18.24\ C_L\left[\frac{1}{A} - \frac{1}{B}\right]$$

The difference in total drags will be the same as the difference in induced drags. Then for the two wings of the preceding paragraph, A and B being the two aspect ratios and the wings being the same airfoil section and same area, the total drags for each will be

$$C_{D(A)} = C_{D0(A)} + C_{Di(A)}$$

$$C_{D(B)} = C_{D0(B)} + C_{Di(B)}$$

The profile drags, being independent of aspect ratio, are the same for both wings. Then

$$C_{D(A)} - C_{D(B)} = C_{Di(A)} - C_{Di(B)}$$

$$= \frac{C_L^2}{\pi A} - \frac{C_L^2}{\pi B}$$

$$= \frac{C_L^2}{\pi}\left(\frac{1}{A} - \frac{1}{B}\right)$$

Example. An airfoil, with aspect ratio of 6, at an angle of attack of 3°, has a $C_L = 0.381$ and $C_D = 0.0170$; find, for the same airfoil section, the angle of attack and the C_D that will correspond with the C_L of 0.381 if the aspect ratio is 4.

Solution.

Approximate:

$$\begin{aligned} \alpha_6 - \alpha_4 &= 18.24 \times 0.381 \left(\tfrac{1}{6} - \tfrac{1}{4}\right) \\ &= -0.579 \\ 3^\circ - (-0.579) &= 3.579^\circ = \text{angle of attack for A.R. 4} \\ C_{D(6)} - C_{D(4)} &= \frac{0.381^2}{\pi} \times \left(\tfrac{1}{6} - \tfrac{1}{4}\right) \\ &= -0.00385 \\ 0.0170 - (-0.00385) &= 0.0208 \ C_D \text{ for A.R. 4} \end{aligned}$$

Exact (using Glauert corrections):

$$\begin{aligned} \alpha_6 - \alpha_4 &= 18.24 \times 0.381 \left(\frac{1.18}{6} - \frac{1.14}{4}\right) \\ &= -0.611 \\ C_{D(6)} - C_{D(4)} &= \frac{0.381^2}{\pi} \left(\frac{1.054}{6} - \frac{1.033}{4}\right) \\ &= -0.0038 \end{aligned}$$

Coefficients for Infinite Aspect Ratio. With infinite aspect ratio, there is neither induced angle of attack nor induced drag. Lately it has become customary to give the characteristics of an airfoil as if the airfoil were of infinite aspect ratio. When they are so given, the angle of attack is identical with the effective angle of attack and the drag coefficient is the profile drag coefficient. To find the characteristics for a finite aspect ratio, to the effective angle of attack is added the induced angle of attack for that aspect ratio to give the geometric angle of attack, and to the profile drag coefficient is added the induced drag coefficient for that aspect ratio to give the total drag coefficient.

Example. For a certain airfoil of infinite aspect ratio at 9° angle of attack, the C_L is 1.03 and C_D is 0.067. Find corresponding characteristics for an aspect ratio of 8.

Solution.

Approximate:

$$\begin{aligned} \alpha &= \alpha_0 + \alpha_i \\ &= 9^\circ + \frac{18.24 \times 1.03}{8} \\ &= 9^\circ + 2.3^\circ \\ &= 11.3^\circ \\ C_D &= C_{D0} + C_{Di} \\ &= 0.067 + \frac{1.03^2}{\pi \times 8} \\ &= 0.067 + 0.042 \\ &= 0.109 \end{aligned}$$

Wing of aspect ratio 8 at angle of attack of 11.3° will have lift coefficient of 1.03 and drag coefficient of 0.109.

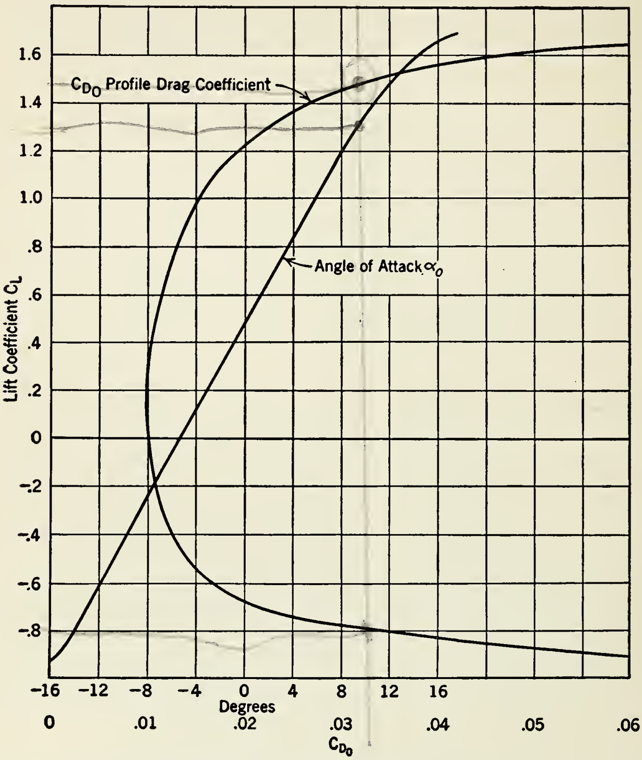


FIG. 38. Characteristics of Clark Y airfoil, infinite aspect ratio.

Example. From Fig. 18, the lift and drag coefficients for the Göttingen 398 airfoil for an aspect ratio of 6, at an angle of attack of 2°, is $C_L = 0.293$ and $C_D = 0.031$. Find corresponding characteristics for infinite aspect ratio.

Solution.

$$\alpha_{(A.R.6)} = \alpha_{0(A.R.6)} + \alpha_{i(A.R.6)}$$

$$2^\circ = \alpha_0 + \frac{18.24 \times 0.293}{6}$$

$$\alpha_0 = 2^\circ - 0.89$$

$$= 1.11^\circ, \text{ angle of attack for infinite aspect ratio to give } C_L \text{ of } 0.293.$$

$$C_{D(\text{A.R.6})} = C_{D0} + C_{Di(\text{A.R.6})}$$

$$0.031 = C_{D0} + \frac{\overline{0.293}^2}{\pi \times 6}$$

$$C_{D0} = 0.031 - 0.0045$$

$$= 0.026 \text{ drag coefficient for infinite aspect ratio}$$

$$\text{when} \quad C_L = 0.293 \text{ (i.e., angle of attack } 1.11°\text{).}$$

Example. A certain airfoil with aspect ratio of 6, $K_y = 0.00245$ and $K_x = 0.00017$, at 6° angle of attack; with aspect ratio of 9, what will be the angle of attack for K_y of 0.00245, and what will be K_x?

$$\alpha_{(\text{A.R.6})} - \alpha_{(\text{A.R.9})} = 7130 \times 0.00245 \left(\tfrac{1}{6} - \tfrac{1}{9}\right)$$

$$= 0.97°$$

$$\alpha_{(\text{A.R.9})} = 5.03° \text{ for } K_y = 0.00245$$

$$K_{x(\text{A.R.6})} - K_{x(\text{A.R.9})} = 125 \times \overline{0.00245}^2 \left(\tfrac{1}{6} - \tfrac{1}{9}\right)$$

$$= 0.0000417$$

$$K_{x(\text{A.R.9})} = 0.00017 - 0.00004$$

$$= 0.00013$$

By using these formulas, if the characteristic curves for one aspect ratio are known, the characteristic curves for any other aspect ratio can be calculated. Figure 37 shows the lift and drag coefficient curves for an aspect of 8, as calculated from data for an aspect ratio of 6.

Note that variation in the aspect ratio has no effect on the angle of zero lift.

Problems

1. An airfoil of infinite aspect ratio has a C_D of 0.01 when C_L is 0.7. (*a*) What is the C_D for a similar airfoil with aspect ratio of 9, when C_L is 0.7? (*b*) What is the L/D ratio?

2. For the same airfoil section as in problem 1, but with an aspect ratio of 8: (*a*) what is C_D when C_L is 0.7; (*b*) what is the L/D ratio?

3. For the same airfoil section as in problem 1, but with an aspect ratio of 6: (*a*) what is C_D when C_L is 0.7; (*b*) what is the L/D ratio?

4. An airfoil with aspect ratio of 6, at 2° angle of attack has C_L = 0.78 and $C_D = 0.047$. (*a*) At what angle of attack will this airfoil have the same C_L if the aspect ratio is infinity? (*b*) What will be the C_D under these conditions?

5. An airfoil with aspect ratio of 6, at 10° angle of attack has C_L = 1.32 and $C_D = 0.110$. (*a*) With an aspect ratio of 9, at what angle of attack will $C_L = 1.32$? (*b*) What will be C_D? (*c*) What will be L/D?

6. An airfoil with aspect ratio of 6, at 8° angle of attack has $C_L = 0.85$ and $C_D = 0.046$. (*a*) With aspect ratio of 8, at what angle of attack will $C_L = 0.85$? (*b*) What will be C_D?

7. An airfoil with aspect ratio of 6, at 0° angle of attack has $C_L = 0.64$ and $C_D = 0.035$. (*a*) With aspect ratio of 8.5, at what angle of attack will $C_L = 0.64$? (*b*) What will be C_D?

8. An airfoil with aspect ratio of 6, at 1° angle of attack has $C_L = 0.58$ and $C_D = 0.031$. (*a*) With aspect ratio of 4, at what angle of attack will $C_L = 0.58$? (*b*) What will be C_D?

9. An airfoil of aspect ratio of 6, at angle of attack of 8°, has $K_y = 0.0025$ and $K_x = 0.000175$. (*a*) With aspect ratio of 8, at what angle of attack will $K_y = 0.0025$? (*b*) What will be K_x?

10. From Fig. 28 obtain the data and plot the lift and drag coefficient curves for an aspect ratio of 8.5.

Total Drag of Monoplane Wings from Model Data. In practical calculations, it is often desired to find the drag of a wing from the characteristics of the model, without knowing the corresponding velocities. The L/D of the monoplane can be found, and then use is made of the fact that, in level flight, lift equals weight.

The formula for induced drag

$$D_i = \frac{L^2}{\pi q b^2}$$

may be rewritten as

$$D_i = \frac{L\left(C_L \frac{\rho}{2} S V^2\right)}{\pi \frac{\rho}{2} V^2 b^2}$$

$$= \frac{L \times C_L}{\pi \frac{b^2}{S}}$$

$$= \frac{L \times C_L}{\pi \text{ (A.R.)}}$$

The drag being composed of two parts, profile drag independent of aspect ratio and induced drag dependent on aspect ratio, the drag of any monoplane may be written as follows

$$C_D \text{ (monoplane)} = C_D \text{ (model)} - C_{Di} \text{ (model)} + C_{Di} \text{ (monoplane)}$$

$$= C_D \text{ (model)} - \frac{C_L^2}{\pi \text{ (A.R.}_{\text{model}})} + \frac{C_L^2}{\pi \text{ (A.R.}_{\text{monoplane}})}$$

then

$$\left(\frac{L}{D}\right) \text{monoplane} = \frac{1}{\left(\frac{D}{L}\right)_{\text{model}} - \frac{C_L}{\pi}\left(\frac{1}{\text{A.R.}_{\text{model}}} - \frac{1}{\text{A.R.}_{\text{monoplane}}}\right)}$$

Knowledge of L/D of the monoplane wing enables the drag to be found quickly, since, in level flight, lift substantially equals weight.

Example. An airplane weighs 2,000 lb.; the wing area is 180 sq. ft.; the wing span is 39 ft. What is the wing drag at 4° angle of attack? Wing section is U.S.A.-35A.

Solution. From Fig. 27 at 4° angle of attack $C_L = 0.84$, $C_D = 0.058$.

$$\left(\frac{D}{L}\right) \text{model} = \frac{0.058}{0.84} = 0.069$$

$$\text{A.R. monoplane} = \frac{\overline{39}^2}{180} = 8.45$$

$$\frac{L}{D} = \frac{1}{0.069 - \frac{0.84}{\pi}\left(\frac{1}{6} - \frac{1}{8.45}\right)}$$

$$= \frac{1}{0.069 - 0.0129}$$

$$= \frac{1}{0.0561}$$

Since lift is equal to weight

$$\frac{2{,}000}{D} = \frac{1}{0.0561}$$

$$D = 2{,}000 \times 0.0561$$

$$= 112.2 \text{ lb.}$$

It is to be noted that span is always the distance from wing tip to wing tip. The term chord used in connection with the aspect ratio of a tapered wing refers to the average or mean arithmetic chord and should not be confused with the mean geometric or mean aerodynamic chords. The wing area is total area including ailerons and is assumed to extend through the fuselage.

Problems

1. A monoplane weighing 3,000 lb. has a U.S.A.-35A wing, of 35-ft. span, 4-ft. chord. What is the wing drag at 2° angle of attack?

2. A monoplane weighing 2,600 lb. has a U.S.A.-35A wing, of 27-ft. span, $3\frac{1}{2}$-ft. chord. What is the wing drag at 6° angle of attack?

3. A monoplane weighing 700 lb. has a U.S.A.-35A wing of 36-ft. span and 4-ft. chord. What is the wing drag at 8° angle of attack?

4. A monoplane weighing 2,700 lb. has a rectangular wing of 42-ft. 5-in. span and 6-ft. 0-in. chord. If Göttingen 398 airfoil is used, what is the wing drag at 5° angle of attack?

5. A monoplane weighing 9,300 lb. has a Göttingen 398 wing 574 sq. ft. in area. The span is 66 ft. What is the wing drag at 8° angle of attack?

6. A monoplane whose weight is 5,400 lb. has a C-80 wing 285 sq. ft. in area. The span is 44 ft. What is the wing drag at the same angle of attack for which model airfoil (A.R. 6) has a C_L of 0.7?

7. A monoplane whose weight is 4,360 lb. has a Clark Y wing 292 sq. ft. in area. The span is 42 ft. 9 in. What is the wing drag at 8° angle of attack?

8. A monoplane weighing 7,000 lb. has a U.S.A.-35A wing 363 sq. ft. in area. Span is 48 ft. What is the wing drag at 10° angle of attack?

9. A racing plane weighing 6,500 lb. has a C-80 wing of 211 sq. ft. in area. Span is 34 ft. 3 in. What is the wing drag at 2° angle of attack?

10. What is the wing drag of the airplane in problem 9 at 10° angle of attack?

Effect of Taper on Induced Drag. Whenever the term area has been used in connection with aspect ratio, it refers to the total area of a wing including ailerons. Nothing is subtracted for fuselage or nacelle; the leading edge is assumed to continue across in a straight line from where the leading edges of each wing intersect the fuselage, and the same assumption is made for the trailing edge.

With the airfoil section decided upon, the area is fixed by the landing speed. The largest possible span will, of course, give the least possible induced drag. The long wing spars necessary for a large span will have to be very deep to prevent sagging or hogging, so that there are very practical limits to the length of span. Rectangular wings are cheapest to manufacture since each rib is identical; however, there are many advantages to tapering a wing.

There are three ways of tapering wings. The first is tapering in plan form only. This means using the same airfoil section throughout the wing. The chord is lessened from the root to the tip; this entails decreasing the thickness from root to tip. The second method is to taper in thickness only. This means a rectangular wing, the chord remaining the same from root to tip. The airfoil section is changed from a thick wing at the root to a thin wing at the tip. The third method is a combination of the first two.

The first method of tapering in plan form is the only one commonly used. Besides constructional advantages and improvement in performance, if the upper surface is flat, the under surface slopes outward and upward, which gives an effective dihedral. This feature is explained in the chapter on stability.

By decreasing the chord near the tip, the lift and drag loads are made small at the outer extremities of the wing spars. This makes it possible to design a spar that is properly effective, being deep where the greatest stresses occur.

Example. When $C_L = 0.9$, find the difference in induced drag for a rectangular wing of 36-ft. span and 6-ft. chord and a tapered wing of the same area, the tapered wing having a 6-ft. chord at the root, the leading and trailing edges being tangent to a circle of 2-ft. radius at the tip, and the fuselage being 3 ft. wide. The airspeed is 100 miles per hour.

Solution.

Area $= S = 36 \times 6 = 216$ sq. ft.

Rectangular wing:

$$D_i = \frac{C_L^2 \frac{\rho}{2} S V^2}{\pi \times \text{A.R.}}$$

$$= \frac{\overline{0.9}^2 \times 0.00256 \times 216 \times \overline{100}^2}{\pi \times 6}$$

$$= 236 \text{ lb.}$$

By laying-out tapered wing conforming to above description, span is found to be 44 ft.

$$D_i = \frac{\overline{0.9}^2 \times 0.00256 \times 216 \times \overline{100}^2}{\pi \times \frac{\overline{44}^2}{216}}$$

$$= 158 \text{ lb.}$$

Problems

1. When C_L is 1.2, what is the induced drag coefficient of a wing which has a span of 40 ft.? The chord at the root is 5 ft. Both leading and trailing edges are tangent to the arc of a circle of $1\frac{1}{2}$-ft. radius at the tip. The fuselage is 3 ft. wide.

2. When C_L is 0.9, what is induced drag coefficient of a wing of 74-ft. 0 in. span? The chord at root is 15 ft. 0 in. Both leading and trailing edges are tangent to circular tips, 3-ft. 6 in. radius. Fuselage is 4 ft. wide.

3. When C_L is 0.8, what is induced drag coefficient of a Douglas Transport whose span is 85 ft. 0 in.? The center section of the wing is 28 ft. 0 in. wide and has a 17-ft. 0 in. chord. The trailing edge is straight. The leading edge sweeps back. Both leading and trailing edges are tangent to circular tips of 3-ft. 0 in. radius.

4. When C_L is 0.85, what is induced drag coefficient of a General Aviation Transport wing of 53-ft. 0 in. span? Fuselage is 4 ft. 6 in. wide. Root chord is 11 ft. 0 in. Wing is tapered from fuselage out, both leading and trailing edges being tangent to circular tip of 2-ft. 0 in. radius.

5. When C_L is 0.9, what is induced drag coefficient of Northrop Delta wing? Span is 48 ft. 0 in. Rectangular center section is 11 ft. 0 in. wide. Root chord is 9 ft. 0 in. Wing tapers from center section to tip, leading and trailing edges being tangent to circular tip of 2-ft. 0 in. radius.

Correction Factor for Aspect Ratio. By making certain approximations, a correction factor may be obtained by which a correction may be applied to the angle of attack of a model airfoil to obtain the angle of attack with the same lift coefficient of an airfoil with a differing aspect ratio.

Earlier in this chapter it was pointed out that the change in angle of attack for a change in aspect ratio is due to the change in the induced angle of attack.

$$\alpha' = \alpha - \frac{57.3\, C_L}{\pi\,\text{A.R.}} + \frac{57.3\, C_L}{\pi\,\text{A.R.}'}$$

α = original angle of attack (in degrees)
α' = angle of attack for new aspect ratio
A.R. = original aspect ratio
A.R.′ = new aspect ratio

It will be noticed that, when C_L is plotted against angle of attack, the curve is a straight line from the angle of zero lift up to near the burble point. For this portion of the curve the following is true

$$C_L = \frac{\Delta C_L}{\Delta \alpha} \times \alpha_{Z.L.}$$

$\frac{\Delta C_L}{\Delta \alpha}$ = slope of lift curve

$\alpha_{Z.L.}$ = angle of attack measured from angle of zero lift

Substituting this value of C_L in the previous equation:

$$\alpha'_{Z.L.} = \alpha_{Z.L.} - \frac{57.3\left(\frac{\Delta C_L}{\Delta \alpha}\right)\alpha_{Z.L.}}{\pi \text{ A.R.}} + \frac{57.3\left(\frac{\Delta C_L}{\Delta \alpha}\right)\alpha_{Z.L.}}{\pi \text{ A.R.}'}$$

Examining a large number of lift curves for airfoils with aspect ratio of 6, it will be found that the slope is practically the same and $\Delta C_L/\Delta\alpha = 0.0718$. Substituting this value of slope and the value of 6 for original aspect ratio the equation becomes

$$\begin{aligned}
\alpha'_{Z.L.} &= \alpha_{Z.L.} - \frac{57.3 \times 0.0718 \times \alpha_{Z.L.}}{\pi\, 6} + \frac{57.3 \times 0.0718 \times \alpha_{Z.L.}}{\pi \text{ A.R.}'} \\
&= \alpha_{Z.L.} - \frac{57.3 \times 0.0718 \times \alpha_{Z.L.}}{\pi}\left(\frac{1}{6} - \frac{1}{\text{A.R.}'}\right) \\
&= \alpha_{Z.L.} - 1.311\, \alpha_{Z.L.}\left(\frac{1}{6} - \frac{1}{\text{A.R.}'}\right) \\
&= \alpha_{Z.L.}\left[1 - 1.311\left(\frac{1}{6} - \frac{1}{\text{A.R.}'}\right)\right] \\
&= \alpha_{Z.L.}\left(0.782 + \frac{1.311}{\text{A.R.}'}\right) \\
&= \frac{\frac{\alpha_{Z.L.}}{1}}{0.782 + \frac{1.311}{\text{A.R.}'}} \\
&= \frac{\frac{\alpha_{Z.L.}}{\text{A.R.}'}}{0.782 \text{ A.R.}' + 1.311}
\end{aligned}$$

or

$$\alpha'_{Z.L.} = \frac{\alpha_{Z.L.}}{F_{A.R.}} \qquad \text{where} \qquad F_{A.R.} = \frac{\text{A.R.}'}{0.782 \text{ A.R.}' + 1.311}$$

This factor $F_{A.R.}$ is much used in stability computations, especially in connection with finding the reactions at the tail surfaces. It is rarely necessary to consider the condition of large angle of attack of the tail surfaces, and the small errors incidental to the use of this approximation are unimportant. The factor $F_{A.R.}$ is plotted against aspect ratio in Fig. 39.

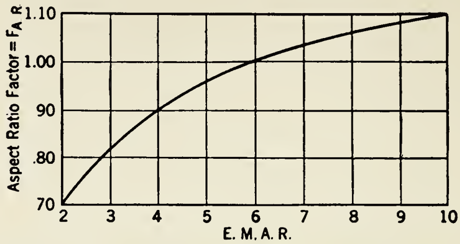


FIG. 39. $F_{A.R.}$ plotted against aspect ratio.

This factor, $F_{A.R.}$, is used by the Army Air Corps and is based on the average slope of a large number of airfoils being 0.0718, where α is measured in degrees. The Department of Commerce advocates a slightly different value, based on theoretical results deduced for thin airfoils.

The Department of Commerce uses the symbol m to represent the slope of the lift curve: $m = \Delta C_L/\Delta\alpha$, where α is measured in radians. From the theory of thin airfoils, the slope of the lift curve for a wing of infinite aspect ratio is 2π.

$$\frac{\Delta C_L}{\Delta\alpha_\infty} = m_\infty = 2\pi$$

Then
$$\Delta C_L = 2\pi\,\Delta\alpha_\infty$$

but
$$\alpha = \alpha_\infty + \frac{C_L}{\pi \times \text{A.R.}}$$

or
$$\Delta\alpha = \Delta\alpha_\infty + \frac{\Delta C_L}{\pi \times \text{A.R.}}$$

Therefore
$$\Delta\alpha = \Delta\alpha_\infty + \frac{2\pi\,\Delta\alpha_\infty}{\pi \times \text{A.R.}}$$

$$= \Delta\alpha_\infty\left(1 + \frac{2}{\text{A.R.}}\right)$$

Then, for aspect ratio of 6,

$$\Delta\alpha_6 = \Delta\alpha_\infty\left(1 + \frac{2}{6}\right)$$

$$= \frac{4}{3}\Delta\alpha_\infty$$

or
$$\Delta\alpha_\infty = \frac{3}{4}\Delta\alpha_6$$

Then $m_6 = \frac{3}{4} m_\infty = \frac{3}{4}(2\pi)$ if α is in radians, or $m_6 = \frac{3\pi}{2 \times 57.3}$ $= 0.0822$ if α is in degrees.

Returning to the earlier equation

$$\Delta\alpha = \Delta\alpha_\infty \left(1 + \frac{2}{\text{A.R.}}\right)$$

Substituting from above

$$\Delta\alpha = \frac{3}{4}\Delta\alpha_6\left(1 + \frac{2}{\text{A.R.}}\right)$$

$$= \Delta\alpha_6 \frac{3 + \frac{6}{\text{A.R.}}}{4}$$

Then

$$\frac{\Delta C_L}{\Delta\alpha} = \frac{\Delta C_L}{\Delta\alpha_6} \frac{4}{3 + \frac{6}{\text{A.R.}}}$$

using the Department of Commerce notation

$$m = m_6 \times \frac{4}{3 + \frac{6}{R}}$$

Example. A certain airfoil has a lift coefficient of 0.76 at 8° angle of attack when the aspect ratio is 6. The angle of zero lift is −2°. What is the slope of the lift curve for an aspect ratio of 9?

Solution.

$$m_6 = \frac{0.76}{8^\circ - (-2)^\circ}$$

$$= 0.076$$

$$m = m_6 \left(\frac{4}{3 + 6/R}\right)$$

$$= 0.076 \left(\frac{4}{3 + 6/9}\right)$$

$$= 0.0829$$

Problems

1. An airfoil has a lift-curve slope of 0.0712 for aspect ratio of 6. What is slope of lift curve for aspect ratio of $4\frac{1}{2}$?

2. An airfoil has a lift-curve slope of 0.073 for aspect ratio of 6. What is slope of lift curve for infinite aspect ratio?

3. With aspect ratio of 6, an airfoil has a C_L of 0.7 at 5° angle of attack. Angle of zero lift is $-3°$. What is C_L at 3° angle of attack when aspect ratio is 8?

4. With aspect ratio of 6, slope of lift curve is 0.076; what is C_L for 4° when aspect ratio is $8\frac{1}{2}$, if angle of zero lift is $-2°$?

5. For a symmetrical airfoil (angle of zero lift = 0°), slope of lift curve for aspect ratio 6 is 0.074; what is C_L at 5° angle of attack when aspect ratio is $9\frac{1}{2}$?

Chord and Beam Components. In analyzing the stresses in spars, it is necessary to know the force produced by the air flowing around the wing. Whereas, for performance calculations, components are used which are perpendicular and parallel to the relative wind (lift and drag components), for stress calculations, components are desired which are perpendicular and parallel to the chord.

If the lift and drag components are known, the components about different axes may be found by resolution. Since the resultant at high angles of attack is only a small angle back of the vertical, the chord component at high angles of attack actually acts in the direction of the leading edge.

Calling forces upward and rearward positive, the chord component coefficient C_C and the beam or normal coefficient C_N may be expressed in terms of lift and drag coefficients, as follows,

$$C_C = C_D \cos \alpha - C_L \sin \alpha$$
$$C_N = C_D \sin \alpha + C_L \cos \alpha$$

Department of Commerce Method of Aspect-Ratio Correction. In correcting angle of attack and drag coefficients for the effect of aspect ratio, instead of using the formulas

$$\alpha_A - \alpha_B = 18.24\, C_L\left(\frac{1}{A} - \frac{1}{B}\right)$$

and
$$C_{D_A} - C_{D_B} = \frac{C_L^2}{\pi}\left(\frac{1}{A} - \frac{1}{B}\right)$$

since it is customary for data to be given for airfoils with aspect ratio of 6, the Department of Commerce uses formulas in the form

$$\alpha = \alpha_6 + 18.24\, KC_L$$

and
$$C_D = C_{D6} + 0.318\, KC_L^2$$

α = angle of attack
C_D = drag coefficient for aspect ratio of the wing

and $$K = \frac{1}{R} - \frac{1}{6}$$
$$= \frac{1}{R} - 0.1667$$

The Department of Commerce uses a standard form for tabulating the correctional data. Below appears a data sheet worked out for a Clark Y monoplane, aspect ratio of 9, the data for the second, fifth, and sixteenth lines being taken from Fig. 17. K is equal to $1/9 - 0.1667$ or -0.0556; a is 24.1 per cent chord.

TABLE IV

Computation of Airfoil Characteristics

Department of Commerce Method

1	C_L	0.2	0.4	0.6	0.8	1.0	1.2	1.4
2	α_6	−2.2	0.6	3.4	6.2	9.0	12.2	15.4
3	$\Delta\alpha$	−0.2	−0.4	−0.6	−0.8	−1.0	−1.2	−1.4
4	α	−2.4	0.2	2.8	5.4	8.0	11.0	14.0
5	C_{D6}	0.012	0.019	0.030	0.046	0.068	0.098	0.132
6	ΔC_{Di}	−0.0007	−0.0028	−0.0064	−0.0113	−0.0177	−0.0256	−0.0346
7	C_D	0.0113	0.0162	0.0236	0.0347	0.0503	0.0724	0.0974
8	$\cos\alpha$	0.9991	1.0000	0.9988	0.9955	0.9903	0.9816	0.9703
9	$\sin\alpha$	−0.0419	0.0035	0.0489	0.0941	0.1392	0.1908	0.2419
10	$C_L\cos\alpha$	0.200	0.400	0.599	0.796	0.990	1.178	1.358
11	$C_D\sin\alpha$	−0.0005	0.0001	0.0012	0.0033	0.0070	0.0138	0.0235
12	C_N	0.2005	0.4001	0.6002	0.7993	0.9970	1.1918	1.3815
13	$C_L\sin\alpha$	−0.0084	0.0014	0.0293	0.0752	0.1392	0.2290	0.3387
14	$C_D\cos\alpha$	0.0113	0.0162	0.0235	0.0346	0.0497	0.0711	0.0945
15	C_C	0.0193	0.0148	−0.0055	−0.0406	−0.0895	−0.1579	−0.2445
16	C.P.	0.60	0.41	0.35	0.32	0.31	0.30	0.29
17	$C_{Mc/4}$	−0.070	−0.064	−0.0599	−0.0559	−0.0598	−0.0596	−0.0553
18	C_{Ma}	−0.0718	−0.0676	−0.0654	−0.0631	−0.0688	−0.0702	−0.0677
19	C_{Di}	0.0014	0.0056	0.0128	0.0226	0.0354	0.0512	0.0692
20	C_{D0}	0.0099	0.0106	0.0108	0.0121	0.0149	0.0212	0.0282

Explanation

Item 2, from characteristic curve, Fig. 17.
Item 3, $\Delta\alpha = 18.24\ KC_L$.
Item 4, $\alpha = (2) + (3)$.
Item 5, from characteristic curve, Fig. 17.
Item 6, $\Delta C_{Di} = 0.318\ KC_L^2$.
Item 7, $C_D = (5) + (6)$.
Item 8, $\cos\alpha = \cos(4)$.

Item 9, $\sin \alpha = \sin$ (4).
Item 10, $C_L \cos \alpha =$ (1) $\times$ (8).
Item 11, $C_D \sin \alpha =$ (7) $\times$ (9).
Item 12, $C_N =$ (10) + (11).
Item 13, $C_L \sin \alpha =$ (1) $\times$ (9).
Item 14, $C_D \cos \alpha =$ (7) $\times$ (8).
Item 15, $C_C =$ (14) − (13).
Item 16, C.P. is for aspect ratio 6, from Fig. 17.
Item 17, $C_{Mc/4} =$ (0.25 − (16)) $\times$ (12).
Item 18, $C_{Ma} =$ (17) + (a − 0.25) $\times$ (12).
Item 19, $C_{Di} = (6)/KR = \dfrac{0.318\, C_L^2}{R} = \dfrac{C_L^2}{\pi R}$.
Item 20, $C_{D0} =$ (7) − (19).

NOTE: Item 12, the beam or normal component, and item 15, the chord component, are used in stress analysis but are not needed in performance calculations.

Problems

1. Prepare a table similar to above for a Clark Y wing with an aspect ratio of 8.

2. Prepare a table similar to above for a wing with an aspect ratio of 7.5.

TABLE V

Aerodynamic Characteristics of Airfoils

Airfoil	$C_{L\max.}$ Maximum Lift Coefficient	$C_{Mc/4}$ Moment Coefficient about $\frac{1}{4}$ Chord Point where $C_L = 0$	a Aerodynamic Center in Percentage of Chord	M $\frac{dC_L}{d\alpha}$ for A.R.6
Clark				
Y	1.56	−0.068	24.2	0.0716
YM-15	1.58	−0.068	24.1	0.0722
YM-18	1.49	−0.065	23.6	0.0716
Curtiss				
C-72	1.62	−0.084	23.8	0.0739
Göttingen				
387	1.56	−0.95	23.9	0.0745
398	1.57	−0.083	24.4	0.0734
N-22	1.60	−0.074	25.0	0.0743
N.A.C.A.				
0006	0.88	0	24.3	0.0748
0012	1.53	0	24.1	0.0743
2212	1.60	−0.029	24.6	0.0753
2409	1.51	−0.044	24.7	0.0753
2412	1.62	−0.044	24.6	0.0743
2415	1.55	−0.040	24.3	0.0743
2418	1.43	−0.037	24.0	0.0726
4412	1.65	−0.089	24.5	0.0736
CYH	1.47	−0.027	24.5	0.0740
M-6	1.40	0.002	25.0	0.0744
M-12	1.25	−0.022	25.0	0.0705
R.A.F.				
15	1.21	−0.052	23.2	0.0719
U.S.A.				
27	1.59	−0.077	23.7	0.0718
35-A	1.48	−0.111	23.4	0.0730
35-B	1.69	−0.076	24.5	0.0749

From tests in the variable density tunnel of the National Advisory Committee for Aeronautics.

CHAPTER VI

INDUCED DRAG OF BIPLANES

Biplane Mutual Interference. The air flowing over and under a wing causes the pressure to be less than atmospheric on the upper side of the wing and slightly more than atmospheric on the under side of the wing. If another wing is placed over the first wing, the gap being relatively small, the low-pressure area on the upper side of the lower wing will be affected by the high-pressure area on the under side of the upper wing, and vice versa. Owing to the proximity of the upper wing, the pressure on the upper side of the lower wing will not be as low as if there were no upper wing. The pressure on the under side of the upper wing will not be as high as if there were no lower wing.

In addition the vortices on each wing have interaction on each other, so that added drag is produced on each wing by the presence of the other wing. This has the same effect as reducing the aspect ratio.

Equivalent Monoplane Aspect Ratio. Prandtl has proved that in a biplane the added induced drag on one of the wings caused by the other wing is

$$\frac{\sigma L_1 L_2}{\pi q b_1 b_2}$$

where L_1 and b_1 are the lift and span respectively of one wing, L_2 and b_2 are the lift and span of the other wing, q is $(\rho/2)V^2$, and σ is a dimensionless factor dependent on the ratio of gap to average span and on the ratio of the shorter to longer span. σ is called the Prandtl interference factor.

The added drag on the upper wing produced by the lower wing is the same as the added drag on the lower wing produced by the upper wing. The total added drag has twice the value of that for a single wing. Then the total induced drag of a biplane is

$$D_i = \frac{L_1^2}{\pi q b_1^2} + \frac{2\,\sigma L_1 L_2}{\pi q b_1 b_2} + \frac{L_2^2}{\pi q b_2^2}$$

If c_1 and c_2 are the two chords, the subscripts corresponding with the subscripts for lift and span, then, assuming there is no decalage, so that the angles of attack and consequently the lift coefficients are the same for both wings,

$$D_i = \frac{\left(C_L \frac{\rho}{2} c_1 b_1 V^2\right)^2}{\pi q b_1^2} + \frac{2\,\sigma\left(C_L \frac{\rho}{2} c_1 b_1 V^2\right)\left(C_L \frac{\rho}{2} c_2 b_2 V^2\right)}{\pi q b_1 b_2} + \frac{\left(C_L \frac{\rho}{2} c_2 b_2 V^2\right)^2}{\pi q b_2^2}$$

$$= \frac{C_L^2 \frac{\rho}{2} c_1^2 V^2}{\pi} + \frac{2\,\sigma C_L^2 \frac{\rho}{2} c_1 c_2 V^2}{\pi} + \frac{C_L^2 \frac{\rho}{2} c_2^2 V^2}{\pi}$$

but since $D_i = C_{Di} \frac{\rho}{2} S V^2$ where S = wing area

then $C_{Di} = \frac{C_L^2}{\pi S} (c_1^2 + 2\,\sigma c_1 c_2 + c_2^2)$

this bears a strong resemblance to the expression for induced drag coefficient for a monoplane, viz.,

$$C_{Di} = \frac{C_L^2}{\pi \text{ A.R.}}$$

The expression $\frac{S}{c_1^2 + 2\,\sigma c_1 c_2 + c_2^2}$ is therefore called the equivalent monoplane aspect ratio of a biplane, abbreviated as E.M.A.R.

$$\text{E.M.A.R.} = \frac{S}{c_1^2 + 2\,\sigma c_1 c_2 + c_2^2}$$

Where the chords of both wings are the same length, $c_1 = c_2$, and the expression becomes

$$\text{E.M.A.R.} = \frac{S}{2\,c^2\,(1 + \sigma)}$$

Figure 40 gives values of σ plotted against ratio of gap to mean span for various values of μ where μ is the ratio of the shorter to longer span. Using values of Prandtl's interference factor from this figure, the E.M.A.R. may be found.

It is to be noted that the area, S, in expressions for E.M.A.R. is the total area of wings including ailerons. Where there is a fuse-

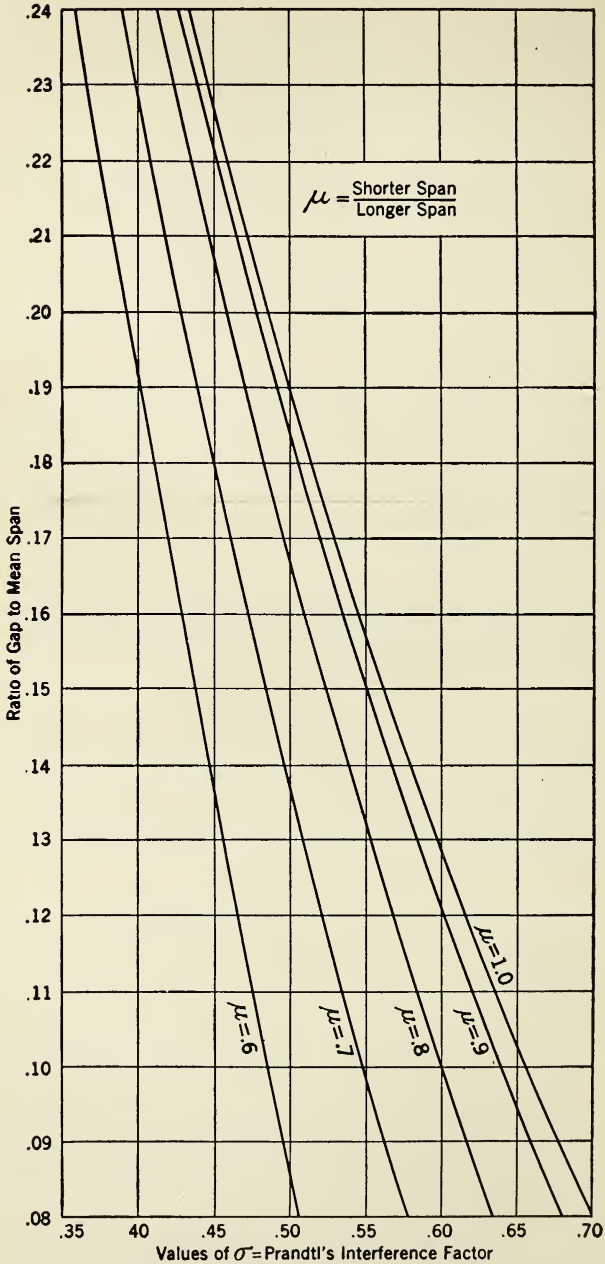


FIG. 40. Prandtl's interference factor.

lage or nacelles, the wing is assumed to be continuous through the fuselage or nacelle, the leading and trailing edges being considered as continuing across in straight lines from where they intersect the fuselage.

Another expression for E.M.A.R. is derived as follows. The ratio of the lift of the wing with the shorter span (L_2) to the lift of the wing with the longer span (L_1) is called r. Then $L_2 = rL_1$ and the total lift $L = L_1 + rL_1 = L_1(1 + r) = L_2 + L_2/r$. The ratio of the shorter span b_2 to the longer span b_1 is called μ: $\mu = b_2/b_1$; μ is never greater than unity.

Making these substitutions in the original expression for induced drag of a biplane

$$
\begin{aligned}
D_i &= \frac{L_1^2}{\pi q b_1^2} + \frac{2\,\sigma L_1 L_2}{\pi q b_1 b_2} + \frac{L_2^2}{\pi q b_2^2} \\
&= \frac{1}{\pi q}\left[\frac{\left(\frac{L}{1+r}\right)^2}{b_1^2} + \frac{2\,\sigma\left(\frac{L}{1+r}\right)\left(\frac{rL}{1+r}\right)}{b_1(\mu b_1)} + \frac{\left(\frac{rL}{1+r}\right)^2}{(\mu b_1)^2}\right] \\
&= \frac{L^2}{\pi q b_1^2}\left[\left(\frac{1}{1+r}\right)^2 + \frac{2\,\sigma r}{\mu(1+r)^2} + \frac{r^2}{\mu^2(1+r)^2}\right] \\
&= \frac{L^2}{\pi q b_1^2}\left(\frac{\mu^2 + 2\,\sigma\mu r + r^2}{\mu^2(1+r)^2}\right)
\end{aligned}
$$

The coefficient of induced drag for a biplane may then be found.

$$C_{Di} q S = \frac{(C_L q S)^2}{\pi q b_1^2}\,\frac{(\mu^2 + 2\,\sigma\mu r + r^2)}{\mu^2(1+r)^2}$$

$$C_{Di} = \frac{C_L^2 S}{\pi b_1^2}\left(\frac{\mu^2 + 2\,\sigma\mu r + r^2}{\mu^2(1+r)^2}\right)$$

The expression $\frac{b_1^2}{S}\left(\frac{\mu^2(1+r)^2}{\mu^2 + 2\,\sigma\mu r + r^2}\right)$ is another form of writing the E.M.A.R. and is more widely used than the one previously given.

Example. Find the equivalent monoplane aspect ratio of a biplane with rectangular wings; upper span 40 ft., upper chord 4 ft. 10 in., lower span 32 ft., lower chord 3 ft. 9 in., gap 4 ft. 6 in.

Solution.

$$\mu = \frac{32}{40} = 0.8$$

$$\frac{\text{Gap}}{\text{Mean span}} = \frac{4.5}{36} = 0.125$$

From Fig. 40, $\sigma = 0.56$

Area, upper wing $= 40 \times 4.83 = 193.2$ sq. ft.
Area, lower wing $= 32 \times 3.75 = 120.2$ sq. ft.

Total area $= 313.4$ sq. ft. $= S$

$$\text{E.M.A.R.} = \frac{313.4}{\overline{4.83}^2 + 2 \times 0.56 \times 4.83 \times 3.75 + \overline{3.75}^2}$$
$$= \frac{313.4}{23.4 + 20.3 + 14.0}$$
$$= 5.4$$

Problems

1. Find E.M.A.R. of a biplane with rectangular wings; upper span 31 ft. $6\frac{1}{2}$ in., upper chord 4 ft. 8 in., lower span 28 ft. $4\frac{1}{2}$ in., lower chord 4 ft. 0 in., and gap 4 ft. 6 in.

2. Find E.M.A.R. of a biplane with rectangular wings; upper span 28 ft. 0 in., upper chord 4 ft. 0 in., lower span 25 ft. 3 in., lower chord 3 ft. 6 in., and gap 50 in.

3. Find E.M.A.R. of a sesquiplane with rectangular wings; upper span 25 ft. 0 in., upper chord 4 ft. 6 in., lower span 15 ft. 0 in., lower chord 3 ft. 0 in., gap 45 in.

4. Find E.M.A.R of a biplane with rectangular wings; upper span 27 ft. 0 in., upper chord 4 ft. 6 in., lower span 22 ft. 0 in., lower chord 4 ft. 6 in., gap 4 ft. 6 in.

5. Find E.M.A.R. of a biplane with rectangular wings; upper span 38 ft. 0 in., upper chord 9 ft. 0 in., lower span 38 ft. 0 in., lower chord 9 ft. 0 in., gap 9 ft. 0 in.

Best Lift Distribution in a Biplane. The two wings of a biplane cause mutual induced drag, and the amount of this drag depends on several factors. The best performance will be secured when these factors are so selected that the induced drag is the least.

The reasons for not having an airplane designed as a monoplane are several. If the weight is too great to be carried on a single wing, the designer resorts to a biplane. A biplane has greater maneuverability than a monoplane. Probably some airplanes are built as biplanes merely because the designer has a predilection for biplanes.

The simplest form of biplane would be one having equal spans and equal areas. It is sometimes thought advisable to have wing panels interchangeable, so that an upper right wing, for example, may be used as a lower right wing, etc. It is also desirable in some designs to make the lower wing smaller to improve the visibility in landing. Sometimes the reason for making the wings of unequal size is to aid stability.

The greater the gap between the wings the smaller will be the interference between the wings. The biplane having an infinite gap between the wings will have no mutual wing interference and the induced drag will be solely the induced drag of each wing. Practically, of course, the gap must be finite. Each wing might be built as a cantilever monoplane wing, but the internal bracing would be excessive. Very simple struts between wings will permit very moderate wing spans to be used. Struts act as columns, and, if the gap is unduly big, the struts are excessively long and the structure is weak. It is usual to make the gap approximately equal to the mean chord. If the gap/chord ratio is greater than 1 the structure becomes weak; if the gap/chord ratio is less than 1 the mutual drag interference becomes excessive. Any departure from gap/chord ratio of unity is usually for reasons of visibility.

The expression for E.M.A.R. $= \frac{b_1^2}{S}\left(\frac{\mu^2(1+r)^2}{\mu^2 + 2\,\sigma\mu r + r^2}\right)$ contains three factors μ, r, and σ. To find the relation of these variables which will give the minimum induced drag, the expression for drag may be differentiated. If it is differentiated with respect to μ, the ratio of spans, the solution for r, ratio of lift, is inapplicable. Differentiating with respect to r, the following results.

$$\frac{dD_i}{dr} = \frac{L^2}{\pi q b_1^2}\,\frac{d}{dr}\left(\frac{\mu^2 + 2\,\sigma\mu r + r^2}{\mu^2(1+r)^2}\right)$$

$$= \frac{L^2}{\pi q b_1^2}\left[\frac{\mu^2(1+r)^2(2\,\sigma\mu + 2\,r) - \mu^2(\mu^2 + 2\,\sigma\mu r + r^2)(2 + 2\,r)}{\mu^4(1+r)^4}\right]$$

$$= \frac{L^2}{\pi q b_1^2}\left[\frac{2}{\mu^2}\,\frac{(1+r)(\sigma\mu + r) - (\mu^2 + 2\,\sigma\mu r + r^2)}{(1+r)^3}\right]$$

$$= \frac{L^2}{\pi q b_1^2}\left[\frac{2}{\mu^2}\,\frac{(\sigma\mu + r + \sigma\mu r + r^2 - \mu^2 - 2\,\sigma\mu r - r^2)}{(1+r)^3}\right]$$

$$= \frac{L^2}{\pi q b_1^2}\left[\frac{2}{\mu^2}\,\frac{r(1-\sigma\mu) - (\mu^2 - \sigma\mu)}{(1+r)^3}\right]$$

$$= \frac{L^2}{\pi q b_1^2}\left[\frac{\frac{2}{\mu^2}(1-\sigma\mu)\left(r - \frac{\mu^2 - \sigma\mu}{1-\sigma\mu}\right)}{(1+r)^3}\right]$$

This differential of the induced drag $\frac{(dD_i)}{dr}$ is equal to zero when r = infinity or when

$$r = \frac{\mu^2 - \sigma\mu}{1 - \sigma\mu}$$

Mathematically when r has this value the induced drag may be either a maximum or a minimum. It should be noted that μ is the ratio of the smaller to the larger span, so that μ is always either unity or less than unity. Prandtl's interference factor σ is always less than unity. Then the product $\sigma\mu$ is always less than unity and the quantity $(1 - \sigma\mu)$ is always positive. If r is greater in value than $\frac{\mu^2 - \sigma\mu}{1 - \sigma\mu}$, the expression $\left(r - \frac{\mu^2 - \sigma\mu}{1 - \sigma\mu}\right)$ is positive and $\frac{dD_i}{dr}$ is positive; that is, the slope of D_i plotted against r is positive. If r is less than $\frac{\mu^2 - \sigma\mu}{1 - \sigma\mu}$, the expression $\left(r - \frac{\mu^2 - \sigma\mu}{1 - \sigma\mu}\right)$ is negative, and the slope of D_i plotted against r is negative. Therefore, when $r = \frac{\mu^2 - \sigma\mu}{1 - \sigma\mu}$, the induced drag of a biplane is a minimum.

In the expressions for induced drag and induced drag coefficient, r is the ratio of lifts of the two wings. If the lift coefficients of the two wings are the same, then the ratio of lifts is the same as the ratio of the areas. Actually, even if the same airfoil section is used for both wings, there will be interaction between the two wings, so that neither C_{LU}, the lift coefficient for the upper wing, nor C_{LL}, the lift coefficient for the lower wing, are the same as C_L, the lift coefficient of the biplane.

$$\begin{aligned} C_{LU} &= C_L \pm \Delta C_{LU} \\ C_{LL} &= C_L \mp \Delta C_{LL} \\ &= C_L \mp \Delta C_{LU} \frac{S_U}{S_L} \end{aligned}$$

When both wings are the same area, the increments ΔC_{LU} and ΔC_{LL} are equal and of opposite sign.

$$\Delta C_{LU} = K_1 + K_2 C_L$$

where K_1 and K_2 are functions of stagger, overhang, gap/chord, decalage, and wing thickness. The reader is referred to N.A.C.A. Report 458 for details of relative wing loading.

Common practice ignores this correction to lift coefficients and assumes that r, the ratio of lifts, is also the ratio of wing areas. Then if unequal spans have been decided upon for a biplane, the areas should be divided so that there is least induced drag, that is, r is made equal to $\dfrac{\mu^2 - \sigma\mu}{1 - \sigma\mu} = \dfrac{\mu - \sigma}{1/\mu - \sigma}$. If unequal areas have been chosen, the spans should be so selected that, when the ratio of the smaller to larger span, less the interference factor, σ, is divided by the ratio of the larger to smaller span, less the interference factor σ, the quotient is equal to the ratio of smaller to larger wing area. This ratio being known, the proper chords can be found.

Example. A biplane is to have a 30-ft. span on the upper wing and a 27-ft. span on the lower wing; the gap is to be 4 ft. 6 in. What should be the ratio of lower wing to upper wing? If rectangular wings are used, what should be the chords?

$$\frac{b_2}{b_1} = 0.9 \qquad \frac{b_1}{b_2} = 1.11$$

$$\frac{\text{Gap}}{\text{Mean span}} = \frac{4.5}{28.5} = 0.158$$

From Fig. 40,

$$\sigma = 0.538$$

$$r = \frac{0.9 - 0.538}{1.11 - 0.538}$$
$$= 0.632$$

Therefore the area of the lower wing should be 0.632 times the area of the upper wing. The area of both the wings will be 1.632 times the area of the upper wing, or the area of the upper wing will be 1/1.632 or 0.613 of the total area.

$$\frac{b_2c_2}{b_1c_1} = 0.632$$

$$\frac{c_2}{c_1} = 0.632\frac{b_1}{b_2}$$

$$\frac{c_2}{c_1} = 0.632 \times 1.11$$
$$= 0.702$$

Therefore lower chord should be 0.702 times the upper chord.

Problems

1. A biplane with rectangular wing is to have upper span 30 ft., lower span 24 ft., and gap 4 ft. What should be the ratio of areas and chords?

2. A biplane is to have 45-ft. upper span, 38-ft. lower span, gap 55 in. What should be the ratio of areas?

3. A biplane is to have 40-ft. upper span, 24-ft. lower span, gap 6 ft. (*a*) What should be the ratio of areas? (*b*) For rectangular wings, what should be the ratio of chords?

4. A biplane is to have 28-ft. upper span, 20-ft. lower span, and gap 45 in. What should be the ratio of areas?

5. A biplane is to have 28-ft. upper span, 20-ft. lower span, and gap 5 ft. What should be the ratio of areas?

E.M.A.R. for Best Lift Distribution. Of the two expressions for equivalent monoplane aspect ratio, namely,

$$\text{E.M.A.R.} = \frac{S}{c_1^2 + 2\,\sigma c_1 c_2 + c_2^2} \qquad \mu = \frac{b_2}{b_1} = \text{ratio of spans}$$

and

$$\text{E.M.A.R.} = \frac{b_1^2}{S}\left[\frac{\mu^2(1+r)^2}{\mu^2 + 2\,\sigma r\mu + r^2}\right] \qquad r = \frac{L_2}{L_1} = \frac{S_2}{S_1} = \text{ratio of areas}$$

The second may be modified by introducing the condition for best lift distribution, namely, that

$$r = \frac{\mu - \sigma}{1/\mu - \sigma} = \frac{\mu^2 - \sigma\mu}{1 - \sigma\mu}$$

Making this substitution the expression becomes

$$\text{E.M.A.R.}_{(\text{Min. }Di)}$$

$$= \frac{b_1^2}{S}\left[\frac{\mu^2\left(1 + \dfrac{\mu^2 - \sigma\mu}{1 - \sigma\mu}\right)^2}{\mu^2 + 2\,\sigma\mu\left(\dfrac{\mu^2 - \sigma\mu}{1 - \sigma\mu}\right) + \left(\dfrac{\mu^2 - \sigma\mu}{1 - \sigma\mu}\right)^2}\right]$$

$$= \frac{b_1^2}{S}\left[\frac{\mu^2\left(\dfrac{1 - 2\,\sigma\mu + \mu^2}{1 - \sigma\mu}\right)^2}{\dfrac{\mu^2(1 - \sigma\mu)^2 + 2\,\sigma\mu^2(\mu - \sigma)(1 - \sigma\mu) + \mu^2(\mu^2 - 2\,\sigma\mu + \sigma^2)}{(1 - \sigma\mu)^2}}\right]$$

$$= \frac{b_1^2}{S}\left[\frac{(1 - 2\,\sigma\mu + \mu^2)^2}{(1 - \sigma\mu)^2 + 2\,\sigma(\mu - \sigma)(1 - \sigma\mu) + (\mu^2 - 2\,\sigma\mu + \sigma^2)}\right]$$

$$= \frac{b_1^2}{S}\left[\frac{(1 - 2\,\sigma\mu + \mu^2)^2}{1 - 2\,\sigma\mu + \sigma^2\mu^2 + 2\,\sigma\mu - 2\,\sigma^2 - 2\,\sigma^2\mu^2 + 2\,\sigma^3\mu + \mu^2 - 2\,\sigma\mu + \sigma^2}\right]$$

$$= \frac{b_1^2}{S}\left[\frac{(1 - 2\,\sigma\mu + \mu^2)^2}{1 - 2\,\sigma\mu + \mu^2 - \sigma^2 + 2\,\sigma^3\mu - \sigma^2\mu^2}\right]$$

$$= \frac{b_1^2}{S}\left[\frac{1 - 2\,\sigma\mu + \mu^2}{1 - \sigma^2}\right]$$

Since $\sigma < \mu$

then $0 < \mu - \sigma$

$0 < \mu^2 - 2\,\sigma\mu + \sigma^2$

and $1 - \sigma^2 < 1 - 2\,\sigma\mu + \mu^2$

Therefore the fraction $\dfrac{1 - 2\,\sigma\mu + \mu^2}{1 - \sigma^2}$ is always greater than 1.

Comparing a monoplane with a biplane whose greater span is the same as the monoplane, and both monoplane and biplane having the same lift (area), the equivalent monoplane aspect ratio of the biplane will be greater than the aspect ratio of the monoplane, since the E.M.A.R. of the biplane is the A.R. of the monoplane multiplied by a fraction which is greater than 1. Since the induced drag depends on the aspect ratio, the biplane will have less induced drag than the monoplane.

The above expression for E.M.A.R. $= \dfrac{b_1^2}{S}\left[\dfrac{(1 - 2\,\sigma\mu + \mu^2)}{1 - \sigma^2}\right]$ is applicable only for biplanes which have the best lift distributions between the wings.

If $\dfrac{b_2}{b_1} = \mu$ be substituted in the equation, it becomes

$$\text{E.M.A.R.}_{(\text{Min. } Di)} = \frac{b_1^2}{S}\left(\frac{1 - \dfrac{2\,\sigma b_2}{b_1} + \dfrac{b_2^2}{b_1^2}}{1 - \sigma^2}\right)$$

$$= \frac{b_1^2 - 2\,\sigma b_1 b_2 + b_2^2}{S(1 - \sigma^2)}$$

If spans are equal $b_1 = b_2$ and

$$\text{E.M.A.R.}_{(\text{Min. } Di)} = \frac{b^2 - 2\,\sigma b^2 + b^2}{S(1 - \sigma^2)}$$

$$= \frac{2\,b^2(1 - \sigma)}{S(1 - \sigma^2)}$$

$$= \frac{2\,b^2}{S(1 + \sigma)}$$

Also if $b_1 = b_2$, $\dfrac{b_2}{b_1} = \mu = 1$, and the ratio of wing areas for best lift distribution becomes

$$r = \frac{\mu^2 - \sigma\mu}{1 - \sigma\mu}$$

$$= \frac{1 - \sigma}{1 - \sigma}$$

$$= 1$$

The spans being equal and the areas (lift) being equal, the chords of each wing are equal and the aspect ratios of each individual wing are the same. The area of one wing is one-half the total area. The actual aspect ratio of each wing is $2\,b^2/S$. The equivalent monoplane aspect ratio of the biplane is the aspect ratio of one wing divided by $(1 + \sigma)$, where σ is a factor depending on the gap.

If the spans are not equal, μ is less than 1. Then for any ratio of areas, the aspect ratio will be less and the induced drag more than for the same ratio of areas with equal spans.

Although it is always desirable to have equal spans, it is sometimes necessary for various design reasons to have one wing, usually the lower, shorter span than the other. With unequal spans, the best efficiency is obtained not with equal areas but with the wing of greater span having the greater area.

Example. A biplane has a span of 40 ft. for upper wing and 36 ft. for lower wing and a gap of 5 ft. The total wing area is 400 sq. ft. Assuming that the area of each of the wings has been selected for best lift distribution, what is the E.M.A.R.?

Solution.

$$\frac{b_2}{b_1} = \frac{36}{40} = 0.9$$

$$\frac{\text{Gap}}{\text{Mean span}} = \frac{5}{38} = 0.132$$

From Fig. 40, $\sigma = 0.580$

$$\text{E.M.A.R.} = \frac{\overline{40}^2}{400}\,\frac{(1 - 2 \times 0.580 \times 0.9 + 0.9^2)}{1 - \overline{0.580}^2}$$

$$= \frac{\overline{40}^2}{400} \times 1.16$$

$$= 4.64$$

Problems

1. Assuming that wing areas are for best distribution, what is E.M.A.R. of a biplane with total area of 380 sq. ft., upper wing 35-ft. span, lower wing 28-ft. span, gap 5 ft.?

2. Assuming wing areas for best distribution, what is E.M.A.R. of a biplane with total wing area of 350 sq. ft., upper span 30 ft., lower span 25 ft., and gap 4 ft.?

3. What is the E.M.A.R. of a biplane with equal spans of 24 ft., each wing having an area of 90 sq. ft., gap being 3 ft.?

4. What is the E.M.A.R. of a biplane whose upper span is 31 ft. 7 in., lower span 29 ft. 5 in., total area 298 sq. ft., gap 75 in.? Assume best lift distribution.

5. What is the E.M.A.R. of a biplane the span of both upper and lower wings of which is 30 ft., the chord of each wing 5 ft. 3 in., and the gap 4 ft. 10 in.?

6. What is the E.M.A.R. of a biplane whose total area is 245 sq. ft., upper span 30 ft. 0 in., lower span 26 ft. 4 in., and gap 4 ft. $10\frac{1}{2}$ in.?

7. What is the E.M.A.R. of a biplane whose total area is 379 sq. ft., upper span 38 ft., lower span 35 ft., and gap 64 in.?

8. What is the E.M.A.R. of a biplane whose total area is 376 sq. ft., upper span 40 ft., lower span 38 ft. 5 in., and gap 61 in.?

9. What is the E.M.A.R. of a biplane whose total area is 1,182 sq. ft., both spans being 79 ft. 8 in. and gap 110 in.?

10. What is the E.M.A.R. of a biplane whose total area is 1,596 sq. ft., both spans being 90 ft. 0 in. and gap 126 in.?

Equivalent Monoplane Span. The aspect ratio for a monoplane being b^2/S and the equivalent monoplane aspect ratio of a biplane being $\frac{b_1^2}{S}\frac{[\mu^2(1+r)^2]}{\mu^2+2\sigma r\mu+r^2}$, the expression $\sqrt{\frac{\mu^2(1+r)^2}{\mu^2+2\,\sigma r\mu+r^2}}$ is called the "apparent span factor" or "Munk's span factor" and is usually represented by the symbol K.

$$K = \sqrt{\frac{\mu^2(1+r)^2}{\mu^2+2\,\sigma r\mu+r^2}}$$

$$= \frac{\mu(1+r)}{\sqrt{\mu^2+2\,\sigma r\mu+r^2}}$$

Using this factor K, the expression for E.M.A.R. of a biplane becomes

$$\text{E.M.A.R.} = \frac{(Kb_1)^2}{S}$$

The expression for induced drag coefficient becomes

$$C_{Di} = \frac{C_L^2 S}{\pi K^2 b_1^2}$$

The expression for induced drag becomes

$$D_i = \frac{L^2}{\pi q \times K^2 b_1^2}$$

Being used in these expressions exactly in the same manner as b, the span in the case of a monoplane, the term (Kb_1) is called the " apparent span " or " equivalent monoplane span."

Example. Find equivalent monoplane span of a biplane whose upper span is 32 ft., lower span 29 ft., gap 4.63 ft., area upper wing 152 sq. ft., and area lower wing 120 sq. ft.

Solution.

$$\mu = \frac{29}{32} = 0.879$$

$$\frac{\text{Gap}}{\text{Mean span}} = \frac{4.63}{30.5} = 0.152$$

From Fig. 40, $\sigma = 0.54$

$$r = \frac{120}{152} = 0.79$$

$$\begin{aligned} K &= \frac{0.879 \times 1.79}{\sqrt{\overline{0.879}^2 + 2 \times 0.54 \times 0.79 \times 0.879 + \overline{0.79}^2}} \\ &= 1.07 \\ Kb_1 &= 1.07 \times 32 \\ &= 34.2 \text{ ft.} \end{aligned}$$

Problems

1. Find equivalent monoplane span of an MB-3 airplane; area upper wing 124 sq. ft., area lower wing 122.9 sq. ft., upper span 26 ft., lower span 24 ft. 6 in., gap 4 ft. 5 in.

2. Find equivalent monoplane span of a PW-8 airplane; area upper wing 172 sq. ft., area lower wing 125 sq. ft., upper span 32 ft., lower span 32 ft., gap 4 ft. $7\frac{1}{2}$ in.

3. Find equivalent monoplane span of a Vought training plane, area upper wing 143 sq. ft., area lower wing 140.7 sq. ft., upper and lower spans each 34 ft. $5\frac{1}{2}$ in., gap 4 ft. 8 in.

4. Find equivalent monoplane span of a Curtiss observation airplane; area upper wing 214 sq. ft., area lower wing 150 sq. ft., upper span 38 ft., lower span 35 ft., gap 5.35 ft.

5. Find equivalent monoplane span of a Curtiss bomber; area of each wing 795 sq. ft., upper and lower spans each 90 ft., gap 9 ft. 3 in.

6. Find equivalent monoplane span of a Sperry Messenger; area of each wing 77 sq. ft., length of each span 20 ft., gap 3 ft. 10 in.

7. Find equivalent monoplane span of a Douglas observation plane; area upper wing 192 sq. ft., area lower wing 186 sq. ft., upper span 40 ft., lower span 38 ft. 5 in., gap 6 ft.

8. Find equivalent monoplane span of a Stearman biplane; area upper wing 150 sq. ft., area lower wing 135 sq. ft., upper span 32 ft., lower span 27 ft. 10 in., gap 5 ft.

9. Find equivalent monoplane span of a Stearman sportster; area upper wing 175 sq. ft., area lower wing 150 sq. ft., upper span 33 ft., lower span 26 ft., gap 4 ft. 10 in.

10. Find equivalent monoplane span of a Consolidated Courier; total wing area 295 sq. ft., upper and lower spans 34 ft. $5\frac{3}{4}$ in., upper and lower chords equal, gap 4 ft. $11\frac{1}{2}$ in.

Effect on Aspect Ratio of Gap Variation. In previous paragraphs, the gap has been specified and changes in aspect ratio with different areas studied. It was shown that with a given gap, the least induced drag (i.e., maximum aspect ratio) would be obtained with equal spans and areas. It is evident, however, from a study of Fig. 40 that σ is a function of gap as well as both spans, and with both spans fixed, a decrease in gap will mean an increase in σ. As the gap is decreased the mutual interference increases. When the gap has decreased to zero, σ becomes 1, the two wings of the biplane merge and coincide, becoming a monoplane; the spans will then be equal, the expression for E.M.A.R. = $\frac{2\,b^2}{S(1+\sigma)}$ becomes $\frac{2\,b^2}{S(1+1)} = \frac{b^2}{S}$.

Induced Drag of a Biplane. The equivalent monoplane aspect ratio of a biplane being known, the induced drag may be found from the formula

$$D_i = \frac{C_L{}^2 \frac{\rho}{2} S V^2}{\pi(\text{E.M.A.R.})}$$

$$= \frac{L^2}{\pi q (K b_1)^2} \qquad \text{where } q = \frac{\rho}{2} V^2,\ V \text{ in feet per second}$$

For level flight, lift must equal the total weight of the airplane.

Example. A biplane is to weigh 4,000 lb. It has been decided that a wing area of 250 sq. ft. will be sufficient to give proper landing speed and that the lower span should be 0.8 of the upper span, which is to be 40 ft. What are best chords, and what is the E.M.A.R., if the gap is chosen as one-sixth of the longer span? What is the induced drag at sea-level, at airspeed of 100 miles per hour?

Solution.

$$\frac{b_2}{b_1} = 0.8 = \mu$$

$$\frac{G}{b_1} = 0.167$$

then

$$\frac{G}{\frac{1}{2}(b_1 + b_2)} = \frac{0.167}{0.9} = 0.185$$

From Fig. 40, $\sigma = 0.477$

For best lift distribution

$$\begin{aligned} r &= \frac{(0.8)^2 - 0.477 \times 0.8}{1 - 0.477 \times 0.8} \\ &= \frac{0.64 - 0.382}{1 - 0.382} \\ &= 0.417 \\ 0.417 &= \frac{c_2}{c_1}\mu \\ \frac{c_2}{c_1} &= 0.522 \\ \text{Total area } (S) &= 250 \text{ sq. ft.} \\ S_1(1 + r) &= 250 \\ S_1 &= \frac{250}{1.417} \\ &= 176.5 \text{ sq. ft.} \\ S_2 &= 0.417 \times 176.5 \\ &= 73.5 \text{ sq. ft.} \\ b_1 &= 40 \text{ ft.} \\ c_1 &= \frac{176.5}{40} \\ &= 4.41 \text{ ft.} \\ b_2 &= 0.8 \times 40 \\ &= 32 \text{ ft.} \\ c_2 &= \frac{73.5}{32} \\ &= 2.3 \text{ ft.} \\ K^2 &= \frac{1 - 2 \times 0.477 \times 0.8 + \overline{0.8}^2}{1 - \overline{0.477}^2} \\ &= 1.138 \end{aligned}$$

$$\text{E.M.A.R.} = \frac{\overline{40}^2}{250} \times 1.138$$
$$= 7.28$$
$$D_i = \frac{\overline{4000}^2}{\pi \times 0.00119 \times \overline{147}^2 \times 1.138 \times \overline{40}^2}$$
$$= 109 \text{ lb.}$$

It is to be noted that the formula $K^2 = \frac{1 - 2\sigma\mu + \mu^2}{1 - \sigma^2}$ is applicable only when the ratio of areas is such that the induced drag is minimum; if the areas are arbitrarily chosen without regard to this, the general formula $K^2 = \frac{\mu^2(1 + r)^2}{\mu^2 + 2\sigma r\mu + r^2}$ must be used.

Example. A biplane weighs 4,000 lb.; the upper span is 40 ft., lower span 32 ft., upper chord 3.45 ft., lower chord 3.5 ft., and gap 6.67 ft. What is the induced drag at 100 miles per hour at sea-level?

Solution.

$$\mu = \frac{32}{40}$$
$$= 0.8$$
$$\frac{G}{\frac{1}{2}(b_1 + b_2)} = \frac{2 \times 6.67}{40 + 32}$$
$$= 0.185$$

From Fig. 40, $\sigma = 0.477$

$$r = \frac{32 \times 3.5}{40 \times 3.45}$$
$$= \frac{112}{138}$$
$$= 0.81$$
$$K^2 = \frac{(0.8)^2 \times (1 + 0.81)^2}{(0.8)^2 + 2 \times 0.477 \times 0.81 \times 0.8 + (0.81)^2}$$
$$= 1.096$$
$$D_i = \frac{\overline{4{,}000}^2}{\pi \times 0.00119 \times \overline{147}^2 \times 1.096 \times \overline{40}^2}$$
$$= 113 \text{ lb.}$$

Problems

1. A biplane weighing 4,000 lb. has a total wing area of 250 sq. ft., upper span 40 ft., lower span 32 ft., upper chord 4.41 ft., lower chord 2.3 ft., gap one-seventh of longer span; what is induced drag at air-speed of 100 miles per hour at sea-level?

2. What is the induced drag for the airplane in problem 1, if gap is one-ninth of longer span?

3. What is the induced drag for the airplane in problem 1, if gap is one-eighth of longer span?

4. A biplane weighs 4,000 lb. and has a total wing area of 250 sq. ft., upper span 40 ft., lower span 32 ft., gap one-seventh of longer span, and chords such as to give minimum drag. What is the induced drag at 100 miles per hour at sea-level?

5. What would be the induced drag for the airplane in problem 4 if the gap were one-eighth of the longer span and the chords such as to give minimum drag?

6. What is the induced drag for the airplane in problem 4 if gap is one-fifth of longer span, and the chords are chosen to give minimum drag?

7. A biplane weighing 4,000 lb. has an upper span of 40 ft., lower span 32 ft., total wing area 250 sq. ft., gap one-seventh of larger span. What should be chords and E.M.A.R. for minimum drag, and what is drag at airspeed of 100 miles per hour at sea-level?

8. A biplane weighing 2,932 lb. has a total wing area of 257 sq. ft., the upper span 31.58 ft., lower span 26 ft.; chords are chosen for minimum drag, and gap is 4.19 ft. What is drag at 150 miles per hour at an altitude of 10,000 ft.?

9. A biplane weighing 10,350 lb. has a total wing area of 1,154 sq. ft., upper and lower spans are each 66.5 ft., the chords are equal, and the gap is 9.17 ft. What is the drag at 120 miles per hour at sea-level?

10. The Curtiss Condor weighs 17,370 lb., its total wing area is 1,510 sq. ft., each span is 91 ft. 8 in., the gap is 10 ft. 6 in., the chords are equal. What is the induced drag at 110 miles per hour at sea-level?

Biplane Characteristics with Engineering Coefficients. The expressions for induced drag coefficient

$$C_{Di} = \frac{C_L^2(c_1^2 + 2\,\sigma c_1 c_2 + c_2^2)}{\pi S}$$

and

$$C_{Di} = \frac{C_L^2 S}{\pi b_1^2}\left[\frac{\mu^2 + 2\,\sigma r\mu + r^2}{\mu^2(1 + r)^2}\right]$$

may be transformed by substituting $C_{Di} = 391\,K_{xi}$ and $C_L = 391\,K_y$

$$K_{xi} = \frac{391\,K_y^2(c_1^2 + 2\,\sigma c_1 c_2 + c_2^2)}{\pi S}$$

$$= \frac{125\,K_y^2(c_1^2 + 2\,\sigma c_1 c_2 + c_2^2)}{S}$$

$$K_{xi} = 125\ K_y{}^2 \frac{S}{b_1{}^2}\left[\frac{\mu^2 + 2\ \sigma r\mu + r^2}{\mu^2(1 + r)^2}\right]$$
$$= 125\ K_y{}^2 \frac{S}{(Kb_1)^2}$$

The expressions for E.M.A.R., equivalent monoplane span, and span factor are functions of areas and lengths and are the same whether absolute or engineering coefficients are used.

The expression for induced drag is as follows.

$$D_i = K_{xi} S V^2 \qquad V \text{ in miles per hour}$$
$$= \left[125\ K_y{}^2 \frac{S}{(Kb_1)^2}\right] S V^2$$
$$= 125 \frac{K_y{}^2 S^2 V^4}{(Kb_1)^2 V^2}$$
$$= \frac{125\ L^2}{(Kb_1)^2 V^2} \qquad V \text{ in miles per hour}$$
$$\text{H.P.}_{Di} = \frac{D_i \times V}{375}$$
$$= \frac{1}{3V}\left(\frac{L}{Kb_1}\right)^2$$

It will be noted that the expressions for induced drag and for horsepower to overcome induced drag are independent of lift and drag coefficients and are therefore the same whether absolute or engineering coefficients are used in the airfoil data.

Total Drag of Biplane Wings from Model Data. With the velocity not known, the total drag may be found from model data. The simplest method, devised by F. W. Herman, is first to find the L/D of the combination of wings and then make use of the fact that, in level flight, lift equals weight.

$$D_i = \frac{L^2}{\pi q (Kb_1)^2}$$
$$= \frac{L\left(C_L \frac{\rho}{2} S V^2\right)}{\pi \frac{\rho}{2} V^2 (Kb_1)^2}$$
$$= \frac{L \times C_L}{\pi \frac{(Kb_1)^2}{S}}$$
$$= \frac{L \times C_L}{\pi(\text{E.M.A.R.})}$$

Let D_{ib} be the induced drag of the biplane wings and D_b the total drag of the biplane wings.

$$D_b = D_{\text{model}} - D_{i\,\text{model}} + D_{ib}$$
$$= D_{\text{model}} - \frac{L \times C_L}{\pi(\text{A.R.})_{\text{model}}} + \frac{L \times C_L}{\pi(\text{E.M.A.R.})_{\text{biplane}}}$$

$$\left(\frac{L}{D}\right)_{\text{biplane}} = \frac{1}{\left(\frac{D}{L}\right)_{\text{model}} + \frac{C_L}{\pi}\left(\frac{1}{\text{E.M.A.R.}_{\text{biplane}}} - \frac{1}{\text{A.R.}_{\text{model}}}\right)}$$
$$= \frac{1}{\left(\frac{D}{L}\right)_{\text{model}} + \frac{C_L}{\pi}\left(\frac{S}{(Kb_1)^2} - \frac{1}{\text{A.R.}_{\text{model}}}\right)}$$

With engineering coefficients, this becomes

$$\left(\frac{L}{D}\right)_{\text{biplane}} = \frac{1}{\left(\frac{D}{L}\right)_{\text{model}} + 125\,K_y\left(\frac{S}{(Kb_1)^2_{\text{biplane}}} - \frac{1}{\text{A.R.}_{\text{model}}}\right)}$$

In level flight, lift is practically equal to weight; so that, if the weight and the L/D at that angle of attack are known, the drag may be found.

Example. A biplane weighs 3,116 lb.; the upper wing area is 154.8 sq. ft., lower wing area 103.67 sq. ft., upper span 30.1 ft., lower span 24.33 ft., gap 4.7 ft., wing is Clark Y. What is the total wing drag at 6° angle of attack?

Solution.

$$\mu = \frac{b_2}{b_1} = \frac{24.33}{30.1} = 0.808$$
$$r = \frac{103.67}{154.8} = 0.67$$
$$\frac{2G}{b_1 + b_2} = \frac{2 \times 4.7}{24.33 + 30.1}$$
$$= 0.173$$

From Fig. 40, $\sigma = 0.493$

$$K^2 = \frac{\mu^2(1+r)^2}{\mu^2 + 2\sigma r\mu + r^2}$$
$$= \frac{\overline{0.808}^2\,(1 + 0.67)^2}{\overline{0.808}^2 + 2 \times 0.493 \times 0.67 \times 0.808 + \overline{0.67}^2}$$
$$= 1.03$$
$$\text{E.M.A.R.} = \frac{1.03 \times (30.1)^2}{103.7 + 154.8}$$
$$= 3.61$$

Clark Y, A.R.6, at 6° angle of attack; $C_L = 0.791$; $L/D = 17.5$.

$$\left(\frac{L}{D}\right)_{\text{biplane}} = \frac{1}{\frac{1}{17.5} + \frac{0.791}{\pi}\left(\frac{1}{3.61} - \frac{1}{6}\right)}$$
$$= \frac{1}{0.0849}$$
$$D = 3{,}116 \times 0.0849$$
$$= 268 \text{ lb.}$$

If the spans are equal, the E.M.A.R. is $\frac{2b^2}{S(1 + \sigma)}$ or the aspect ratio of one wing divided by $(1 + \sigma)$. This simplifies the expression for total drag to

$$D_{\text{biplane}} = D_{\text{model}} + \frac{L \times C_L}{\pi}\left(\frac{(1 + \sigma)}{\text{A.R.}_{\text{(one wing)}}} - \frac{1}{\text{A.R.}_{\text{model}}}\right)$$

Example. Find wing drag of a D.H. at 6° angle of attack. Weight, 5,000 lb.; total wing area, 450 sq. ft.; equal spans of 42 ft.; gap 6 ft., Clark Y section.

Solution.

$$\mu = 1$$
$$\frac{\text{Gap}}{\text{Span}} = \frac{6}{42}$$
$$= 0.143$$

From Fig. 40, $\sigma = 0.574$

$$\text{A.R. (one wing)} = \frac{\overline{42}^2}{225}$$
$$= 7.84$$
$$\frac{L}{D} = \frac{1}{\frac{1}{17.5} + \frac{0.791}{\pi}\left(\frac{1.57}{7.84} - \frac{1}{6}\right)}$$
$$= \frac{1}{0.0655}$$
$$D = 5{,}000 \times 0.0655$$
$$= 328 \text{ lb.}$$

Problems

1. Biplane weighs 3,000 lb. Wing section, Clark Y. Area, upper wing 120 sq. ft., lower wing 100 sq. ft., upper span 30 ft., lower span 24 ft., gap 3.5 ft. What is total wing drag at 8° angle of attack?

2. A Thomas Morse observation airplane weighs 4,104 lb. It uses Clark Y airfoil section. Area upper wing 212 sq. ft., area lower wing 146.7 sq. ft., upper span 39.75 ft., lower span 37.6 ft., gap 5.64 ft. What is wing drag at 6° angle of attack?

3. A Consolidated training plane weighs 2,445 lb. and uses a Clark Y airfoil. Total wing area is 300.6 sq. ft., equal spans are 34.46 ft., gap is 4.72 ft. What is total wing drag at 8° angle of attack?

4. Plot L/D versus C_L for the D.H. biplane. Wings from data in sample problem.

CHAPTER VII

PARASITE DRAG

When an object moves through air, a resistance is encountered. The air flowing around the object exerts a force on the object. This force always has a component acting parallel to the direction of movement of the body with respect to the air. This component is drag.

Struts, wires, landing gear and other parts of the airplane offer resistance to forward motion of the airplane. The force of the air on these parts is all drag. There is no upward or lift component. This is called parasite drag.

Formerly the total drag of an airplane was considered as being composed of two parts, the wing drag and the parasite drag. Classified in this way the parasite drag was the drag of all parts of the airplane except the wings.

The wing drag, however, is composed of two parts, the induced drag and the profile drag. It is now the custom to consider the profile drag of the wing as part of the parasite drag. Parasite drag being drag that is not associated with the production of useful lift, it is quite correct to consider profile drag as parasite.

Parasite drag is caused not only by skin friction but is also due to turbulence. Every care must therefore be exercised in the design to eliminate protuberances or sharp corners. To this end, " fairing " of balsa wood or sheet metal is frequently used. Wherever possible the airplane parts that would cause parasite drag are made streamline in shape.

Sometimes, but rarely, parasite drag is referred to as structural drag.

Units of Measurement of Parasite Drag. It was formerly the custom to give data on parasite drag in terms of equivalent flat plate area. That is, the actual drag in pounds at a given airspeed being known, the area of a flat plate normal to the wind, which at the same airspeed would have the same drag, would be calculated.

This unnecessarily complicated the computation, and the convention of referring to a fictitious flat plate has been discarded.

Some data of old tests are still recorded as being in "equivalent flat plate area." The resistance in pounds of a flat plate at sea-level is $0.0032\, aV^2$, where a is the area in square feet and V is in miles per hour; or the resistance is $1.28 \frac{\rho}{2} aV^2$, where ρ is the air density in slugs per cubic foot, a is the area in square feet, and V is in feet per second.

Another method of stating the drag of an airplane part is to give the drag of that part at some defined airspeed. If the drag is given at an airspeed of 1 mile per hour, this drag multiplied by the square of the airspeed is the drag at that airspeed. Since the drag of minor parts of an airplane is usually quite small, the drag is often recorded for an airspeed of 100 miles per hour. The drag at any other airspeed will be the drag at 100 miles per hour multiplied by the square of the airspeed and divided by 10,000.

Another method of designating drag is to use a parasite drag coefficient.

The total drag of the wing, $C_D \frac{\rho}{2} SV^2$, is made up of two parts: the profile drag $C_{D_0} \frac{\rho}{2} SV^2$, and the induced drag $C_{Di} \frac{\rho}{2} SV^2$. The total drag of the airplane is the wing drag plus the parasite drag. The calculation of drag is therefore much simplified if the parasite drag coefficient is put in such a form that when multiplied by $\frac{\rho}{2} SV^2$, the product is parasite drag. The parasite drag coefficient in this form can then be added directly to the profile drag coefficient and the induced drag coefficient.

If a is the equivalent flat plate area of all the surfaces except wings, which produce drag, then the coefficient of parasite drag is

$$C_{Dp} = \frac{1.28 \times a}{S} \qquad S \text{ is wing area in square feet}$$

Then the total drag, exclusive of wing drag, is

$$D_p = C_{Dp} \frac{\rho}{2} SV^2 \qquad V \text{ in feet per second}$$

and

$$D_p = 0.00256\, C_{Dp} SV^2 \qquad V \text{ in miles per hour}$$

If the drag of an airplane part is given in pounds at an airspeed of 100 miles per hour, the coefficient of parasite drag is

$$C_{Dp} = \frac{D_{100\text{ m.p.h.}}}{0.00256\, S \times 10{,}000}$$
$$= \frac{0.0392 \times D_{100\text{ m.p.h.}}}{S}$$

Streamlining. The purpose of streamlining is to reduce as much as possible the turbulence in the rear of objects moving through the air. The air flowing along the sides of a streamlined object experiences no sudden change in direction. If the object always met the air in a direction parallel to the axis of the object, there might be merit in having the nose of the object pointed or knife-shaped. A very slight change in direction of an object whose entering surface is so shaped would increase the resistance or turbulence enormously. It is therefore best to have a blunt nose. Since an object whose rear end is blunt leaves a wake when passing through air, and the air rushes into this region of low pressure in a turbulent manner, the contour of a streamlined body must taper back to a " tail."

Any non-streamlined body can have its resistance or drag greatly reduced merely by the addition of a blunt nose and a tapered tail. If the contour is also a continuous curve the shape approaches the ideal streamline.

Fineness. Fineness is the ratio of the length of a body, i.e., parallel to the usual direction of motion, to its maximum thickness. Best results can be obtained with a fineness ratio of about 4. A fineness less than 4 shows a big increase in drag coefficient. A fineness ratio greater than 4 gives a slight increase probably due entirely to the added skin friction.

The preceding remarks about streamlining and fineness apply both to solids of revolution like the gas-bag of dirigibles and to the cross-section of objects like struts about which the flow is two-dimensional.

Graphical Method for Streamline Shape. The following procedure describes a method of obtaining a Joukowski symmetrical airfoil section, which serves as a very good contour for a streamline section. If the section which is obtained appears to be too thin at the rear for strength, it may be arbitrarily thickened there and, provided this change is not too great, there will be little increase in the low air resistance of the section.

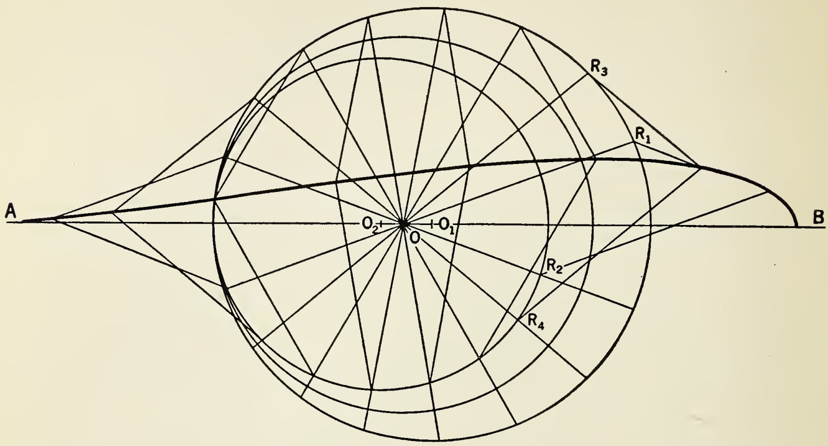


Fig. 41. Graphical method for Joukowski symmetrical airfoil.

If D is the depth from front to back of the desired streamline section and F is the fineness ratio, the maximum thickness is D/F. Before starting the graphical construction it is first necessary to calculate three dimensions: a, b, and r.

$$\frac{a}{r} = 0.6495\,F - \sqrt{0.4219\,F^2 - 1}$$

$$a = \frac{D/F}{3\sqrt{3}}$$

$$b = \frac{ar}{2\,a + r}$$

For example, it is desired to have a streamline section 4 in. deep with a fineness ratio of 5; then the maximum thickness will be $\frac{4}{5}$ in. or 0.8 in., and

$$\frac{a}{r} = 0.6495 \times 5 - \sqrt{0.4219 \times 25 - 1}$$

$$= 0.1578$$

$$a = \frac{0.8}{5.196}$$

$$= 0.1540 \text{ in.}$$

$$r = \frac{0.1540}{0.1578}$$
$$= 0.976 \text{ in.}$$
$$b = \frac{a}{2\,a/r + 1}$$
$$= \frac{0.154}{2 \times 0.158 + 1}$$
$$= 0.1170 \text{ in.}$$

On a baseline AB, with O as center and r as radius, a circle is drawn. This circle is divided into an even number of equal divisions. The distance OO_1 equal to a is laid off on baseline AB to the right of O. With O_1 as center and with $r + a$ as radius, a second circle is drawn. The distance OO_2 equal to b is laid off on baseline AB to the left of O. With O_2 as center and $r - b$ as radius, a third circle is drawn. All three circles will be mutually tangent.

With O as center, radial lines are drawn through the division marks on the first-drawn circle. These radial lines are prolonged to intersect the outer circle.

These radial lines are treated in pairs. The radial line above the baseline, and making a given angle with it, is paired with the radial line below the baseline, and making the same angle with it. In Fig. 41, OR_1 and OR_2 are two such radial lines. Where OR_1 intersects the outer circle, a line is drawn parallel to OR_2. Where OR_2 intersects the inner circle, a line is drawn parallel to OR_1. The intersection of these two lines is a point on the streamline contour. By continuing this construction, sufficient points are obtained through which a smooth curve is drawn. The trailing edge will be a distance of $2\,r$ to the left of O while the leading edge will be a distance of $2\,r(1 + a^2/r^2)$ to the right of O.

Cylinders and Round Wires. Circular cylinders and wires are rarely used in aircraft construction. If round tubes are used, they are faired with falsework of balsa or sheet metal.

The drag of a cylinder in an air stream whose direction is perpendicular to the axis of the cylinder is

$$D = 0.00026 \times d \times V^2$$

D = drag per foot of length
d = diameter in inches
V = airspeed in miles per hour

or

$$D = 0.000121 \times d \times V^2$$

V = airspeed in feet per second

The drag of round wires may be found by the above equations. If the wire is less than 0.2-in. diameter, the result will be slightly high.

Stranded cable will have greater drag than smooth wires, and the following equations should be used:

$$D = 0.00031 \times d \times V^2 \qquad V \text{ in miles per hour}$$
$$D = 0.000144 \times d \times V^2 \qquad V \text{ in feet per second}$$

If two parallel wires are in the air stream and one wire shields the other by being directly behind it, the total drag will be less than for two single wires, and will be approximately $1\frac{1}{2}$ times the drag of a single wire. If the distance center to center of the wires is greater than 3 times the diameter of the wire, the shielding effect is negligible and the drag of each individual wire should be counted separately. Quite often thin strips are fastened on the double wires, in which case the pair of wires has approximately the same resistance as a single wire.

"Streamline" Wire. A solid wire or rod is manufactured commercially which is termed "Streamline" wire. It is not truly streamlined, but it is much better from the standpoint of decreased air resistance than round wire. The drag of this wire is found by the equation:

$$D = 0.000089 \times T \times V^2$$

D is drag in pounds per foot of length
T is thickness in inches
V is airspeed in miles per hour

or

$$D = 0.0000414 \times T \times V^2 \qquad V \text{ is airspeed in feet per second}$$

There is usually present, in flight, some vibration of the wires. Owing to sudden rapid reversal of stresses, hard wire is liable to fatigue and failure with little or no warning. Stranded wire cable will usually stretch, and one or two wires will fray so that the need of repair will be noted.

Struts. Round tubing is the most efficient structural shape, but its drag is high. Nowadays all struts in the air stream have a streamline-shaped cross-section. The drag of a streamline strut depends on the fineness ratio. In air of standard density, the drag in pounds per foot of length of a strut whose leading edge is perpendicular to the direction of the air stream is as follows:

$$D = KwV^2 \qquad \begin{array}{l} w \text{ is width in inches} \\ V \text{ in miles per hour} \end{array}$$

Fineness	K
2.5	0.0000194
3.0	0.0000180
3.5	0.0000175
4.0	0.0000171
4.5	0.0000175

or

$$D = K'wV^2 \qquad V \text{ in feet per second}$$

Fineness	K'
2.5	0.00000902
3.0	0.00000836
3.5	0.00000812
4.0	0.00000795
4.5	0.00000812

By comparing coefficients for round tubes and for streamlined struts, it will be seen that, for the same width, the round tube has approximately 15 times the drag of a streamlined strut.

If a strut is in tension only, its strength depends on its cross-sectional area. The area of a solid circular rod is approximately one-half the area of a solid streamline-section strut of the same width. A streamline-section strut to have the same cross-sectional area as a round rod would have less thickness and more depth; so that the drag of the streamline strut will be about $\frac{1}{25}$ of the solid circular rod of the same area.

Hollow streamline struts are made of tubes rolled to the proper shape. The cross-sectional area of metal will be the same for both shapes and the tensile strength will be the same, but the streamline form will have $\frac{1}{15}$ the resistance.

In direct compression, the strength of a piece depends on area only. Struts as used in aircraft act as columns and are subject to compression and bending. Failure will occur about the axis which has the least moment of inertia. A streamline strut will have a greater moment of inertia about any axis than will a round rod of the same diameter (thickness). If comparison is made between round and streamline rods having the same area and consequently the same weight, the streamline strut will be found to have a greater moment of inertia about its short axis than the round strut; but about its long axis, the streamline strut will have less moment of inertia than the round rod.

Example. Find difference in drag at 100 miles per hour, at sea-level, of four main struts each 6 ft. long if they are round tubes $\frac{3}{4}$ in. in outside diameter or streamline $\frac{5}{8}$ in. in width and have a fineness of 3.5

Solution.

Round:

$$D = (0.00026 \times \tfrac{3}{4} \times \overline{100}^2) \times 4 \times 6$$
$$= 46.8 \text{ lb.}$$

Streamline:

$$D = (0.0000175 \times \tfrac{5}{8} \times \overline{100}^2) \times 4 \times 6$$
$$= 2.6 \text{ lb.}$$

Problems

1. What is resistance in pounds of eight standard streamline wires each $\frac{1}{2}$ in. thick and 21 ft. long, at an airspeed of 90 miles per hour at sea-level?

2. What is the resistance of the wires in problem 1 at 12,000-ft. altitude?

3. What is the drag of the four interplane struts of a biplane if they are streamline in shape with fineness of 3, thickness is $\frac{3}{4}$ in., and length of each is 5 ft. 10 in.? Airspeed 100 miles per hour.

4. What is the drag of 40 ft. of standard "streamline" wire, $\frac{3}{8}$ in. thick, at 125 miles per hour airspeed?

5. A radio antenna mast is a round tube 6 in. in diameter. It is 40 ft. high. What is the force on the mast in a 50-mile wind?

Fittings. At the end of a strut, where it meets the wing or the fuselage, the air flow is distributed by the conjunction of the two surfaces. In a carefully designed plane the end of the strut is buried inside the wing and the interference effect is practically negligible. In the ordinary planes, the socket on the end of the strut and the fitting holding it to the wing surface cause extra drag.

A very good way to allow for the extra resistance is to follow the practice of the U. S. Navy, as follows: For resistance of cables and wires, "Add one foot to length of cable for turnbuckle, and one foot for the eye and fitting. . . . For total resistance of struts, use total length including space occupied by sockets and fittings, and add three feet per strut for the additional resistance of the two end fittings."

Another rule for fittings is to find the projected area on a plane normal to the air stream. This projected area is multiplied by 2 and figured as flat plate area. The reason for figuring on double the area is the interference effect of the nearby surfaces.

Fuselages. The drag of a fuselage is very difficult to predict. There are usually many projections, such as windshields, control wires, and pinned joints, to break up the smooth flow of air. The most accurate way of finding the drag is to build a scale model and actually make wind-tunnel tests. This model to be of value should be an exact reproduction and should have all excrescences such as windshields and filler-cap covers duplicated in miniature.

Building a scale model to exact scale requires meticulous care and is quite costly. If such a proceeding is not feasible, the designer endeavors to ascertain the drag of the fuselage of an already built airplane whose fuselage most nearly resembles his own design. Knowing the cross-sectional area of the fuselage and the airspeed at which the drag was measured, he finds a coefficient he may use to determine the drag of his own-designed fuselage.

Example. An airplane fuselage has been designed which resembles the fuselage of a Vought (VE-9) training plane. The Vought fuselage has a cross-sectional area of 8.9 sq. ft.; at 110 miles per hour its drag is 102 lb. The unknown fuselage has a cross-sectional area of 12 sq. ft.; what is its drag at 145 miles per hour?

Solution.

From data on Vought:

$$\begin{aligned}\text{Coefficient} &= \frac{102}{8.9 \times \overline{110}^2} \\ &= 0.00095\end{aligned}$$

For newly designed airplane fuselage:

$$\begin{aligned}\text{Drag} &= \text{coefficient} \times A \times V^2 \\ &= 0.00095 \times 12 \times \overline{145}^2 \\ &= 239 \text{ lb.}\end{aligned}$$

With wheels, struts, tie wires, etc., there is usually very little difference whether the relative wind is exactly parallel or at a small angle to the longitudinal axis of the airplane; with fuselages, there may be a difference in the drag coefficients with different angles of attack. More exactly, the fuselage drag coefficient depends on the lift coefficient, since a major portion of the fuselage and the tail surfaces is always in the downwash from the wings and the downwash angle relative to the longitudinal axis of the airplane depends on the lift coefficient. For a very rough approximation, the drag coefficient for the fuselage may be assumed constant; but for any degree of accuracy, use should be made of Fig. 42, where a factor F is plotted against the ratio of airspeed

(V) to stalling or minimum speed (V_S). This factor F multiplied by the fuselage drag coefficient at high speed (i.e., low angle of attack) gives the drag coefficient at any airspeed V. It may be considered as the ratio of the flat plate area of the fuselage at any speed to the flat plate area at high speed.

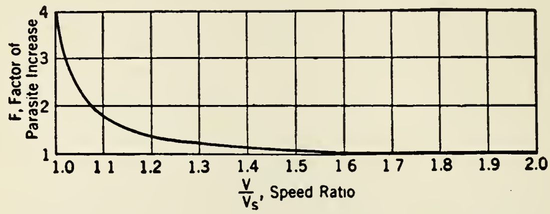


FIG. 42. Parasite drag factor for fuselages, etc.

Example. The top speed of a DH-4 is 124 miles per hour at sea-level; its stalling speed is 61 miles per hour. The wing area is 440 sq. ft.; at a speed of 100 miles per hour, the fuselage has a drag equivalent to a flat plate 3.06 sq. ft. in area. What are the equivalent flat plate area and drag coefficient at stalling speed and at maximum speed?

Solution.

$$\frac{V}{V_S} = \frac{100}{61} = 1.64$$

From Fig. 42, F, at $V/V_S = 1.64$, is 1.02

F at stalling speed is 4.00

F at maximum speed is 1.00

At stalling speed:

$$\text{Equivalent flat plate area} = \frac{3.06 \times 4}{1.02} = 12.0 \text{ sq. ft.}$$

At maximum speed:

$$\text{Equivalent flat plate area} = \frac{3.06 \times 1.00}{1.02} = 3.0 \text{ sq. ft.}$$

At 100 miles per hour:

$$C_{D\ \text{fuselage}} = \frac{3.06 \times 1.28}{440}$$

$$= 0.0089$$

At stalling speed:

$$C_{D\ \text{fuselage}} = \frac{0.0089 \times 4}{1.02}$$

$$= 0.0349$$

At maximum speed:

$$C_{D\ \text{fuselage}} = \frac{0.0089 \times 1.00}{1.02}$$

$$= 0.00874$$

Landing Gear. In the fixed type of landing gear, the drag is composed of the resistance of the struts, the wheels, the mutual interference of the struts and wheels, and the interference caused by the fuselage and wings. Merely estimating the resistance of the individual struts and the wheels will not take into account the interference which may cause a major part of the drag.

If feasible, the landing gear should be incorporated with the wind-tunnel model of the fuselage, and the total drag of fuselage with landing gear measured. If wind-tunnel tests are not obtainable, an approximate idea of the drag may be gained by totaling the resistance of the struts and the wheels, using Table VI.

TABLE VI

High-Pressure Tires and Wheels

Tire size	Drag in pounds per wheel at 100 miles per hour		
	Bare wheel	Usual fairing	Full fairing
26×3	10.8	7.2	3.6
28×4	14.1	9.4	4.7
30×5	18.1	12.0	6.0
32×6	22.8	15.2	7.6
36×8	33.8	22.5	11.2
44×10	51.2	34.1	17.0
54×12	75.9	50.6	25.3
64×14	105.6	70.4	35.2

Within the past few years, it has become common to use low-pressure tires. With a smaller wheel the same landing shock can be taken up as with a larger wheel using high-pressure tires. The low-pressure tires decrease the chances of ground looping since they grip the ground better, they permit a smoother take-off and landing on a rough field, and they afford a better traction on a wet, muddy field. Brakes operate well, and owing to their proportions the drag is less than for a wheel with high-pressure tire to take the same landing load. The $8\frac{1}{2}$ in. by 10 in. low-pressure tire and wheel have become the most widely used. The drag is 6.1 lb. at 80 miles per hour. An extra-low-pressure tire and wheel, sometimes referred to as the "doughnut" wheel, are also used; the tire dimensions are $11\frac{1}{4}$ in. by 25 in.; the drag is 7.1 lb. at 80 miles per hour.

Retractable landing gear is now being employed with remarkable success on transport planes. It was first used in the United States on the Verville-Sperry racing plane which won the Pulitzer Trophy Race in 1921. In flight, the wheels are retracted into the wings or into the fuselage. Lowering the landing gear not only increases the drag but it also lowers the line of action of the total drag force, so that, if the airplane is balanced before lowering, there is a tendency to nose over when the gear is caused to descend. This presents no problem since the landing gear is lowered when the pilot wishes to descend for a landing. During the introduction of retractable landing gear, great difficulty was encountered by pilots' forgetting to lower the gear when landing. Warning devices and the presentation of "dumb-bell" trophies to pilots who land on the belly of the fuselage appear to be of value in aiding the pilot to remember to lower his gear.

Engine Cowling. It is possible to improve the airflow around the engine and thus reduce the parasite resistance, by proper cowling. The Townend ring was developed in England for this purpose. Research at McCook Field by Lieutenants Breene and Greene and later work at the N.A.C.A. produced other forms of cowling. Radial air-cooled engines have considerable drag owing to the eddies around the fins and valves, so that cowling is chiefly beneficial for this type of engine.

It is difficult to predict the decrease in drag which may be attained by cowling the engine, and it is almost impossible in small wind-tunnel models to reproduce in miniature all the little details of the engines which initiate the air disturbances. Full-size flight tests appear to be the only method of finding out exactly the precise value of any particular cowl. Tests would need to be run at several different speeds since a cowling might be helpful at one speed and detrimental at another.

Polar Diagram. The plotting of C_L against C_D has special merit when used in combination with the parasite coefficient, the latter being expressed as a function of the wing area. It was pointed out in Chapter IV that, with C_L plotted against C_D, if a straight line is drawn through the origin tangent to the curve, the point of tangency will locate the maximum L/D for the airfoil alone. Laying off the parasite drag coefficient in the form of 1.28 a/S, to the left of the origin, gives a point such as P in Fig. 43. A line through point P tangent to the curve locates the maximum L/D of the entire airplane. In Fig. 43, it is assumed that the airplane has a Clark Y wing 36 ft. by 6 ft., and that the parasite resistance has an equivalent flat plate area of 8 sq. ft. Then the distance OP is $1.28 \times 8/216$ or 0.047 unit. The point of tangency shows that the angle of attack for this combination of wing and parasite that has maximum L/D_{total} is 8°. The ordinate (C_L) for the point of tan-

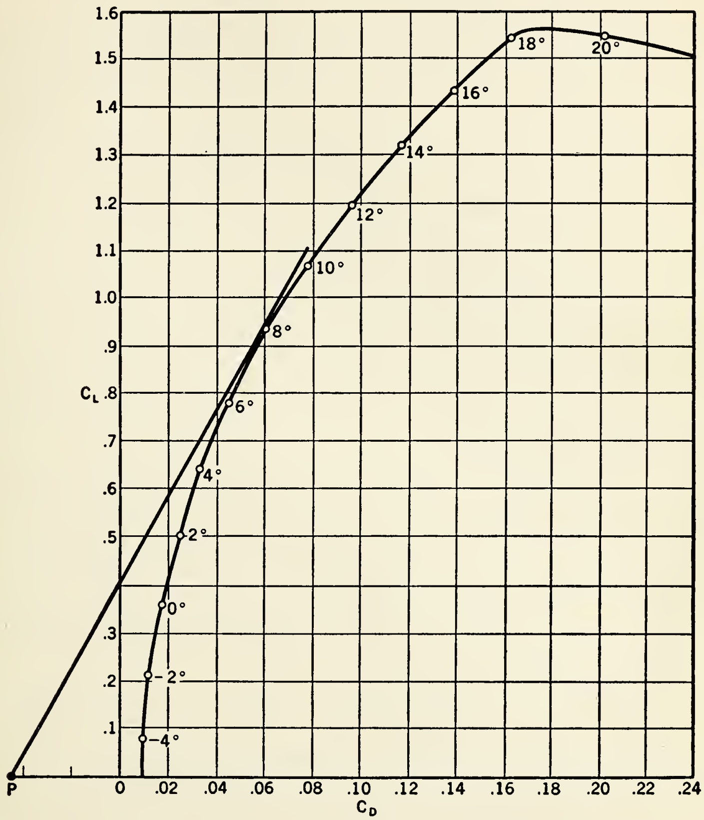


FIG. 43. Polar curve for Clark Y airfoil with parasite drag.

gency is 0.93 and the abscissa, measured from P, ($C_{D\,\text{total}}$), is 0.108, so that the L/D for the entire airplane is 0.93/0.108 or 8.51. This should be compared with Fig. 25, where the tangent drawn from the origin would locate the angle of maximum L/D for the wing alone as 1° and the maximum L/D of the wing alone as 21.5.

CHAPTER VIII

ENGINES

The universally employed aircraft engine is the internal-combustion type. It is either air cooled or liquid cooled. The cylinders may be arranged either radially or vertically in line or in two rows in the form of a letter V.

The essential requirements of an airplane engine are light weight per horsepower, extreme reliability, low fuel consumption, and low frontal area. In principle, aircraft engines do not differ materially from automobile engines. At full throttle, the automobile engine has usually a higher number of revolutions and greater piston speed than the airplane engine, but whereas the average automobile engine rarely is run wide open and the bulk of its operation is at less than half speed, the airplane engine is frequently at top speed and most of its operation is at three-quarter speed or better.

The Department of Commerce before approving a new type of airplane engine requires a 100-hour endurance test. For 50 hours of this test the engine must be run at wide-open throttle, and for the other 50 hours at not less than 75 per cent of rated horsepower. This test is more severe than those required by other countries, and it is not believed that any automobile engine has sufficient reliability to pass it.

The requirement of lightness in weight is a severe handicap in obtaining reliability. It would not be so difficult to obtain dependability if all the parts could be made sturdy and rugged. Reducing weight to a minimum means that the stresses of every part must be carefully analyzed so that there shall be no weakness.

In automobile operation, after the engine is started, the car is slipped into low gear. After the automobile is moving, the speed is slowly increased, and it is usually not till some time has elapsed that the engine develops full power. With airplane operation, the engine is wide open for the take-off and wide open for the climb. It is not till the altitude selected for flight has been reached that the engine is throttled back.

For reliability all engines of more than 100 hp. are required by the Department of Commerce to have dual ignition systems and two spark plugs per cylinder.

Mounting engines at the rear of the wing as in the pusher type airplane brings the center of gravity of the airplane far back and makes the problem of balance quite difficult. It is only in flying boats where the hull extends a good distance forward of the wing to offset to some extent the weight of the engine that the pusher type is permissible. Having the engine in the rear of the pilot presents a hazard in a crash.

To some degree in the pusher type but chiefly in the tractor type is the question of frontal area important. In the early days of aviation, the width of the engine largely determined the width of the fuselage. With improvements in the V-type engine, it became the breadth of the pilot's shoulders that restricted the narrowness of the fuselage. With radial engines, it is still the frontal area of the engine which sets the limit on the decrease in cross-sectional area of the fuselage, as the body is streamlined backwards from the size at the motor.

Air-Cooled versus Liquid-Cooled. The term cooling is somewhat a misnomer since it is not desired to make the engine cold or even moderately cool. In the cylinders, gasoline is burned; and unless the heat is removed, the engine parts would soon reach a temperature at which they would have no strength. The purpose of cooling is to keep the temperature sufficiently low that the engine will function properly.

All engines are air cooled. Some types are cooled directly by air; other types are cooled indirectly by having liquid remove heat from the cylinders and then having the liquid cooled by air. If an air-cooled and a water-cooled engine are of the same power, it is to be expected that the same amount of heat would need to be taken from each. The rate at which heat passes from a surface to air circulating past the surface depends on the difference in temperature of the air and the surface, so that, if the two engines are to run at the same temperature, the same volume of air per minute should go past the cooling fins in one engine and through the radiator in the other. Then, if there were no burbling or eddy currents, it might be expected that the drag of each would be the same.

In order that each cylinder could be properly cooled, the first air-cooled engines had radial cylinders. This arrangement has an advantage of very short length which is desirable in not having the center of gravity of the plane too far forward. In recent years, air-cooled engines have been built both in-line and V-type, and no difficulty has been experienced in keeping the rear cylinders properly cool. Water-cooled engines have the disadvantage of extra weight of the water, the radiator, pump and plumbing — which on a 400-hp. engine would amount to upward of 180 lb. Air-cooled engines save this weight, but in large horsepowers the parasite drag, even with the benefit of cowling, is more than that of water-cooled engines of the same horsepower.

Because of the great weight of water, in the past few years, ethylene glycol has been used as a coolant. The weight of ethylene glycol to cool an engine properly need be only one-quarter the weight of water that would be necessary. The size of radiator needed for ethylene glycol would be about one-quarter that of a water radiator, thus greatly reducing the parasite drag. Because the boiling point of this chemical is considerably above that of water, the engine can be run at a temperature of 300° F. At this temperature the mixture in the intake manifold is absolutely dry, whereas, if water cooled, "wet gas" often is drawn into the cylinders. More horsepower is actually obtained for this reason from an engine when ethylene glycol cooled than when water cooled. Greater care is needed with the plumbing, as this chemical will seep through tiny cracks that would be impervious to water.

There appears to be a definite limit to the horsepower per cylinder of an internal-combustion engine. Good practice dictates that the stroke should be approximately the same length as the diameter of the cylinder. The diameter of the piston is limited by the problem of cooling; with a large piston it is difficult to keep the center of the piston-face from getting too hot. The displacement of each cylinder being restricted and the mean effective pressures on standard engines not exceeding 150 lb. per sq. in., the power obtainable is rarely above 90 hp. per cylinder on present standard engines. To obtain large power means a large number of cylinders, and nine cylinders are the most that can be crowded around the crankcase of a radial engine. Resort has been made, with notable success, to a double bank on one American-made engine.

Engine failures may be classified under four heads: fuel trouble, lubrication trouble, electrical trouble, and cooling trouble. With liquid cooling there is possibility of leaky radiator, water boiling away, hose bursting, pump failing, etc. With air cooling, one of the sources of possible trouble is eliminated.

Steam Power. Langley used a steam engine in his small successful flying model airplane.

In considering the practicability of a steam engine for aircraft use, one must consider the entire unit of boiler, engine, and condenser, since obviously a non-condensing engine is not feasible. Some German torpedo boats have been powered with steam engines, and on the basis of the entire unit, the weights were 13.2 lb. per hp., with a fuel consumption of 0.573 lb. per hp-hr. These were large power plants, and always efficiencies in power and savings in proportionate weight can be effected in large units that cannot be realized in smaller units. Authorities on steam-engine design have, however, announced their belief that the weights of units adapted to aircraft use can be reduced to 9 lb. per hp.

For ordinary flying, this weight would bar out all consideration of steam, since gasoline units have been built weighing less than 1 lb. per hp. These were special jobs, and the average internal-combustion aircraft engine can be taken to weigh around 2 lb. per hp.

Lately considerable thought has been given to stratosphere flying. The steam engine does not lose horsepower with altitude. If there is any difference, the low temperature at high altitudes will give better condensing and there may be a slight increase in power. With unsupercharged gasoline engines, there is a decrease in power with altitude. At 36,000-ft. altitude the horsepower of an unsupercharged gasoline engine is approximately one-fifth of its horsepower at the ground. It has been estimated that at an altitude of 66,000 ft. all the power of an ordinary gasoline engine will be used up in overcoming mechanical losses, so that the effective power will be zero. Supercharging will enable the gasoline engine to deliver full power at moderate altitudes, but there is a definite limit to supercharging. The weight remaining the same with decreasing power, the weight per horsepower increases so that it may safely be said that, at altitudes higher than 40,000 ft., steam engines are lighter than gasoline engines.

With a steam power plant, energy is obtained from the fuel and stored up as pressure energy, to be drawn upon later. This is considered extremely hazardous as high-pressure superheated steam is very dangerous. Against the objectionable features of greater weight at ordinary altitude and peril from boiler explosion may be weighed the following advantages. A much cheaper fuel may be used, and it may contain more energy per unit weight. A steam engine has no carburetion difficulties, nor has it spark plugs, distributors, or other electrical devices. Eliminating these removes sources of trouble which have caused many forced landings.

Steam turbines have received little consideration since for efficient operation they must be in such large units or must be run at such high speed as to be unsuitable for aircraft use.

Rotary Gasoline Engines. In the early days of gasoline engines, since the ordinary four-stroke cycle means one power stroke per four strokes, it was thought that a flywheel was absolutely necessary. To eliminate extra weight and still have a rotating mass, an engine was built with the cylinders rotating about a stationary crankshaft. The propeller was affixed to the cylinders. The cylinders were arranged radially about the hub and were air cooled.

In order to furnish the gaseous mixture to the cylinders the crankshaft was made hollow. Through the hollow shaft, the mixture passed into the cylinders. The mixture in the crankcase was very dangerous. A backfire might result in the crankcase exploding, tearing the engine apart.

The gyroscopic action of the rotary engine is quite pronounced. On certain wartime airplanes, the pilots found that pushing the rudder for a right turn started a precession so powerful that the plane would be nosed down. Pushing the left rudder caused the plane to nose up to a dangerous stall. As the enemy pilots were not unaware of this action, they would approach these planes from the left side. The pilots of the rotary-engined plane were thus faced with the alternatives of attempting a left turn with its possibility of loss of control or of making a right turn of 270° or more while completely exposed to enemy fire.

Another objection to the rotary engines was that, on stopping, the lower cylinders became filled with oil. Because of the above-listed objections, and because on modern engines it is found that no flywheel action in addition to that of the propeller is needed, rotary engines are no longer manufactured.

Diesel Engines. Engines operating on the Diesel principle have been used successfully as stationary engines and as power plants for large marine vessels. They have always been very heavy in pounds per horsepower compared with engines operating on the Otto-cycle principle.

For reliable performance, the ordinary aircraft engine requires a high-test gasoline. With high compression, special compounds are desirable to prevent pre-ignition. An especially volatile gasoline will cause trouble by vapor-lock in the lines unless the plumbing is specially designed to guard against this.

Because of the high cost of special gasolines for the standard aircraft engines, thought has been given recently to the adaptation of the Diesel cycle to a light-weight aircraft engine. Not only will the Diesel engine use a very cheap oil, but also, since it depends on compression for ignition, electrical troubles are banished. The fuel consumption of a Diesel is a little more than half the pounds-per-horsepower-hour of a gasoline engine.

Two manufacturers have to date produced aircraft Diesel engines, having no greater size than a radial gasoline-fueled engine of the same horsepower. Because of the low weight of fuel consumed per horsepower-hour, an airplane equipped with a Packard-Diesel engine now holds the world's endurance record of 84 hours and 32 minutes.

In the Diesel engine, clean air is drawn into the cylinder and, as the piston moves up, the air is compressed to a pressure of 1,100 lb. per sq. in. While the piston is at the end of its stroke, oil fuel is sprayed into the cylinder under great pressure. Since, owing to compression, the air is at a high temperature, the oil immediately ignites.

As the engine is running at high speed, the time at the end of the compression stroke, during which the oil is injected, is infinitesimally short. During a minute fraction of a second, the drops of oil must be broken up into droplets and mixed with the air. Unless the oil is finely atomized in the brief working stroke, a drop of oil will not burn completely inward to its core and there will be incomplete combustion.

For various reasons, airplane designers have been chary of adopting the Diesel engine as standard equipment. With further development and improvement, however, it undoubtedly will be viewed with more favor.

Brake Horsepower. The power of an engine depends on the force exerted on a piston being transmitted through a connecting rod to a crank, to produce a turning or twisting moment on the shaft. If a pressure-indicating instrument were connected to each cylinder and the pressure measured throughout the cycle it would be possible to calculate the horsepower of the engine. This is the indicated horsepower.

Owing to various friction losses in the engine itself, not all the indicated horsepower can be used for outside work. The indicated horsepower less the power required to overcome the friction in the engine leaves as remainder the brake horsepower, which is the power of the engine to do outside work. The ratio of brake horsepower to indicated horsepower is the mechanical efficiency of the engine, usually about 90 per cent. The indicated horsepower multiplied by the mechanical efficiency gives the brake horsepower.

The mean effective pressure, in pounds per square inch, multiplied by the area of the piston, in square inches, gives the average force acting on the piston during a cycle. If this force is multiplied by the stroke, in feet, the product is the work, in foot-pounds, applied to the piston during a cycle. If the work per cycle is multiplied by the number of cycles per minute, which in the four-cycle engine is one-half the revolutions per minute, the product is the work per minute. This may be expressed as follows

$$\text{Horsepower (indicated)} = \frac{PLANn}{33{,}000}$$

P = mean effective pressure
L = stroke in feet
A = piston area of one cylinder in square inches
N = working strokes per minute = $\frac{1}{2} \times$ revolutions per minute
n = number of cylinders

$$\text{Horsepower (brake)} = \text{Mechanical efficiency} \times \text{Horsepower indicated}$$

Increasing the compression ratio will have the effect of increasing the mean effective pressure and thus increasing the horsepower.

From the foregoing formula, it would appear that horsepower varies directly with revolutions per minute. Within a range close to the rated revolutions per minute, this is true within a reasonable degree of accuracy, below rated power and speed. Above rated speed, the frictional losses get to be very large and the mechanical efficiency is very poor, so that above rated speed the horsepower no longer increases directly as speed.

The variation of horsepower with engine speed has been tested for a large number of engines, both air and water cooled, and the results conform closely with the relation described above. For any particular engine, the manufacturer can furnish information of the relation of horsepower to speed for its type.

The load on the engine is furnished by the propeller. The propeller is chosen for the engine so that at a selected airspeed and rated engine speed the propeller will offer enough torsional resistance to absorb all the horsepower of the engine and the engine will not speed up. At an airspeed less than that for which the propeller is assigned, the propeller will offer more resistance to turning, so that even if the throttle is at the same setting as for full rated power, the revolutions per minute will be less. Slowing up the engine will have the effect of decreasing the brake horsepower. At less than design airspeed, the brake horsepower will be less.

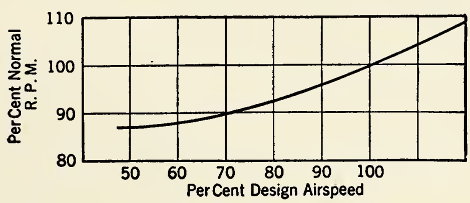


FIG. 44*a*. Variation of revolutions per minute with airspeed.

Effect of Altitude on Horsepower. An engine is an apparatus for burning fuel. If insufficient oxygen is supplied, the fuel cannot burn properly. Each drop of fuel needs a certain quantity of air for complete combustion; that quantity not being supplied, combustion will not be complete. By quantity is meant mass or weight, not volume.

In a carburetor, air picks up gasoline vapor to form a mixture which is drawn into the engine cylinders. Under normal conditions, the mixture drawn into the engine is at sea-level density; and provided the carburetor is adjusted properly, there will be an adequate supply of air to support the combustion of the gasoline vapor.

At altitudes, the air is of less density, so that although the same volume of air is drawn into each cylinder, the mass of air will be less. The mixture will be richer than at ground level, and a good pilot leans his mixture (i.e., changes his carburetor adjustment from the cockpit) and opens his throttle.

The brake horsepower decreases with altitude. Tests of engines have been made in an altitude chamber, which is a room from which air can be pumped so that the air densities in the chamber can simulate various altitudes. As a result of these tests, it appeared that at constant revolutions per minute, i.e., maintaining the same r.p.m. as at the ground:

$$\frac{\text{B.H.P.}}{\text{B.H.P.}_0} = \left(\frac{p}{p_0}\right)^{1.15} \text{ at constant temperature}$$

and

$$\frac{\text{B.H.P.}}{\text{B.H.P.}_0} = \sqrt{\frac{T_0}{T}} \text{ at constant pressure}$$

Then

$$\frac{\text{B.H.P.}}{\text{B.H.P.}_0} = \left(\frac{p}{p_0}\right)^{1.15} \times \left(\frac{T}{T_0}\right)^{-0.5}$$

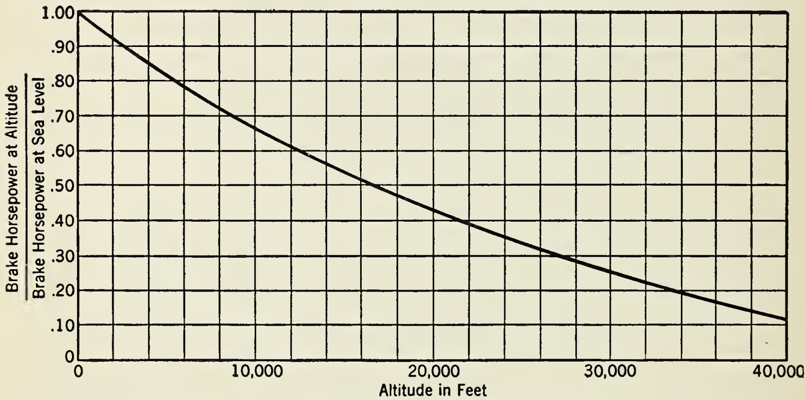


FIG. 44b. Variation of brake horsepower with altitude at constant r.p.m.

In standard atmosphere

$$\frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{4.256}$$

and

$$\frac{\rho}{\rho_0} = \left(\frac{p}{p_0}\right)^{0.81}$$

Substituting these values in the expression for ratio of brake horsepower at altitude to brake horsepower at sea-level

$$\frac{\text{B.H.P.}}{\text{B.H.P.}_0} = \left(\frac{\rho}{\rho_0}\right)^{1.15/0.81} \left(\frac{\rho}{\rho_0}\right)^{-0.5/4.256}$$

$$= \left(\frac{\rho}{\rho_0}\right)^{1.42} \left(\frac{\rho}{\rho_0}\right)^{-0.12}$$

$$= \left(\frac{\rho}{\rho_0}\right)^{1.3}$$

Some slight modifications have been made to this relation at higher altitude, and the variation of horsepower and altitude that is believed to be most nearly correct is given graphically in Fig. 44*b*.

CHAPTER IX

PROPELLERS

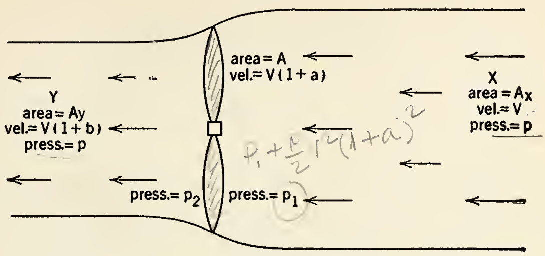
Function. The air screw propeller is the device which changes the torque power of the engine into forward thrust, thus impelling the airplane forward. In the United States it is termed simply the propeller; in Europe, the air screw.

A wood screw forces its way into wood by the back face of the screw thread pressing against the wood. A bolt enters a fixed nut by the back face of the bolt thread sliding against the front face of the nut thread. With a marine propeller, it is the rear face of the blades which pushes against the water. The very first aircraft propellers were built with this viewpoint; very shortly thereafter, however, it was conceived that the blades might be considered as wings, or rather as a number of wings of infinitely short span set end to end. Treating the blades as airfoils makes of primary importance the front face of the blades, i.e., the upper surface of the airfoils.

A propeller should be efficient in transforming the rotary power imparted to it into forward tractive power. Since the propeller is attached to the engine, at whatever speed the engine is run, the propeller must be able to absorb the power furnished by the engine. The propeller cannot be most efficient at all engine speeds and all airspeeds. It is therefore designed to give maximum efficiency at the rated revolutions per minute of the engine and some one airspeed, either maximum speed or whatever is decided upon as cruising speed. At other rates of revolution and airspeeds, the propeller will be less efficient than for the design conditions.

Momentum Theory. The first fundamental propeller theory was the momentum theory developed by R. E. Froude. It deals with the changes in energy of the mass of air affected by the propeller.

In this theory, the propeller is assumed to be a disk which exerts a uniform pressure or thrust over the cross-section of the air column passing through the disk. It is further assumed that the disk imparts no twisting or rotation to the air column, and that flow remains streamline while coming to the disk, passing through it, and beyond it.



$p + \frac{\rho}{2} V^2$

Fig. 45. Momentum theory of propeller action.

Referring to Fig. 45, X is a point in the air stream far enough in front of the propeller so that the pressure is atmospheric; Y is a point in the air stream far enough to the rear of the propeller so that the pressure is atmospheric.

Let
A = propeller disk area.
A_X = cross-sectional area of air column at X.
A_Y = cross-sectional area of air column at Y.
V = velocity at X.
$V(1 + a)$ = velocity at propeller disk.
$V(1 + b)$ = velocity at Y.
p = atmospheric pressure = static pressure at X and Y.
p_1 = static pressure just ahead of propeller disk.
p_2 = static pressure just behind disk.

Since the volume of air flowing through each area in unit time is the same,

$$A_X V = A V(1 + a) = A_Y V(1 + b)$$

No energy is added or subtracted to the air from X to a point immediately in front of the propeller disk, so Bernoulli's equation may be applied, and the sum of static and velocity heads at these two points placed equal.

$$p + \frac{\rho}{2} V^2 = p_1 + \frac{\rho}{2} V^2(1 + a)^2$$

$$p_1 = p + \frac{\rho}{2} V^2[1 - (1 + a)^2]$$

No energy is added or subtracted to the air from a point immediately in the rear of the disk to point Y, so the total heads at these points may likewise be placed equal.

$$p_2 + \frac{\rho}{2} V^2(1 + a)^2 = p + \frac{\rho}{2} V^2(1 + b)^2$$

$$p_2 = p + \frac{\rho}{2} V^2[(1 + b)^2 - (1 + a)^2]$$

It will be noted that the velocity immediately in front is assumed to be the same as the velocity immediately in the rear of the disk, otherwise there would be an instantaneous increase in velocity and an infinite acceleration, which cannot be. The propeller disk does exert thrust on the air column, and the thrust (T) is equal to the difference in pressure created by the propeller times the area of the propeller disk.

$$T = (p_2 - p_1)A$$

Substituting values of p_1 and p_2 already found

$$T = A\left[p + \frac{\rho}{2} V^2([1 + b]^2 - [1 + a]^2) - p - \frac{\rho}{2} V^2[1 - (1 + a)^2]\right]$$

$$= \frac{\rho}{2} A V^2[(1 + b)^2 - 1]$$

$$= \frac{\rho}{2} A V^2(b^2 + 2b)$$

Thrust is also equal to the rate of change of momentum which is the mass of air affected per second times the change in velocity. The mass of air handled per second is the density multiplied by the volume, and the volume is the cross-sectional area at any point times the velocity at the same point. At the propeller disk the area is A and the velocity $V(1 + a)$, so the mass of air passing through the propeller disk in 1 sec. is $\rho A V(1 + a)$. At point X, the velocity is V; at point Y, the velocity is $V(1 + b)$ or $V + bV$; therefore the change in velocity imparted to the air column by the propeller is bV.

Then

$$T = \rho A V(1 + a)bV$$

$$= \rho A V^2(1 + a)b$$

Equating this value for T to the one previously found:

$$\begin{aligned}
\rho A V^2(1 + a)b &= \frac{\rho}{2} A V^2(b^2 + 2\,b) \\
b + ab &= \tfrac{1}{2}(b^2 + 2\,b) \\
2\,b + 2\,ab &= b^2 + 2\,b \\
2\,a &= b \\
a &= \frac{b}{2}
\end{aligned}$$

This means, that, of the total increase in velocity imparted to the air column, one-half is added before the air passes through the propeller disk.

To find the ideal efficiency, the power exerted by the propeller is divided by the power absorbed.

The power exerted by the propeller is the thrust times velocity or

$$\begin{aligned}
\text{Power output} &= TV \\
&= \rho A V^2(1 + a)bV \\
&= \rho A V^3(1 + a)b
\end{aligned}$$

The power absorbed by the propeller is the work done on the air column in unit time, which is the kinetic energy added in unit time to the kinetic energy at X, to give the kinetic energy at Y. Kinetic energy is one-half the mass times the square of the velocity, and the mass can be measured at any point.

The kinetic energy at Y is

$$\begin{aligned}
\text{K.E.}_Y &= \tfrac{1}{2}[\rho A V(1 + a)]\,[V(1 + b)]^2 \\
&= \frac{\rho}{2} A V^3(1 + a)(1 + b)^2
\end{aligned}$$

The kinetic energy at X is

mass

$$\begin{aligned}
\text{K.E.}_X &= \tfrac{1}{2}[\rho A V(1 + a)]V^2 \\
&= \frac{\rho}{2} A V^3(1 + a)
\end{aligned}$$

The difference in kinetic energies at Y and X which represents the work absorbed by the propeller in unit time is the power input.

$$\begin{aligned}
\text{Power input} &= \frac{\rho}{2} A V^3(1 + a)(1 + b)^2 - \frac{\rho}{2} A V^3(1 + a) \\
&= \frac{\rho}{2} A V^3(1 + a)(b^2 + 2\,b)
\end{aligned}$$

Substituting $b = 2a$:

$$\text{Power input} = \frac{\rho}{2} A V^3 (1 + a)(2ab + 2b)$$
$$= \rho A V^3 b (1 + a)^2$$

Therefore

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}}$$
$$= \frac{\rho A V^3 b(1 + a)}{\rho A V^3 b (1 + a)^2}$$
$$= \frac{1}{1 + a}$$

This is the ideal efficiency of a perfect propeller having no losses due to rotation of the slip stream, profile drag of the blades, interference of blades, or radial flow. The only loss considered in the momentum theory is the kinetic energy loss in pure translation. The momentum theory would apply equally well to any device which applies thrust or additional pressure to an air column.

Blade-Element Theory. As the blade-element theory was first put in practical form by Drzewiecki, this conception of the action of the propeller is usually referred to as the Drzewiecki theory. This treatment involves considering the blades as being composed of an infinite number of small airfoils. The span of each airfoil is an infinitely small distance along the length of the blade. All these small airfoils are joined together, wing tip to wing tip, to form one twisted or warped wing which is the propeller blade.

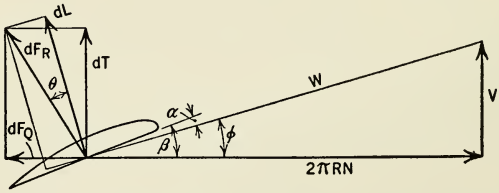


FIG. 46. Blade element theory of propeller action.

Each small section of the blade may be studied separately and the results summed up to get the total action of the blade. Figure 46 shows one of these elements of the blade, which is located a distance of R feet from the axis of the propeller. In one rotation of the propeller, this blade element travels a distance of $2\pi R$ feet around a circle in the plane of the propeller disk. If the propeller is turning over N revolutions per second, the linear velocity of the element in the plane of rotation is $2\pi RN$ feet per second. If the airplane has an airspeed of V feet per second, the propeller is moving forward with that speed. Since the motion of the airplane is practically in the direction of the thrust line of the propeller, here it is sufficiently accurate to state that the motion of the blade element is the resultant of a velocity of $2\pi RN$ feet per second in the plane of the propeller disk combined with a velocity of V feet per second perpendicular to the plane of the propeller disk. This resultant velocity will be in a direction making an angle ϕ with the propeller disk such that

$$\tan \phi = \frac{V}{2\pi RN}$$

The magnitude of this resultant velocity (W) is

$$W = \frac{V}{\sin \phi} = \frac{2\pi RN}{\cos \phi}$$

The blade element has a cross-section which has the contour of an airfoil section, usually modifications of the Clark Y or R.A.F.6 sections. The angle which the reference chord of this section makes with the plane of rotation of the propeller is the blade angle β. The angle of attack α of the blade element is chosen as the angle of maximum L/D of the airfoil section used. It is to be noted that every blade element moves forward with the same velocity V, but the linear speed of rotation is greater for blade elements nearer the tip of the propeller, so the relative velocity W will be greater at the tips. The angle ϕ is less for blade elements near the tip than for those nearer the center. Since

$$\alpha = \beta - \phi$$

if the same angle of attack is maintained along the blade, the blade angle β becomes less as the tip is neared. This is why a propeller blade has its warped or twisted appearance.

The air flows around each element and causes a resultant as with any airfoil. The area of the element is the span, dR, times the chord, which is b, the blade width at that point. The velocity is W. The lift force on the element is

$$dL = C_L \frac{\rho}{2} b \, dRW^2$$

The resultant force is acting in a different direction from the lift component; let the small angle between these two directions be called angle θ. Then

$$\cot \theta = \frac{L}{D}$$

If the propeller blade is so constructed that under design conditions (i.e., normal revolutions per minute and design airspeed) every airfoil section is meeting the air at the same angle of attack (α); then, if the same airfoil section is used throughout, θ will be the same all along the blade. In practice, for strength, sections near the hub are thicker than those near the tip.

The resultant force on the blade element will be

$$dF_R = \frac{dL}{\cos \theta}$$

$$= \frac{C_L \frac{\rho}{2} b \, dR W^2}{\cos \theta}$$

This resultant force, instead of being divided into lift and drag components, is divided in components parallel to the propeller axis and parallel to the plane of rotation. The component parallel to the propeller axis is called the thrust component; that parallel to the plane of rotation is called the tangential or torque component.

The lift component is perpendicular to the direction of the relative wind (W), and the relative wind makes an angle of ϕ with the plane of rotation; therefore the lift component makes the same angle ϕ with the direction of the propeller axis. The resultant force is at angle $(\phi + \theta)$ with the direction of the propeller axis. The thrust component is then

$$dT = dF_R \cos (\phi + \theta)$$
$$= C_L \frac{\rho}{2} b \, dR W^2 \frac{\cos (\phi + \theta)}{\cos \theta}$$

The torque or tangential component is

$$dF = dF_R \sin (\phi + \theta)$$
$$= C_L \frac{\rho}{2} b \, dR W^2 \frac{\sin (\phi + \theta)}{\cos \theta}$$

The torque itself, which is the moment which must be applied, is the tangential force times its moment arm R. Letting Q represent torque

$$dQ = R\,dF_R \sin(\phi + \theta)$$

The power absorbed will be the torque component force times the distance traveled per second:

$$dP_a = 2\,\pi N R\,dF_R \sin(\phi + \theta)$$

The efficiency of each element is the power output divided by the power input or

$$\text{Efficiency} = \frac{V\,dT}{2\,\pi N\,dQ}$$
$$= \frac{V\,dF_R \cos(\phi + \theta)}{2\,\pi N R\,dF_R \sin(\phi + \theta)}$$

Substituting $\dfrac{V}{2\,\pi R N} = \tan\phi$

$$\text{Efficiency} = \frac{\tan\phi}{\tan(\phi + \theta)}$$

Tangent ϕ is the ratio of the airspeed V to the tangential velocity of the blade element. Every element moves forward with the same airspeed, but the tangential velocity is greater nearer the blade tip. For elements near the tip tangent ϕ is smaller than for elements nearer the hub. Cotangent θ is L/D for the blade element. If the same airfoil section is used at the same angle of attack throughout the blade, θ will be constant. By differentiating the efficiency with respect to ϕ and placing result equal to zero

$$\phi = 45° - \frac{\theta}{2}$$

For the Clark Y section, used frequently for propellers, the maximum L/D is 22.5, making θ have a value of $2\frac{1}{2}°$. Then the maximum efficiency will be when $\phi = 45° - 1\frac{1}{4}°$, and the efficiency will be

$$= \frac{\tan 43\frac{3}{4}°}{\tan 46\frac{1}{4}°}$$
$$= \frac{0.957}{1.045}$$
$$= 91.5 \text{ per cent}$$

This will be for the most efficient element. Nearer to or farther from the tip the blade elements will be less efficient, so the total efficiency of the entire blade will be less than the maximum efficiency of the optimum blade element.

Returning to the thrust of a single blade element,

$$dT = \frac{C_L \frac{\rho}{2} b\, dR W^2 \cos(\phi + \theta)}{\cos \theta}$$

The blade width (b) instead of being expressed in feet might be given as a fractional part (b/R) of the radial distance out to that particular blade element. The ratio b/R is a pure number, but this ratio is different for each element. The blade width is then $R(b/R)$.

Also W, the relative velocity, is equal to $2\pi NR/\cos \phi$. The angle θ is always small so that little error is introduced if $\cos \theta$ is assumed to be unity. The distance R to each blade element is expressed as $R/R_1 \times R_1$ or $R/R_1 \times D/2$, where R_1 is the radius of the tip and D is the diameter. Making these substitutions, the expression for thrust becomes

$$dT = \frac{\rho}{2}\left[\frac{\pi^2}{4}\, C_L\, \frac{b}{R}\, \frac{\cos(\phi + \theta)}{\cos^2 \phi} \left(\frac{R}{R_1}\right)^3 d\,\frac{R}{R_1}\right] N^2 D^4$$

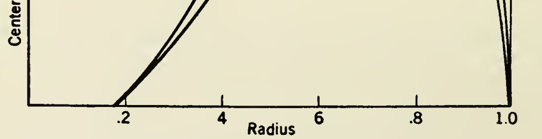


FIG. 47. Typical thrust and torque force curves.

The quantity in the brackets contains terms which vary for each blade element. Values of the bracketed quantity may be plotted against (R/R_1) as in Fig. 47, and a curve of thrust variation along the radius may be drawn. All the terms in the bracket depend on the airfoil section used and its angle of attack, on the variation of blade width with respect to radius, and on the angle ϕ. For geometrically similar propellers, the bracketed quantity depends on the angle ϕ, which is the angle whose tangent is the airspeed divided by the peripheral speed of the tip, since the angle ϕ of any blade element on one propeller will be the same as the angle ϕ of the blade element of a similar propeller located the same fractional distance out from the center. The thrust curve depends therefore on $V/(\pi ND)$. It is common to drop the constant $1/\pi$ and state merely that the thrust curve depends on the $V/(ND)$ of the propeller. V being airspeed in feet per second, N being the revolutions per second, and D being the diameter in feet, the expression $V/(ND)$ is dimensionless.

The area under the thrust curve of Fig. 47 is integrated and multiplied by the number of blades to give the thrust coefficient (T_c) for the entire propeller. The equation for the thrust of the whole propeller is

$$T = \frac{\rho}{2} T_c N^2 D^4$$

The expression for the power absorbed by an individual blade element may be modified in a similar manner.

$$\begin{aligned} dP_a &= 2\,\pi NR\, dF_R \sin(\phi + \theta) \\ &= 2\,\pi NRC_L \frac{\rho}{2} b\, dRW^2 \frac{\sin(\phi + \theta)}{\cos\theta} \\ &= \frac{\rho}{2}\left[\frac{\pi^3}{4} C_L \frac{b}{R} \frac{\cos(\phi + \theta)}{\cos^2\phi} \left(\frac{R}{R_1}\right)^4 d\frac{R}{R_1}\right] N^3 D^5 \end{aligned}$$

The bracketed quantity is plotted against R/R_1 in Fig. 47. Integrating the area under the curve and multiplying by the number of blades gives a power coefficient (P_c). The equation for power absorbed by a propeller becomes

$$P_a = \frac{\rho}{2} P_c N^3 D^5$$

It will be noted, in Fig. 47, that the blade elements inside of 20 per cent R contribute no thrust. To withstand centrifugal force the blades are made extremely thick near the hub and are consequently very poor airfoil sections. This, together with the low relative wind speed and large angle ϕ, makes for small thrust.

The efficiency (η) of a propeller may be found in terms of these coefficients, as follows

$$\eta = \frac{TV}{P}$$

$$= \frac{\left(\frac{\rho}{2} T_c N^2 D^4\right) V}{\frac{\rho}{2} P_c N^3 D^5}$$

$$= \frac{T_c}{P_c} \times \frac{V}{ND}$$

The simple blade-element theory neglects the inflow velocity. It also neglects the fact that air passing around a wing is given a downward velocity, or if circulating around a propeller blade is given a similar flow. The blade of a propeller, instead of meeting a relative wind from the direction W, as shown in Fig. 46, is meeting air disturbed by the previous blade passage. The theory also neglects tip losses.

A combination of the two basic theories, the momentum and blade-element theories, with certain corrections has been found to give excellent results in the design or selection of propellers.

Thrust and Torque Coefficients. The coefficients for thrust (T_c) and power (P_c) described in the simple blade-element theory are used with certain modifications in the complete propeller theory. These coefficients depend not only on the shape and proportions of the blades, but also on the airspeed and speed of rotation; i.e., on the $V/(ND)$ of the conditions under which the propeller is run. They are useful in comparing the thrust developed and power required by different propellers of the same diameter. For use on an airplane, a propeller is selected for a certain V and a certain N, and these coefficients, though suitable for comparing propellers at the same $V/(ND)$, are not applicable for propellers for the same V and N but different D.

The coefficients T_c and P_c are non-dimensional. This can be proved by putting the other terms of the equations for thrust and power into the basic dimensions of mass (M), length (L), and time (t).

$$T = \frac{\rho}{2} T_c N^2 D^4$$

$$\frac{ML}{t^2} = \frac{M}{L^3} \times T_c \times \frac{1}{t^2} \times L^4$$

$$P = \frac{\rho}{2} P_c N^3 D^5$$

$$\frac{ML}{t^2} \times \frac{L}{t} = \frac{M}{L^3} \times P_c \times \frac{1}{t^3} \times L^5$$

For a given propeller (or family of geometrically similar ones), the thrust and power coefficients as well as the efficiency may be plotted against $V/(ND)$. The efficiency is a maximum at one particular value of $V/(ND)$, decreasing for either larger or smaller values. The value of $V/(ND)$ for a given propeller which gives the maximum efficiency is called the design $V/(ND)$. Curves showing how a propeller efficiency changes with $V/(ND)$ are presented in Fig. 48.

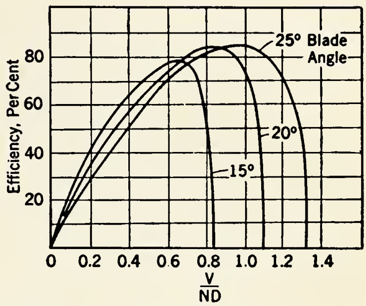


FIG. 48. Typical efficiency curves.

The thrust and power coefficients of a given propeller will vary with the airspeed and revolutions per minute, i.e., with $V/(ND)$. For a typical propeller they are plotted in Fig. 49. This plot will apply to all propellers which are geometrically similar to the one for which the data are plotted.

It will be noted that the thrust coefficient decreases as the $V/(ND)$ increases. At the $V/(ND)$ of 1.3 for this propeller the thrust coefficient becomes zero. This would correspond to a steep dive with power on. At this $V/(ND)$ of zero thrust, the torque coefficient still has a positive value. The thrust coefficient being zero means that the relative wind is meeting the blades at such an angle that the resultant force is parallel to the plane of the propeller disk. Any further increase in airspeed will mean that the relative wind will meet the blades at an angle which will produce negative or backward thrust. The condition of zero thrust corresponds with angle of zero lift for an airfoil and means that the blades are moving forward the greatest possible distance per revolution without developing negative thrust. Pitch being the forward distance per revolution, the $V/(ND)$ for zero thrust, multiplied by $1/\pi$, is called the *experimental mean pitch,* since it is the ideal or maximum value of pitch for the particular propeller.

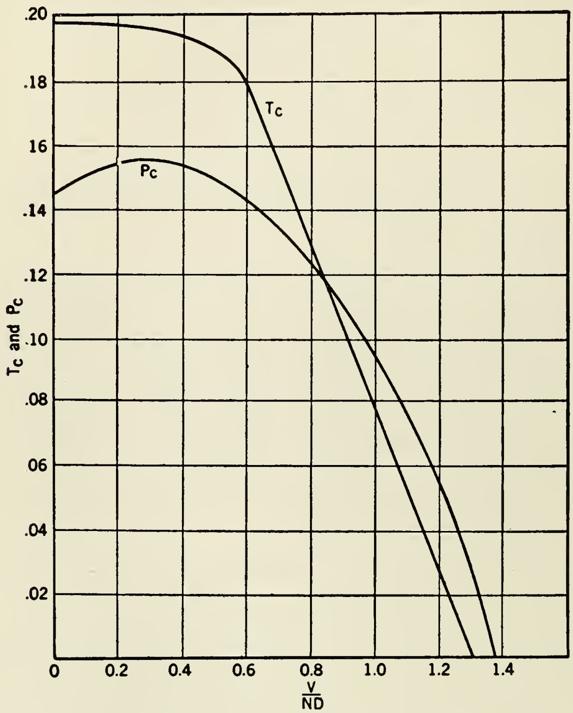


Fig. 49. Typical thrust and power coefficient curves.

When the airplane is stationary, V is zero, making $V/(ND)$ zero. Under this condition, both thrust and power coefficients have large positive values. The value of the thrust coefficient for $V/(ND) = 0$ is a measure of the ability of an airplane to accelerate from a standstill position.

Power-Speed Coefficients. The expression $V/(ND)$ is a nondimensional quantity. For a given propeller (or geometrically similar ones) there are only one T_c, one P_c, and one efficiency for each $V/(ND)$ condition. The coefficient P_c may therefore be properly divided by the corresponding $V/(ND)$ or a power thereof to obtain another coefficient of different magnitude. Using $V/(ND)$ as a divisor, the diameter is eliminated from the equation for power absorbed.

$$P = \frac{\rho}{2} P_c N^3 D^5$$

$$= \frac{\rho}{2}\left[\frac{P_c}{\left(\frac{V}{ND}\right)^5}\right] N^3 D^5 \left(\frac{V^5}{N^5 D^5}\right)$$

$$= \frac{\rho}{2}\left[\frac{P_c}{\left(\frac{V}{ND}\right)^5}\right] \frac{V^5}{N^2}$$

Substituting

$$C_s = \frac{V}{ND}\sqrt[5]{\frac{2}{P_c}}$$

$$P = \frac{\rho V^5}{N^2 C_s^5}$$

The coefficient C_s is called the power-speed coefficient, since, if this coefficient is known together with the forward speed and speed of rotation, the power may be found. Conversely, if the available power (P), the rated revolutions per minute of the engine (N), and the design airspeed of the airplane (V) are known, the C_s can be found. A propeller is then chosen of the proper diameter and pitch to give the maximum efficiency with this power-speed coefficient (C_s).

Pitch and Pitch Ratio. The pitch of a single-thread machine screw is the distance the screw is advanced in one revolution or the distance between threads. Using the convention that a screw is an inclined plane wrapped around a cylinder, the pitch would be the slope of this inclined plane. In applying the terminology of mechanics to an air screw, the difficulty is met that air is not a solid medium and the blade angle differs along the blade.

"Geometric pitch" is the term applied to the calculated distance an air propeller would move forward in one revolution if there were no slip, i.e., if air were solid. This definition cannot be applied without amplification. Each blade element is set at a different blade angle. For uniform geometric pitch the blade angle would decrease as the radius increased. On account of engine interference near the hub the air flow is slower and the pitch is usually made less there. Owing to the non-uniformity of the pitch, the blade element two-thirds the radius was formerly used to measure geometric pitch. For correct performance, it is necessary to have the proper diameter as well as proper pitch, and since it is physically impossible for a propeller manufacturer to stock an infinite number of different diameters as well as an infinite number of pitches, if the user cannot obtain the exact diameter propeller he desires, he saws off and trims the tips of a larger-diameter propeller. Reducing the diameter would change the location of the two-thirds point on the radius, and the measured geometric pitch would be different. It has now become the custom generally to measure the geometric pitch at the blade element at a point 42 in. from the axis and call this the standard or nominal pitch. The geometric pitch is the tangent of the blade angle multiplied by 2π times the radial distance of the blade element from the axis.

"Effective pitch" is the actual distance the propeller travels forward in one revolution and is equal to V/N feet.

"Slip" is the difference, in feet, between the geometric pitch and effective pitch.

"Pitch ratio" is the ratio of the geometric pitch to the propeller diameter.

With a fixed-blade propeller the geometric pitch is permanently fixed. Adjustable-pitch propellers have blades with circular cross-sections near the hub which are gripped in sleeves. By loosening clamping rings, the blades may be turned in their sockets, so that they have a new blade angle and consequently new geometric pitch. An adjustable-pitch propeller can have its blade angles changed only when the engine is at rest, i.e., when the plane is on the ground. A controllable-pitch propeller is one whose blade angles may be changed in flight. This is accomplished by electrical or oil-pressure-operated devices. The term variable-pitch propeller may be applied to either adjustable- or controllable-pitch propellers.

With a fixed-blade propeller, the effective pitch varies: it is zero when the airplane is stationary, and it has a finite value as the airplane moves forward. In level flight, the effective pitch has a value approximately 75 per cent of the geometric pitch; in a dive, it increases still further and may even exceed the geometric pitch.

Propellers are selected for certain design requirements, i.e., for the $V/(ND)$ of the desired flight conditions. While the propeller is operating under these design conditions, the effective pitch is D times the design $V/(ND)$, and the angle of attack (α) of the blade elements is the angle of maximum L/D. The blade angle of a fixed-blade propeller being constant, when the effective pitch is decreased by flight conditions, with the decreased $V/(ND)$ there is an increase in the angle of attack and consequently a decrease in L/D. When the airplane is stationary, the effective pitch and $V/(ND)$ being zero, the angle of attack of each element is equal to the blade angle of the element. For the portion of the blade near the center, the angle of attack is greater than the burble point, and the efficiency is therefore low.

In climbing, the airspeed is low while the engine is running at full r.p.m. so that the $V/(ND)$ is much less than the design $V/(ND)$. The effective pitch during climb is such that the angle of attack α of the blades is greater than the angle of maximum L/D, so the efficiency is low.

Improvement in efficiency which would mean obtaining greater thrust with the same horsepower expenditure can therefore be gained in take-off and in climb by using a controllable-pitch propeller. With a small blade angle in take-off or climb, the angle of attack can be nearly if not exactly the angle of maximum L/D. After the altitude is reached where level flight is to be maintained, the blades can be reset to a new geometric pitch, which will give the correct blade angle for level flight conditions.

Determining Proper Power-Speed Coefficient. The equation for power using the speed-power coefficient C_s is

$$P = \frac{\rho V^5}{N^2 C_s^5}$$

P in foot-pounds per second
ρ in slugs per cubic foot
V in feet per second
N is revolutions per second

In order to employ the more usual units, this formula is modified, as follows.

$$550 \times \text{H.P.} = \frac{\rho \times \left(\dfrac{88\ \text{M.P.H.}}{60}\right)^5}{\left(\dfrac{\text{R.P.M.}}{60}\right)^2 C_s^5}$$

$$\text{H.P.} = \frac{44.5 \times \rho \times \overline{\text{M.P.H.}}^5}{\overline{\text{R.P.M.}}^2 \times C_s^5}$$

For sea-level conditions, where ρ is 0.002378 slug per cubic foot

$$\text{H.P.} = \frac{44.5 \times 0.002378 \times \overline{\text{M.P.H.}}^5}{\overline{\text{R.P.M.}}^2 \times \overline{C_s}^5}$$

$$= \frac{0.106 \times \overline{\text{M.P.H.}}^5}{\overline{\text{R.P.M.}}^2 \times \overline{C_s}^5}$$

Solving for C_s gives

$$C_s = \text{M.P.H.} \sqrt[5]{\frac{0.106}{\text{R.P.M.}^2 \times \text{H.P.}}}$$

$$= \frac{0.638 \times \text{M.P.H.}}{\overline{\text{R.P.M.}}^{\frac{2}{5}} \times \overline{\text{H.P.}}^{\frac{1}{5}}}$$

Therefore, the available horsepower, engine revolutions, and airspeed being known, C_s may be found.

This formula does not lend itself readily to slide-rule operation. It may be solved by logarithms or by use of the chart of Fig. 50.

In using the chart, a line is drawn from the origin O through the intersection of lines representing velocity and revolutions per minute. The intersection of this line with the horsepower line gives the value of C_s.

Example. The engine gives 400 hp. at 1,900 r.p.m. What should be the C_s of the propeller for an airspeed of 150 miles per hour?

Solution. See dotted lines on Fig. 50. From O draw diagonal line through intersection of lines representing 150 miles per hour and 1,900 r.p.m. Where diagonal line intersects line representing power of 400 hp., read on C_s scale 1.40.

Problems

(Use logarithms and check by nomogram.)

1. A Stearman plane uses a Kinner engine which is rated 210 hp. at 1,900 r.p.m. What should be the C_s of a propeller for an airspeed of 100 miles per hour?

2. A Monocoupe uses a Warner engine rated 125 hp. at 2,050 r.p.m. What should be the C_s of the propeller for an airspeed of 112 miles per hour?

3. A Taylor Cub cruises at 65 miles per hour. It is powered with a Continental engine, giving 37 hp. at 2,550 r.p.m. What should be the C_s of the propeller?

4. An Aeronca Collegian cruises at 65 miles per hour, with an engine rated 36 hp. at 2,400 r.p.m. What should be the C_s of the propeller?

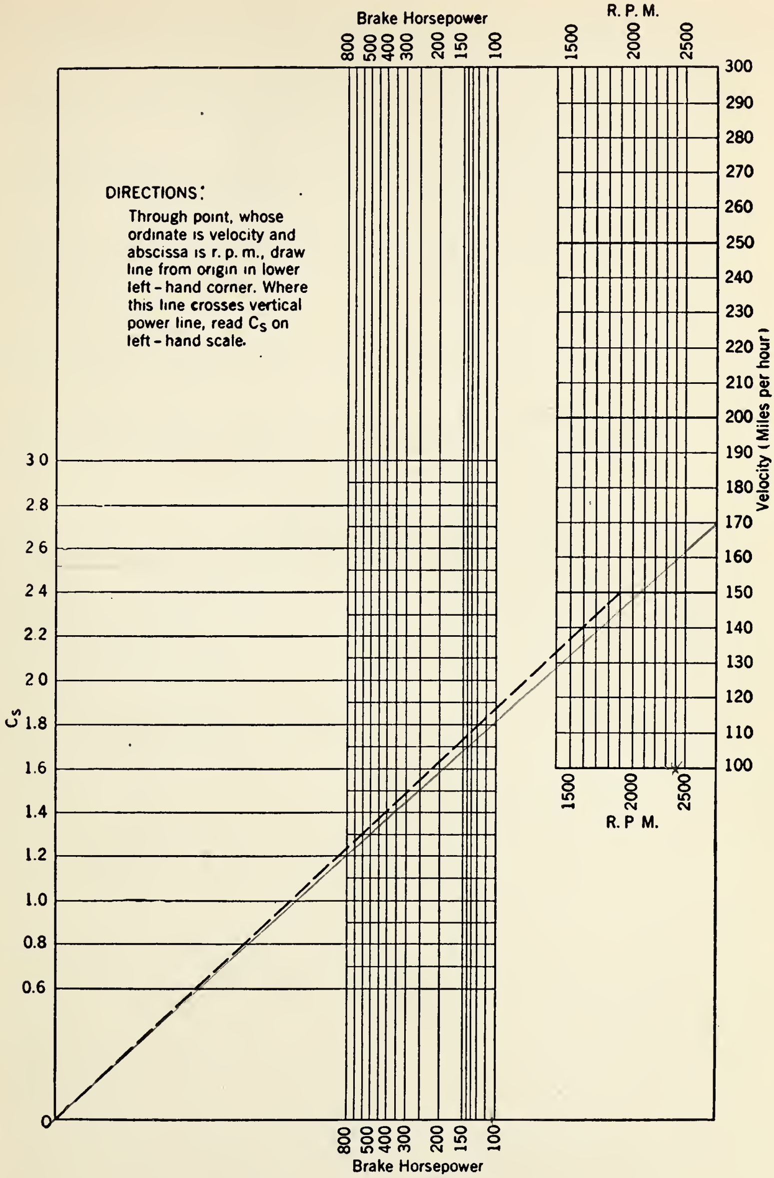


FIG. 50. Chart for finding C_s.

5. A Laird Speedwing is equipped with a Wasp Junior engine of 300 hp. at 2,000 r.p.m. What is the C_s of a propeller suitable for cruising at 150 miles per hour?

6. For the airplane described in problem 5, what is the C_s of a propeller for racing at 190 miles per hour?

7. A Douglas Airliner cruises at 185 miles per hour, equipped with two Wright engines each giving 700 hp. at 1,900 r.p.m. What should be the C_s of suitable propellers?

8. A Lockheed Vega cruises at 150 miles per hour, equipped with a Wasp engine of 420 hp. at 2,000 r.p.m. What is the C_s of a suitable propeller?

9. A Bellanca Pacemaker cruises at 125 miles per hour with a Wasp Junior of 300 hp. at 2,000 r.p.m. What is the C_s of a suitable propeller?

10. A Northrop Delta has a maximum speed of 221 miles per hour with a Wright Cyclone engine of 710 hp. at 1,900 r.p.m. What is the C_s of a suitable propeller?

Determining Blade Angle and Diameter. The design power, airspeed, and revolutions per minute being known, the power-speed coefficient can be found as shown in the preceding section. V and N being known, the effective pitch (V/N) can be found. It is desired to have an efficient propeller for the design conditions, but there are two variables, the diameter and the geometric pitch.

The diameter is limited in size, in that too large a diameter will not allow sufficient ground clearance in landing and taking-off. Also with given number of revolutions per minute, a larger diameter will mean greater tip speeds. With tip speed above 900 ft. per sec. the tip losses are very great and cut down on the efficiency. If design V and N are fixed, increasing the diameter decreases the design $V/(ND)$.

Increasing the $V/(ND)$, by decreasing the diameter, means that a large geometric pitch (large blade angle) will be necessary to secure efficiency under design conditions. A large blade angle will give a low static thrust resulting in poor take-off and poor climb.

Examining Fig. 48, it will be seen that propellers with larger pitch ratios (blade angles) have greater maximum efficiencies than propellers with smaller pitch ratios; also that these greater maximum efficiencies for the large pitch ratio propeller are for greater $V/(ND)$ values. This would seem to indicate that a very small diameter would be desirable accompanied by large pitch ratio. This might be true if the propeller always operated at design

$V/(ND)$. In level flight, throttling the engine to give slightly fewer revolutions per minute will decrease the airspeed approximately the same ratio, and since D is constant, $V/(ND)$ will be approximately the same as design $V/(ND)$. In climb, however, with full throttle, the airspeed will decrease greatly so that $V/(ND)$ will be much less than design $V/(ND)$. Referring to Fig. 48, if a propeller has been picked which has a higher efficiency

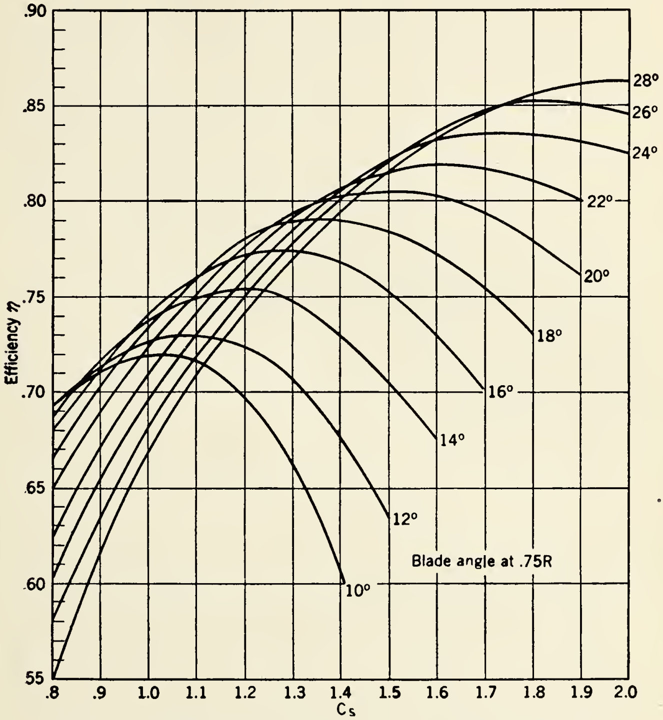


FIG. 51. Efficiency versus C_s for various blade angles.

than any of the other propellers plotted for some given $V/(ND)$, at a smaller $V/(ND)$ a propeller with less pitch will show a higher efficiency.

The efficiency versus $V/(ND)$ curves are practically horizontal near the maximum point, then drop rather sharply. If a propeller is selected which has peak efficiency for the $V/(ND)$ of level flight maximum speed, at the $V/(ND)$ of cruising conditions the propeller will have almost the same efficiency.

In Fig. 51 is shown efficiency η plotted against C_s for a metal propeller with a type J-5 engine mounted in a high-wing cabin monoplane. With a different engine or mounted in the nose of a different type of airplane, owing to the different interference

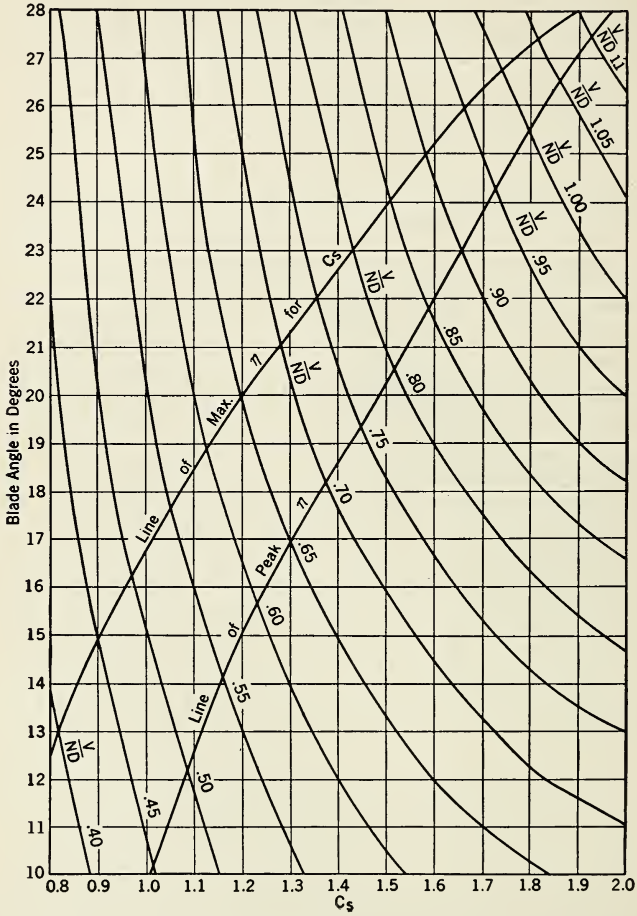


FIG. 52. C_S versus blade angle for various values of $V/(ND)$.

immediately behind the central part of the propeller, the efficiency curves will be altered slightly. The curves of Fig. 51 are quite typically representative, however.

Selecting the peak efficiency of any one blade setting, it will be noted that, at the same value of C_s, a higher efficiency is obtainable with a larger blade angle. If the utmost efficiency is desired for a given C_s then a propeller with the blade angle giving the highest efficiency for that C_s should be chosen. At slightly lower values of C_s, the efficiency of the bigger-blade-angled propeller will have dropped sharply whereas the efficiency of the smaller-blade-angled propeller will have decreased only slightly.

If high speed is the only objective, the C_s being known, the blade angle is chosen which gives the greatest efficiency for that C_s. This will result in an extremely poor take-off.

The blade angle which has peak efficiency for this C_s will permit of a reasonably good take-off but owing to the smaller efficiency will reduce the topspeed.

A propeller with a blade angle which is the mean between those described in the last two paragraphs will result in a take-off approximating that for the peak efficiency while the maximum speed obtainable will be practically that of the propeller selected for high speed. This is the propeller usually selected.

With the blade angle decided on, resort is made to a chart similar to Fig. 52, where C_s is plotted against $V/(ND)$ for various blade angles. Two reference lines are on this diagram, one designating the maximum efficiency for racing planes, the other suitable for general purposes. The use of Figs. 51 and 52 can best be explained by an illustrative example.

Example. A cabin monoplane has a top speed of 150 miles per hour. Its engine gives 400 hp. at 1,900 r.p.m. Find diameter, blade angle, and efficiency of the propeller for general utility. What is its efficiency if airplane flies 100 miles per hour at 1,600 r.p.m.?

Solution. From previous work:

$$C_s = 1.40$$

From Fig. 51, for $C_s = 1.40$; blade angle of 19° is at peak efficiency, $\eta = 0.79$; blade angle of 20° gives efficiency of 0.80, blade angle of 22° gives $\eta = 0.81$.

From Fig. 52, for C_s 1.40 and blade angle 19°

$$\frac{V}{ND} = 0.725$$

but

$$\begin{aligned} V &= 150 \text{ miles per hour} \\ &= 220 \text{ ft. per sec.} \\ N &= 31.7 \text{ r.p.s.} \end{aligned}$$

Then $$D = \frac{220}{31.7 \times 0.725}$$
$$= 9 \text{ ft. } 7 \text{ in.}$$
for $$V = 100 \text{ miles per hour}$$
$$= 146.7 \text{ ft. per sec.}$$
and $$N = 26.7 \text{ r.p.s.}$$
$$\frac{V}{ND} = \frac{146.7}{26.7 \times 9.57}$$
$$= 0.574$$

From Fig. 52 for 19° blade angle and $V/(ND)$ of 0.574
$$C_s = 1.08$$
From Fig. 51 for 19° angle and $C_s = 1.08$
$$\eta = 0.75$$

Problems

1. Find diameter, blade angle, and efficiency for a general-utility propeller for $V = 120$ miles per hour, 200 hp., and 1,800 r.p.m.
2. Find diameter, blade angle, and efficiency for a general utility-propeller for $V = 140$ miles per hour, 200 hp., and 1,800 r.p.m.
3. Find diameter, blade angle, and efficiency for a general-utility propeller for $V = 180$ miles per hour, 200 hp., and 1,800 r.p.m.
4. Find diameter, blade angle, and efficiency for a general-utility propeller for $V = 120$ miles per hour, 400 hp., and 1,900 r.p.m.
5. Find diameter, blade angle, and efficiency for a general-utility propeller for 70 miles per hour, 90 hp., and 1,400 r.p.m.

Finding Blade Angle with Diameter Fixed. It is quite often that the size of the propeller is influenced by the need of having proper ground clearance. Also it is becoming more common to use a variable-pitch propeller, in which case the problem resolves itself into selecting the proper blade setting for a given diameter.

Example. What should be blade setting and efficiency for a variable-pitch propeller 8 ft. 6 in. in diameter for $V = 150$ miles per hour and 400 hp. at 1,900 r.p.m.?

Solution. From previous work $C_s = 1.40$ and
$$\frac{V}{ND} = \frac{220}{31.7 \times 8.5}$$
$$= 0.816$$

From Fig. 52 for $C_s = 1.40$ and $V/(ND) = 0.816$:
$$\text{Blade angle} = 25°$$
From Fig. 51 for $C_s = 1.40$ and blade angle 25°:
$$\eta = 0.81$$

Problems

1. Find blade angle for a 7-ft. propeller for 110 miles per hour, 210 hp., at 2,000 r.p.m.

2. Find blade angle for an 8-ft. propeller for 150 miles per hour, 200 hp., at 1,800 r.p.m.

3. Find blade angle for an 8-ft. propeller for 150 miles per hour, 300 hp., at 2,000 r.p.m.

4. Find blade angle for a 9-ft. propeller for 150 miles per hour, 300 hp., at 2,000 r.p.m.

5. Find blade angle for an 8-ft. propeller for 70 miles per hour, 90 hp., at 1,400 r.p.m.

Propeller Performance at Altitudes. The expressions for thrust, torque, and power all contain the term ρ, the air density. They therefore vary directly with density just as do the lift and drag of a simple wing. Efficiency of a propeller does not depend on density, merely on the $V/(ND)$ condition. As long as the propeller advances the same distance per revolution, the efficiency will remain the same, regardless of altitude. As pointed out in the chapter on engines, the brake horsepower decreases with altitude but at a greater rate than the density decreases.

At the ground, the engine turns over at a certain speed. That speed is the speed at which the torque power or resistance of the propeller just balances the engine power. At that particular speed of rotation, the propeller exerts a definite thrust which pulls the airplane forward at an airspeed which is the velocity at which the total drag, or resistance of the airplane to forward motion, just balances the thrust.

At an altitude, if the throttle is left at the same opening, the engine would tend to turn over faster, because the thinner air would offer less resistance to the propeller; i.e., the power absorbed by the propeller would be less. Speeding up an engine gives greater power, but, since at an altitude the actual power per revolution drops off as the result of lessened volumetric efficiency and this decrease is at a greater rate than the increase in power due to putative increased revolutions per minute, the engine actually runs slower at altitude.

If, at the ground, the airplane is cruising at less than full power, at an altitude the throttle may be opened wider so that the engine speed is the same as it previously was at the ground. In doing this the horsepower delivered by the engine is made equal to the torque horsepower of the propeller at that engine speed at that altitude.

The drag of an airplane in level flight at any one airspeed varies directly with the air density. Drag equals thrust in level flight. At any altitude the thrust required to maintain a given airspeed compared with the thrust required at the ground for that same airspeed will be directly as the ratio of the air densities. The power required to turn the propeller at this airspeed at the altitude would be related to the power required at the ground as the relative densities only if the revolutions per minute were the same. The engine cannot at altitude furnish power proportionate to the relative density compared with the power at the ground at the same revolutions per minute. Therefore, for a given airspeed, the revolutions per minute will be less at altitude than at the ground.

The advance per revolution, or $V/(ND)$, not being the same for a given airspeed at altitude as at the ground, the efficiency of the propeller will be different. The thrust horsepower at altitude compared with the thrust horsepower at the ground for the same airspeed is affected in two ways: first, the brake horsepower of the engine is less; and second, since the engine speed is less the efficiency of the propeller is different.

Propeller Selection for Altitude Flying. Previously in this chapter the method of selecting the proper diameter and blade angle was shown on the basis of flying at or near sea-level. If the engine delivers approximately the same horsepower at altitudes, it is conceivable that the propeller might not offer enough resistance to prevent the engine running overspeed.

The formula used for determining the power-speed coefficient

$$C_s = \frac{0.638 \times \text{M.P.H.}}{\overline{\text{H.P.}}^{1/5} \times \overline{\text{R.P.M.}}^{2/5}}$$

was derived from the more basic formula

$$C_s^5 = \frac{44.5 \times \rho \times \overline{\text{M.P.H.}}^5}{\overline{\text{H.P.}} \times \overline{\text{R.P.M.}}^2}$$

$$C_s = \frac{2.136 \times \rho^{1/5} \times \text{M.P.H.}}{\overline{\text{H.P.}}^{1/5} \times \overline{\text{R.P.M.}}^{2/5}}$$

The power-speed coefficient varies with the fifth root of the air density, provided horsepower, airspeed, and revolutions per minute are unchanged. The nomogram in Fig. 50 being based on sea-level density or

$$C_s = \frac{0.638 \times \text{M.P.H.}}{\overline{\text{H.P.}}^{1/5} \times \overline{\text{R.P.M.}}^{2/5}}$$

$$= \frac{2.136 \times (0.002378)^{1/5} \times \text{M.P.H.}}{\overline{\text{H.P.}}^{1/5} \times \overline{\text{R.P.M.}}^{2/5}}$$

the power-speed coefficient at altitude (C_{sa}) may be obtained by multiplying the power-speed coefficient for sea-level (C_{s0}) by the fifth root of the relative densities (ρ_a/ρ_0).

$$C_{sa} = C_{s0}\left(\frac{\rho_a}{\rho_0}\right)^{1/5}$$

TABLE VII

Altitude in feet	$\sqrt[5]{\frac{\rho_a}{\rho_0}}$	Altitude in feet	$\sqrt[5]{\frac{\rho_a}{\rho_0}}$
0	1.0	10 000	0.941
1 000	0.994	11 000	.935
2 000	.988	12 000	.929
3 000	.982	13 000	.923
4 000	.977	14 000	.917
5 000	.971	15 000	.911
6 000	.965	16 000	.906
7 000	.959	17 000	.900
8 000	.953	18 000	.894
9 000	.947	19 000	.888
10 000	.941	20 000	.882

Example. What should be the blade setting and diameter of the propeller which makes an airspeed of 150 miles an hour at an altitude of 10,000 ft.? The engine is supercharged to give 400 hp. at 1,900 r.p.m. at the altitude of 10,000 ft.

Solution.

$$C_{s10,000} = 0.941 \times C_{s0}$$
$$= 0.941 \times 1.40$$
$$= 1.33$$

From Fig. 52 for $C_s = 1.33$, blade angle $= 19\frac{1}{2}°$, $V/ND = 0.70$,

$$D = \frac{220}{31.7 \times 0.7} = 9.9 \text{ ft.}$$

The propeller, 9.9 ft. in diameter, with a blade angle of $19\frac{1}{2}°$, will absorb the 400 hp. at 10,000-ft. altitude.

If the propeller that was selected previously for 150 miles per hour, 400 hp. at 1,900 r.p.m. at sea-level, with 9.57-ft. diameter and blade angle of 19°, is a fixed-blade propeller, then at 10,000-ft. altitude, since V and N are the same, the $V/(ND)$ will be the same as at the ground. The C_s which is a function of the $V/(ND)$ and blade angle will be the same. The power absorbed by this propeller at 10,000-ft. altitude is

$$\begin{aligned} \text{H.P.} &= \frac{44.5 \times \rho_{10,000} \times \overline{\text{M.P.H.}}^5}{\overline{\text{R.P.M.}}^2 \times \overline{C_s}^5} \\ &= \frac{44.5 \times 0.001756 \times \overline{150}^5}{\overline{1900}^2 \times \overline{1.40}^5} \\ &= 295 \end{aligned}$$

This horsepower is not sufficient to hold back the 400-hp. engine so that the engine would tend to race if not throttled.

A propeller suitable for flying at altitude is not efficient near the ground. A good propeller for ground conditions is not suitable for altitude work. For this reason a variable-pitch propeller is highly desirable.

Problems

1. Find the blade angle for a 9-ft. propeller for flying at 15,000-ft. altitude, with an airspeed of 110 miles per hour, 210 hp. at 2,000 r.p.m. at that altitude.

2. Find the blade angle for a 9-ft. propeller for flying at 15,000-ft. altitude with an airspeed of 150 miles per hour, 300 hp. at 2,000 r.p.m. at that altitude.

3. Find the blade angle for a 9-ft. propeller for flying at 20,000-ft. altitude with an airspeed of 150 miles per hour, 300 hp. at 2,000 r.p.m. at that altitude.

4. Find the blade angle for a 9-ft. propeller for flying at 15,000-ft. altitude with an airspeed of 150 miles per hour, 250 hp. at 1,800 r.p.m. at that altitude.

5. Find the blade angle for a 9-ft. propeller for flying at 20,000-ft. altitude, with an airspeed of 150 miles per hour, 250 hp. at 1,800 r.p.m. at that altitude.

Geared Propellers. Propellers with high geometric pitch, i.e., large blade angle, have high efficiencies. A high geometric pitch postulates a large $V/(ND)$. The horsepower of airplane engines is proportional to the speed of revolution, so that, to keep down the size and weight per horsepower, the revolutions per second should be high. With a fixed-diameter propeller, for a given airspeed, increasing N has the effect of decreasing $V/(ND)$. The increase in horsepower with increased revolutions per minute increases the denominator of $\frac{0.638 \times \text{M.P.H.}}{\text{H.P.}^{1/5} \times \text{R.P.M.}^{2/5}}$ and therefore decreases the value of C_s. In addition, high revolution speed with large-diameter propeller means high tip speeds with consequent loss of efficiency. High engine speed is desirable, but high propeller speed is undesirable.

Geared propellers have the following disadvantages. There is added weight and loss of power in gear friction. Owing to high engine speed and low propeller speed, cooling of the engine becomes a problem. The necessary greater diameter involves greater landing-gear height. There is an added cost.

The advantages of gearing-down propellers are as follows. There is increase in performance of the airplane due to greater efficiency of the propeller consequent on larger pitch ratio, on the absence of tip losses, and on the larger diameter involving a proportionately less interference effect from the fuselage and engine. A geared propeller is practically noiseless owing to the low tip speed.

With gearing, the propeller must be mounted outboard of the bearings, and since the propeller shaft is very short, it must be lined up perfectly or the wear on the gears will be excessive. The gear teeth must be cut accurately, and the gears must be well lubricated at all times.

The effect of gearing is best shown by an illustrative problem. In a previous illustration, it was worked out for a plane making 150 miles per hour, with an engine of 400 hp. at 1,900 r.p.m., that the efficiency was 80.5 per cent. Considering that gearing will give greater efficiency and therefore greater horsepower let it be assumed that the airspeed will be increased to 160 miles per hour.

Example. A cabin monoplane has a speed of 150 miles with a 400-hp. engine at 1,900 r.p.m. If the propeller is geared 2 to 1, what will be C_s, D, and efficiency?

Solution.

$$C_s = \frac{0.638 \times 150}{950^{2/5} \times 400^{1/5}}$$
$$= 1.86$$

From Figs. 51 and 52, for $C_s = 1.86$, $V/(ND) = 1.08$, blade angle = 27°, $\eta = 0.86$:

$$D = \frac{220}{15.8 \times 1.08}$$
$$= 12.9 \text{ ft.}$$

Summarizing for the geared and direct-driven propellers gives this comparison:

Gear ratio	Direct	2 : 1
R.p.m. of propeller	1,900	950
C_s	1.41	1.86
$V/(ND)$	0.76	1.08
Diameter, feet	9.1	12.9
Blade angle	21°	27°
Efficiency	0.805	0.86
Tip speed, feet per second	920	650

Body Interference. The presence of the engine and fuselage immediately in the rear of the propeller modifies the flow of air through the propeller disk. The air coming through the central part of the disk is diverted outward, causing a change in the direction of stream flow back of the propeller.

The net effect of this interference is to increase the C_s slightly for all values of $V/(ND)$. It also causes the peak value of efficiency to occur at a slightly higher $V/(ND)$ than it would without the interference.

Propeller Performance. The brake horsepower of an airplane engine varies almost directly with the speed of rotation. The amount of power which a propeller is capable of absorbing depends on its speed-power coefficient and its $V/(ND)$. The full-throttle engine speed is dependent on the torque resistance offered by the propeller.

The thrust horsepower of an engine-propeller unit is the brake horsepower of the engine multiplied by the efficiency of the propeller. It is the thrust horsepower that produces and maintains the velocity of the airplane.

The speed of the engine may be reduced either directly by throttling or by reducing the airspeed by pulling back on the control stick. At a reduced airspeed, the numerator of the expression $V/(ND)$ becomes less. With smaller $V/(ND)$, the power coefficient of the propeller becomes greater (see Fig. 49). For a two-bladed propeller $C_s = V/(ND \sqrt[5]{P_c})$, if the power coefficient

P_c is increased, the fifth root of the reciprocal of the power coefficient will be less. With a decreased airspeed, the power coefficient will decrease at a faster rate than the airspeed. The power required to rotate the propeller being

$$\text{H.P.} = \frac{0.106}{\text{R.P.M.}^2}\left(\frac{\text{M.P.H.}}{C_s}\right)^5$$

and C_s decreasing faster than the miles per hour, the horsepower absorbed would increase if the revolutions per minute remained the same. Actually putting more load on the engine will slow it down.

Propeller performance is the effectiveness with which the propeller changes engine power into thrust power at all airspeeds. Propeller performance is either stated in the form of a table or as a curve of thrust horsepower at various airspeeds.

It usually is assumed that brake horsepower varies directly as revolutions per minute, or, letting B.H.P._0 be rated horsepower and R.P.M._0 be rated speed, for unsupercharged engines,

$$\frac{\text{B.H.P.}}{\text{B.H.P.}_0} = \frac{\text{R.P.M.}}{\text{R.P.M.}_0}$$

Airspeeds are chosen for 10-mile-per-hour intervals throughout the range of airspeed it is expected that the airplane will fly. Since at airspeeds less than design airspeed, the engine will run slower than design revolutions per minute, to find the engine speed at each airspeed it is necessary to interpolate.

At each airspeed at least three engine speeds are assumed in the neighborhood of which the engine speed might be expected to be. For each of these engine speeds a value of C_s is found. These calculations are for a fixed-pitch propeller so that the blade angle does not change. If the C_s corresponding to each engine speed is known, the $V/(ND)$ may be found from Fig. 52 for each engine speed. With V and D known, N may be found giving a second value of engine speed for each engine speed originally assumed. By plotting the originally assumed engine speeds against C_s and the calculated engine speeds against C_s and locating the intersection of these two plots, the actual revolutions per minute and C_s are found for this airspeed. An illustrative example follows.

Example. A cabin monoplane has an engine rated 400 hp. at 1,900 r.p.m. The propeller has a blade angle of 21° and the diameter is 9.1

ft. The design airspeed is 150 miles per hour. What are the revolutions per minute and thrust horsepower at 120 miles per hour?

Solution. For 1,850 r.p.m.:

$$\text{H.P.} = 400 \times \frac{1,850}{1,900}$$
$$= 389.5$$
$$C_s = \frac{0.638 \times 120}{1,850^{2/5} \times 389.5^{1/5}}$$
$$= 1.146$$

From Fig. 52, $V/(ND) = 0.63$ for 21° angle and $C_s = 1.146$.

$$N = \frac{120 \times 88}{9.1 \times 0.63}$$
$$= 1,842 \text{ r.p.m.}$$

For 1,800 r.p.m.:

$$\text{H.P.} = 400 \times \frac{1,800}{1,900}$$
$$= 379$$
$$C_s = \frac{0.638 \times 120}{1,800^{2/5} \times 379^{1/5}}$$
$$= 1.165$$

From Fig. 52, $V/(ND) = 0.64$ for 21° angle and $C_s = 1.165$.

$$N = \frac{120 \times 88}{9.1 \times 0.64}$$
$$= 1813$$

For 1,700 r.p.m.:

$$\text{H.P.} = \frac{400 \times 1,700}{1,900}$$
$$= 358$$
$$C_s = \frac{0.638 \times 120}{1,700^{2/5} \times 358^{1/5}}$$
$$= 1.205$$

From Fig. 52, $V/(ND) = 0.66$ for 21° angle and $C_s = 1.205$.

$$N = \frac{120 \times 88}{9.1 \times 0.66}$$
$$= 1,758 \text{ r.p.m.}$$

In Fig. 53 are plotted the assumed revolutions per minute against the corresponding C_s values and a line is drawn through these points. The calculated revolutions per minute are also plotted against C_s and a line is drawn. The intersection gives 1,830 r.p.m. for an airspeed of 120 miles per hour, and corresponding to these conditions C_s is 1.153.

From Fig. 51, the efficiency for this C_s and a blade angle of 21° is 76 per cent.

$$\begin{aligned} \text{B.H.P.} &= 400 \times \frac{1{,}830}{1{,}900} \\ &= 385 \\ \text{Thrust H.P.} &= \text{B.H.P.} \times \eta \\ &= 385 \times 0.76 \\ &= 293 \text{ hp.} \end{aligned}$$

The foregoing method when worked out for each 10-mile-per-hour interval is a series of calculations of some magnitude. A less exact method, often employed, makes use of the assumption that for most propellers the percentage of maximum efficiency is the same at the same percentage of design $V/(ND)$. The curve in Fig. 55*a* shows this relation for metal propellers. If wooden propellers are used the curve has a somewhat different slope. The ordinates of this curve are *not* efficiencies but percentages of the maximum efficiency; likewise the abscissas are *not* values of $V/(ND)$ but percentages of design $V/(ND)$. The use of this graph will be illustrated in performance calculations later in this book.

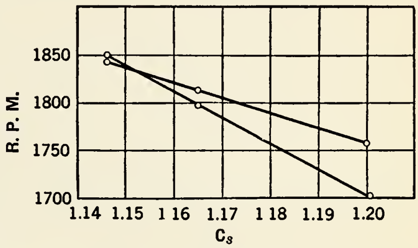


FIG. 53. Interpolation for illustrative example.

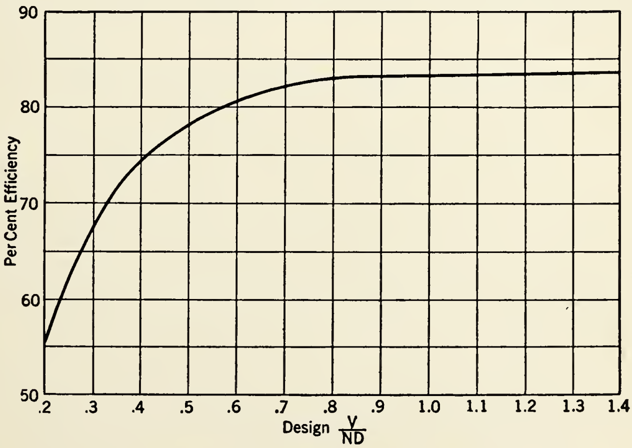


FIG. 54. Maximum efficiency versus design $V/(ND)$.

In preliminary calculations of performance where meticulous accuracy is not required, an approximate idea of the maximum efficiency of a propeller may be obtained by use of the graph in Fig. 54. This gives the maximum efficiency for various values of design $V/(ND)$.

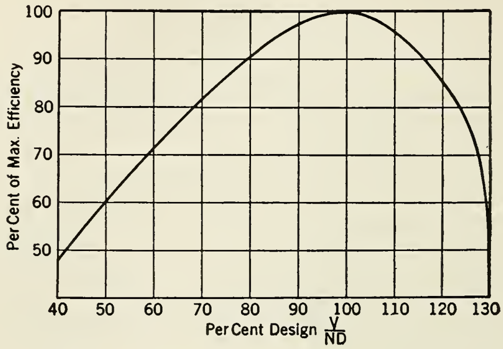


FIG. 55*a*. Variation of efficiency with $V/(ND)$.

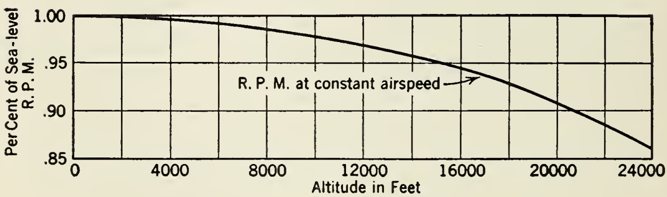


FIG. 55*b*. Variation of R.P.M. with altitude.

Construction. The first airplane propellers were made of wood. This material offers the simplest form of construction. Many different kinds of wood have been tried. Walnut, oak, and birch are considered the best, birch usually being thought superior to the others. The propeller is not made from a single block of wood, but for greater strength is made of several layers firmly glued together under pressure. These laminations extend across from one blade to the other, being continuous through the hub. The tips are covered with thin sheet brass, this protective material extending inward along the leading edge for some distance.

Wood and the glue uniting the laminations are both affected by heat and dampness. Also the blades have to be thicker if made of wood than if made of stronger material; being thick they cannot be made an efficient shape in cross-section. Wooden propellers find their chief use in small, inexpensive airplanes.

The first advance in propeller material was the use of Micarta. Micarta is stronger than wood; it is made of layers of canvas strongly impregnated with a Bakelite composition, united in one piece and formed under heavy pressure in a mold. Each blade is made separately, the inner end being cylindrical with a shoulder or projecting rim on the extreme butt. The inner blade ends are firmly held in metal sockets attached to the metal hub. Micarta is not affected by moderate heat or by dampness and is somewhat lighter in weight than a wooden propeller.

One of the first forms of metal propellers was made by shaping a flat piece of aluminum sheet and twisting it near the hub to the desired pitches while cold, then heat-treating to relieve the stresses. The hub section was reinforced by clamping a round block of wood on each side. This type of propeller was a vast improvement over the wooden one. If a plane equipped with a wooden propeller nosed over in landing, the propeller invariably shattered. With a thin metal propeller, the tip might be bent at right angles in nosing-over without breaking. An airplane has even been flown with a blade in this shape, but the blade can be roughly hammered flat to be later sent to the manufacturer for straightening.

Later one-piece duralumin propellers were made by forging. Magnesium alloy propellers are now being employed to some extent. Hollow steel propellers that are seamless and have no welds have been made. They are fabricated by drawing a seamless steel tube through rollers to taper it. The small end is then spun closed, and the other end is peined out to form a shoulder which can be gripped in the hub. After machining to obtain proper wall thickness, the tapered tube is squeezed into final shape in iron dies. Since the forming is done while cold, the blades are heat-treated to relieve stresses.

When a propeller blade whirling at high speed strikes a drop of rain, the rain drop acts as if it were a solid bullet. A wooden propeller flown through a rainstorm will be badly pitted. Metal propellers withstand the action of rain much better. Aluminum parts are adversely affected by salt air or salt spray unless they are protected by a special treatment.

The Department of Commerce requires that any new type of propeller must pass a whirl test. This consists of 10 hours of whirling at an overload speed as specified below:

$N_T = C_A N_A$

or

$N_T = C_G N_G$

N_T = revolutions per minute during test
N_A = r.p.m. in level flight at which propeller will absorb rated horsepower
N_G = r.p.m. on ground at which propeller will absorb rated horsepower
C_A and C_G = constants from following table

	Tested on Electric Meter		Tested on Internal Combustion Engine	
	C_A	C_G	C_A	C_G
Non-metallic (fixed pitch).....	1.05	1.20	1.00	1.20
Metal (fixed pitch)............	1.15	1.30	1.10	1.25
Variable pitch (any material) .	1.15	1.30	1.10	1.25

A further requirement of the Department of Commerce is that propellers should be so designed that they will limit the speed of the engine at full throttle to 105 per cent of the official rated engine speed.

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CHAPTER X

AIRPLANE PERFORMANCE

The performance of an airplane is its ability to do certain things. Its functioning in respect to some characteristics is difficult to measure quantitatively, and the stability, maneuverability, and ease of control can be described in only a general or qualitative manner. Other features of its behavior in the air which can be specified quite exactly are termed the performance. The items usually listed as performance characteristics are maximum and minimum speed at sea-level and at various altitudes, absolute ceiling, service ceiling, time to climb to various altitudes, best climbing speeds, angle of glide, radius of glide, range, and endurance.

The present state of our knowledge permits the prediction, with a high degree of accuracy, of the performance of an airplane while in its design stage. This is especially true if wind-tunnel measurements have been made of the drag of the fuselage and landing-gear assembly.

The stalling speed of an airplane can be found if only the wing loading and the maximum lift coefficient of the wing section are known. All other performance characteristics of an airplane require that data on the engine and further aerodynamic data on the airplane be known. In studying performance, there are two primary conditions: first, the power required to move the airplane through the air; secondly, the power that the engine can furnish while the airplane is executing that movement.

Horsepower Required at Sea-Level. The total horsepower required to move the airplane forward through the air is the sum of the horsepowers required to move the various parts of the airplane through the air. The parts may be grouped in various ways.

If a graph is available giving the characteristics of the airfoil section data for the aspect ratio of the actual wing, it is probably simpler to find the horsepower needed for the wing and that for the parasite. The sum of these two is the total horsepower required.

It is not common, however, to have at hand characteristic curves for the proper aspect ratio. Curves for airfoils of infinite aspect ratio, which usually are obtainable, serve in calculating the profile drag of the wing. This may be joined with the parasite drag of the struts, landing gear, and wheels, etc., which like the profile wing drag does not vary with the angle of attack. The horsepower required to overcome this combined drag varies only as the cube of the airspeed. As the parasite drag of the fuselage and tail surfaces is presumed to change with angle of attack as well as with the cube of the velocity, the horsepower needed for this is computed separately. The horsepower to overcome the induced wing drag is calculated by itself.

Since the increased drag of the fuselage and landing gear with angle of attack is manifest only at high angles, i.e., low airspeeds, where ample engine horsepower is available, little error is produced in the final result if these drags are assumed to change not with angle of attack but only with velocity. Making this assumption permits all the parasite resistance to be grouped and simplifies the computation. This approximation causes very small error at high speeds but introduces inaccuracy at low speed. Each method is exemplified in an example.

Example 1. (First method.) Find the horsepower required for a monoplane weighing 2,000 lb. and having a Clark Y rectangular wing 36 ft. by 6 ft. The parasite drag has an equivalent flat plate area of 3.8 sq. ft. Result to be given as plot of horsepower versus velocity.

Solution. From Fig. 17, read off and, as shown in table below, tabulate C_L and C_D for various angles of attack from near the zero lift angle to beyond the burble point.

The velocity is found by using the formula

$$V = \frac{\sqrt{\dfrac{W}{0.00256\,S}}}{\sqrt{C_L}}$$

For any particular airplane, $\sqrt{\dfrac{W/S}{0.00256}}$ is constant throughout the series of calculations.

For this airplane:

$$\text{Wing loading} = \frac{2{,}000}{216}$$
$$= 9.26 \text{ lb. per sq. ft.}$$
$$\sqrt{\frac{W/S}{0.00256}} = \sqrt{\frac{9.26}{0.00256}}$$
$$= 60.2$$
$$V = \frac{60.2}{\sqrt{C_L}}$$

Horsepower required for the wing is determined for each angle of attack by the formula

$$\text{H.P.}_{\text{wing}} = \frac{D \times V}{375}$$
$$= \frac{C_D \times 0.00256 \times S \times V^3}{375}$$
$$= C_D V^3 \times \left(\frac{0.00256\, S}{375}\right)$$

For any given problem, the term in parentheses is constant; for this problem

$$\frac{0.00256\, S}{375} = \frac{0.00256 \times 216}{375}$$
$$= 0.00147$$

and $$\text{H.P.}_{\text{wing}} = 0.00147\, C_D V^3$$

For flat plates

$$\text{H.P.} = \frac{0.00327 \times a \times V^3}{375}$$

For this airplane

$$\text{H.P.}_{\text{par.}} = \frac{0.00327 \times 3.8 \times V^3}{375}$$
$$= 0.0000331\, V^3$$

A variation of the above method is to make use of the L/D curve of Fig. 17. Since in level flight lift is equal to weight:

$$\text{Drag} = \frac{\text{Weight}}{L/D}$$

TABLE VIII

1	2	3	4	5	6	7
α	C_L	C_D	V	H.P.$_{\text{req.wing}}$	H.P.$_{\text{req.par.}}$	H.P.$_{\text{req.total}}$
−4	0.07	0.010	228	174	392	566
−3	0.14	.010	161	61	138	199
−2	0.215	.012	130	38	73	111
−1	0.285	.014	112	29	46.7	76
0	0.36	.017	100	26.5	33.6	60
4	0.645	.033	74.9	20.3	13.9	34
8	0.93	.060	62.3	21.4	8.0	29
12	1.19	.095	55.2	23.6	5.5	29
16	1.435	.139	50.2	25.6	4.2	30
18	1.545	.164	48.1	29.5	3.7	31
19	1.560	.180	48.1	29.5	3.7	33
20	1.540	.206	48.5	34.5	3.7	38

Columns 2 and 3 obtained from Fig. 17.
Columns 5, 6, and 7 plotted against column 4 in Fig. 56.

Velocity is found as above. Horsepower for the wing is

$$\text{H.P.}_{\text{wing}} = \frac{DV}{375}$$
$$= \frac{WV}{375\ L/D}$$

For this airplane

$$\text{H.P.}_{\text{wing}} = \frac{2{,}000}{375} \times \frac{V}{L/D}$$
$$= 5.33 \times \frac{V}{L/D}$$

Horsepower required for the parasite is found as above.

TABLE IX

1	2	3	4	5
α	C_L	L/D	V	$H.P._{req.wing}$
−4	0.07	7.0	228	174
−3	0.14	14.0	161	61
−2	0.215	17.9	130	38
−1	0.285	20.4	112	29
0	0.36	21.2	100	26.5
4	0.645	19.5	74.9	20.3
8	0.93	15.5	62.3	21.4
12	1.19	12.5	55.2	23.6
16	1.435	10.3	50.2	25.6
18	1.545	8.7	48.1	29.5
19	1.560	8.6	48.1	29.5
20	1.540	7.5	48.5	34.5

Example 2. (Second method.) Find the horsepower required for a monoplane weighing 2,000 lb. and having a rectangular wing 43.2 ft. by 5 ft. The parasite assumed to vary with angle of attack, consisting of fuselage and tail surface, has an equivalent flat plate area of 1.6 sq. ft. The parasite not varying with angle of attack, namely, struts, landing gear, and wheels, etc., has an equivalent flat plate area of 2.2 sq. ft.

Solution.

$$\text{Aspect ratio} = \frac{43.2}{5}$$
$$= 8.64$$
$$C_{Di} = \frac{C_L^2}{\pi \times 8.64}$$
$$= 0.0368\, C_L^2$$

From Fig. 17 or Fig. 25, $C_{L\,max.} = 1.56$

$$V_{min.} = \sqrt{\frac{2{,}000}{0.00256 \times 1.56 \times 216}}$$
$$= 48.1 \text{ miles per hour}$$

Equivalent flat plate area of parasite drag varying with angle of attack, A_e = 1.6 sq. ft.

$$\text{At high speed, } C_{DP_1} = \frac{1.6 \times 1.28}{216}$$
$$= 0.00948$$

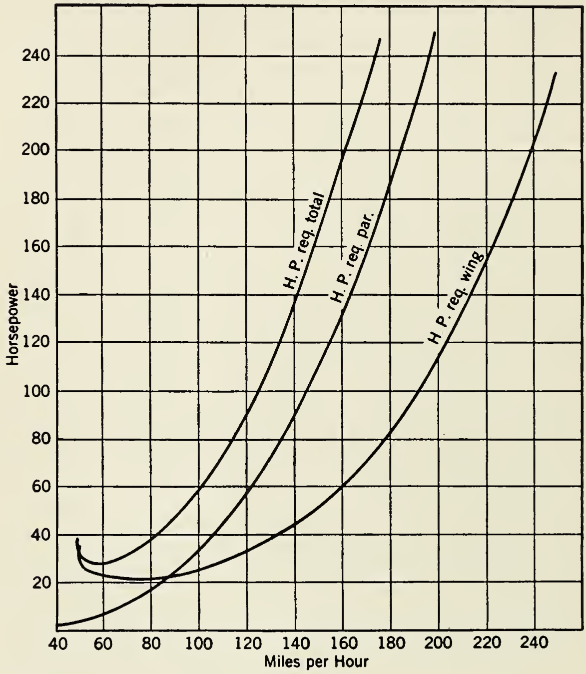


FIG. 56. Horsepower required for various airspeeds. (Example 1.)

Equivalent flat plate area of parasite drag constant with angle of attack, A_e = 2.2 sq. ft.

$$C_{DP_2} = \frac{2.2 \times 1.28}{216}$$
$$= 0.0130$$

The velocity being assumed, the corresponding value of C_L can be found by

$$C_L = \frac{W}{0.00256 \times SV^2}$$

$$\text{For this problem, } C_L = \frac{2{,}000}{0.00256 \times 216 \times V^2}$$
$$= \frac{3{,}640}{V^2}$$

Since the coefficient of parasite drag is expressed as a function of wing area S, it may be added to the coefficients of induced and profile drag to give an all-inclusive drag coefficient, C_D.

$$\text{H.P.}_{\cdot\text{total}} = \frac{C_D \times 0.00256 \times S V^3}{375}$$

For this problem

$$\text{H.P.}_{\cdot\text{total}} = \frac{0.00256 \times S \times C_D V^3}{375}$$
$$= 0.00147\, C_D V^3$$

TABLE X

1	2	3	4	5	6	7	8	9	10
V	V/V_m	C_L	C_{Di}	C_{D_0}	F	C_{Dp_1}	C_{Dp}	C_D	H.P.$_{\cdot\text{req.}}$
48	1.00	1.58	0.0920	0.0446	4.0	0.0380	0.0410	0.1876	30.5
50	1.04	1.45	.0773	.0310	2.5	.0237	.0367	.1450	26.7
60	1.25	1.01	.0376	.0140	1.3	.0123	.0253	.0769	24.4
70	1.46	0.74	.0202	.0120	1.1	.0104	.0234	.0556	28.0
80	1.67	0.57	.0118	.0105	1.0	.0095	.0225	.0448	33.8
90	1.87	0.45	.0074	.0100	1.0	.0095	.0225	.0399	42.8
100	2.08	0.36	.0048	.0100	1.0	.0095	.0225	.0373	54.8
110	2.29	0.30	.0033	.0099	1.0	.0095	.0225	.0357	69.9
120	2.50	0.25	.0023	.0098	1.0	.0095	.0225	.0346	87.4
130	2.71	0.215	.0017	.0099	1.0	.0095	.0225	.0341	110.5
140	2.92	0.186	.0013	.0099	1.0	.0095	.0225	.0337	135.8

Explanation of Table

Column 2 obtained by dividing items in column 1 by $V_{\text{min.}}$ (= 48).
Column 3 obtained by dividing 3,640 by items in column 1 squared.
Column 4 obtained by multiplying items in column 3 squared by 0.0368.
Column 5 obtained from Fig. 38.
Column 6 obtained from Fig. 42.
Column 7 obtained by multiplying C_{Dp_1}H.S. (= 0.0095) by items in column 6.
Column 8 obtained by adding C_{Dp_2} (= 0.0130) to items in column 7.
Column 9 obtained by adding items in columns 4, 5, and 8.
Column 10 obtained by multiplying 0.00147 by items in column 9 by items in column 1 cubed.

In Fig. 57 velocity (column 1) is plotted against horsepower required (column 10).

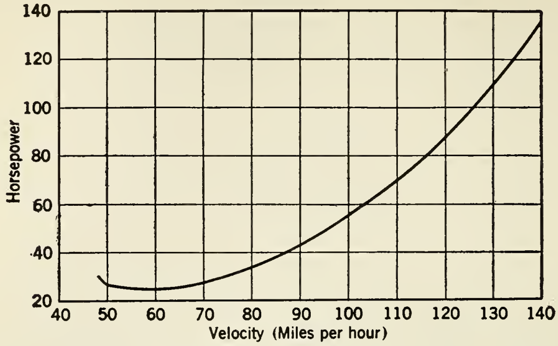


FIG. 57. Horsepower required for various airspeeds. (Example 2.)

Example 3. Find the horsepower required by a biplane weighing 4,225 lb. The upper span is 38 ft., the lower span is 35 ft., the gap is 5.35 ft. The area of upper wing is 214 sq. ft., the area of lower wing is 150 sq. ft. Both wings are Clark Y airfoils. At high speed, the parasite has an equivalent flat plate area of 9.4 sq. ft. of which 3.2 sq. ft. varies with angle of attack.

Solution.

$$\mu = \frac{b_2}{b_1} = \frac{35}{38} = 0.92$$

$$\frac{\text{Gap}}{\text{Mean span}} = \frac{5.35}{36.5} = 0.146$$

From Fig. 40, $\sigma = 0.56$

$$r = \frac{150}{214} = 0.737$$

$$K^2 = \frac{(0.92)^2\,(1 + 0.737)^2}{(0.92)^2 + 2 \times 0.56 \times 0.737 \times 0.92 + (0.737)^2} = 1.185$$

$$\text{E.M.A.R.} = \frac{K^2 b_1^2}{S} = \frac{1.185 \times \overline{38}^2}{150 + 214} = 4.7$$

$$C_{Di} = \frac{C_L^2}{\pi \times 4.7}$$
$$= 0.0677\, C_L^2$$

$$V_{\text{min.}} = \sqrt{\frac{W}{0.00256\, C_{L\text{max.}} S}}$$
$$= \sqrt{\frac{4225}{0.00256 \times 1.56 \times 364}}$$
$$= 54 \text{ miles per hour}$$

Equivalent flat plate area of parasite drag varying with angle of attack, $A_e = 3.2$ sq. ft.

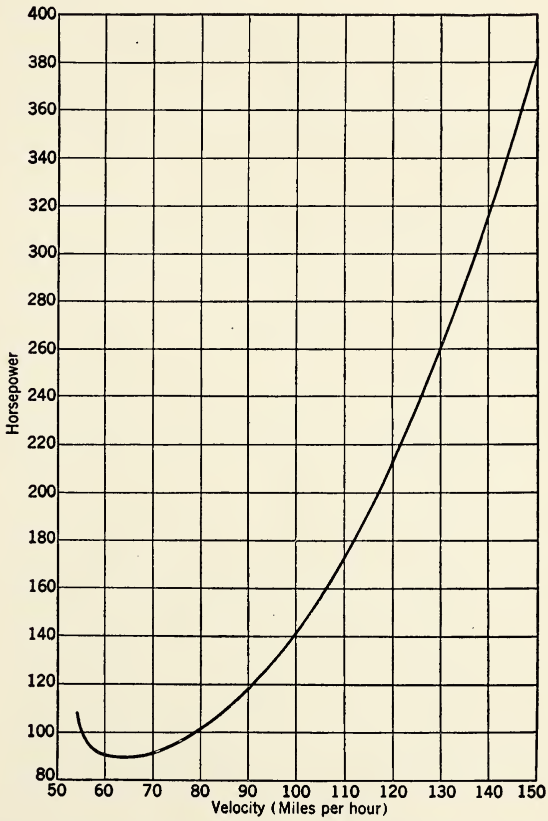


FIG. 58. Horsepower required for various airspeeds. (Example 3.)

At high speed, $$C_{Dp1(\text{H.S.})} = \frac{3.2 \times 1.28}{364}$$
$$= 0.0113$$

Equivalent flat plate area of parasite drag constant with angle of attack, A_e = 6.2 sq. ft.

$$C_{Dp_2} = \frac{6.2 \times 1.28}{364}$$
$$= 0.0218$$

If velocity is assumed, corresponding value of C_L can be found by

$$C_L = \frac{W}{0.00256 \times S \times V^2}$$

For this problem

$$C_L = \frac{4{,}225}{0.00256 \times 364 \times V^2}$$
$$= \frac{4{,}550}{V^2}$$

For this problem

$$\text{H.P.}_{\text{req total}} = \frac{C_D \times 0.00256 \times S \times V^3}{375}$$
$$= \frac{C_D \times 0.00256 \times 364 \times V^3}{375}$$
$$= 0.00248\ C_D V^3$$

TABLE XI

1	2	3	4	5	6	7	8	9	10
V	$V/V_{\text{min.}}$	C_L	C_{Di}	C_{D_0}	F	C_{Dp_1}	C_{Dp}	C_D	H.P.$_{\text{req.}}$
54	1.00	1.56	0.1640	0.0446	4.00	0.0452	0.0670	0.2756	104
60	1.10	1.26	.1070	.0200	1.72	.0194	.0412	.1682	90
70	1.30	0.93	.0588	.0135	1.20	.0136	.0354	.1077	91
80	1.47	0.71	.0342	.0120	1.08	.0122	.0340	.0802	102
90	1.67	0.56	.0212	.0105	1.00	.0113	.0331	.0648	117
100	1.85	0.45	.0140	.0100	1.00	.0113	.0331	.0571	142
110	2.04	0.37	.0095	.0100	1.00	.0113	.0331	.0526	174
120	2.22	0.32	.0070	.0099	1.00	.0113	.0331	.0500	214
130	2.40	0.27	.0049	.0098	1.00	.0113	.0331	.0478	260
140	2.59	0.232	.0036	.0099	1.00	.0113	.0331	.0466	317
150	2.78	0.202	.0028	.0099	1.00	.0113	.0331	.0458	384

In Fig. 58, velocity (column 1) is plotted against horsepower required (column 10).

Horsepower Available. The horsepower available for moving the airplane is the brake horsepower of the engine multiplied by the efficiency of the propeller. The propeller is selected for one airspeed, either maximum, cruising, or sometimes best climbing speed. At any other speed, the propeller will be less efficient; and because proportionately more torque load is put on the engine, so that the engine runs slower, less brake horsepower is developed. This means that, in an airplane flying at less than design airspeed, the engine will develop less than its rated horsepower even though the throttle is full open.

The first requirement is to select the most suitable propeller. Propeller choice depends on rated horsepower, speed of the engine, and design airspeed. Engine speed and power will have to be furnished in order to make the performance curves, but the maximum airspeed is not known exactly until the horsepower-available curve is drawn.

Two procedures may be followed. An arbitrary top speed may be assumed based solely on similarity to existing airplanes of the airplane for which performance curves are being drawn. On the basis of this tentative maximum speed, a propeller is chosen, enabling the horsepower available to be drawn. The completion of this curve will give a maximum airspeed, which is the top speed which can be obtained using the chosen propeller. If the top speed obtained agrees closely with the assumed airspeed, the propeller is the correct one to use. If the assumed airspeed is different from that indicated by the curves, the propeller selected on the basis of the assumed airspeed is not suited for the engine-airplane combination, and higher airspeeds will be obtained by substituting another propeller. The second method of procedure differs only in that arbitrarily a design efficiency is assumed for the propeller. Usually 82 per cent efficiency is selected as this is about the highest efficiency obtainable from propellers. Applying this factor to the rated brake horsepower gives a tentative thrust horsepower. The total horsepower required curve is then consulted to find the velocity at which the horsepower required corresponds to this thrust horsepower.

The velocity obtained from the horsepower required curve is assumed to be the design airspeed. With this trial airspeed the propeller is selected as described above.

With design airspeed and propeller tentatively chosen, the effect of changes in airspeed on engine speed is next found. An exact method was described in the chapter on propellers, but it is usually considered sufficiently accurate to make use of the curve of Fig. 44*a*.

If the engine manufacturer furnishes curves or other information as to the drop in horsepower with decrease in engine speed, they should be utilized. Otherwise use should be made of $\frac{\text{R.P.M.}_1}{\text{R.P.M.}_2} = \frac{\text{B.H.P.}_1}{\text{B.H.P.}_2}$, which is representative of most engines. This gives the brake horsepower.

At each airspeed there will be a corresponding engine speed, and these determine the $V/(ND)$ of the propeller. At each value of $V/(ND)$, the propeller efficiency is found, either from Fig. 48 or more quickly but less exactly by Fig. 55*a*.

The thrust horsepower or horsepower available is the product of the propeller efficiency and the brake horsepower. The method is illustrated by the following example.

Example. For the airplane described in example 1 in this chapter, an engine rated at 150 hp. at 1,800 r.p.m. is installed. Find the data for horsepower available versus velocity curve.

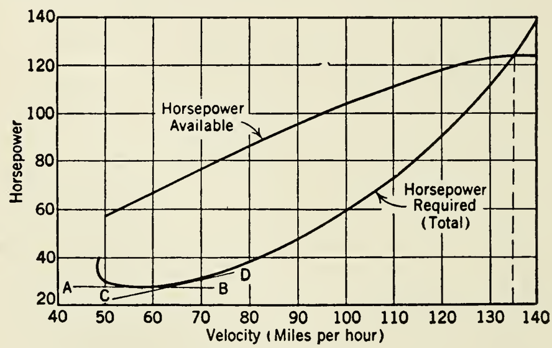


FIG. 59. Horsepower required and available curves. (Example 1.)

Solution. Assume η for propeller as 82 per cent.

First approximation of $\text{H.P.}_{\text{avail.}} = 150 \times 0.82 = 123$.

From Fig. 56, 123 hp. is required at 135 miles per hour.

$$C_s = \frac{0.638 \times \text{M.P.H.}}{(\text{H.P.})^{1/5} \times (\text{R.P.M.})^{2/5}}$$

$$= \frac{0.638 \times 135}{150^{1/5} \times 1{,}800^{2/5}}$$

$$= 1.57$$

From Fig. 52, a propeller is chosen with a blade angle of 23° with $V/(ND)$ of 0.87 (η = 82.5 per cent).

$$D = \frac{V}{N \times 0.87}$$

$$= \frac{135 \times 88}{1{,}800 \times 0.87}$$

$$= 7.59 \text{ ft.}$$

TABLE XII

1	2	3	4	5	6	7	8	9	10
V m.p.h.	% design V	% design N	N r.p.m.	Brake H.P.	V/ND	% design V/ND	% max. η	η	H.P. avail.
50	37.1	87.0	1 565	131	0.371	42.6	52.0	0.430	56.3
60	44.5	87.0	1 565	131	.445	51.1	61.5	.507	66.5
70	51.9	87.2	1 570	131	.517	59.5	70.5	.580	76.0
80	59.3	88.0	1 583	132	.587	67.5	79.0	.651	86.0
90	66.7	89.0	1 600	134	.653	75.0	86.0	.710	95.2
100	74.0	90.8	1 633	136	.710	81.6	91.8	.757	103.0
110	81.5	93.0	1 673	139	.759	87.2	95.8	.790	109.9
120	88.9	97.5	1 751	146	.795	91.5	98.0	.808	118.0
130	96.2	99.0	1 781	149	.848	97.5	99.5	.820	122.1
135	100.0	100.0	1 800	150	.870	100.0	100.0	.825	123.8

Explanation of Table

Column 2 is the items of column 1 divided by design speed (135 miles per hour).

Column 3 is obtained from Fig. 44*a*. (Note: A more accurate method is described in the chapter on propellers.)

Column 4 is the items in column 3 multiplied by design r.p.m. (1,800).

Column 5 is the items in column 3 multiplied by design horsepower (150) (assuming power varies as engine speed).

Column 6 is the items in column 1 multiplied by 88 and divided by the corresponding items in column 4 and the diameter (7.59).

Column 7 is the items in column 6 divided by the design $V/(ND)$ (0.870).

Column 8 is obtained from Fig. 55*a*.

Column 9 is the items in column 8 multiplied by design efficiency (0.825).

Column 10 is the items in column 5 multiplied by corresponding items in column 9.

Items in column 10 are plotted against corresponding items in column 1 on the same graph as the horsepower-required curve; see Fig. 59. It will be noted that the horsepower-required curve and horsepower-available curve intersect at 135 miles per hour. This is, then, the correct maximum airspeed. If the assumed maximum airspeed, on which the propeller was picked, differs materially from the maximum airspeed found by the intersection of the two curves a second set of calculations must be gone through. It must be borne in mind that if an incorrect assumption was made of maximum airspeed, whether *too high* or *too low*, the propeller selected was unsuited to this airplane engine combination. The maximum airspeed obtained by the intersection of the curves is low, since the horsepower-available curve is based on using an improper, poor-performing propeller. With a more nearly suitable propeller the horsepower-available curve will have larger ordinates and the intersection with the horsepower available will show a higher top speed. Allowance should be made for this in a second assumption of maximum airspeed to select the more suitable propeller.

Maximum Speed at Sea-Level. The maximum speed is found by the intersection of the curves of horsepower available and horsepower required. At velocities less than the maximum, the ordinate of the horsepower-required curve is less than the ordinate of the horsepower-available curve, which means that more horsepower is available than is needed for level flight so that the engine may be partially throttled. At velocities greater than the maximum, the ordinate of the horsepower-required curve is larger than the ordinate of the horsepower-available curve, meaning that more horsepower is needed at this speed than is available. The airplane cannot fly at this speed, and if it is placed at the angle of attack corresponding to this velocity, the thrust horsepower will not be sufficient to equal the drag at this speed. Since this velocity cannot be attained in level flight the lift will not equal the weight, and the airplane will lose altitude.

Velocities for Minimum Fuel Consumption at Sea-Level. It can be proven quite easily that the minimum horsepower is required for a wing when flying at the angle of attack where $C_D/C_L^{3/2}$ is the minimum. For the entire airplane, minimum horsepower is required for level flight when $\dfrac{C_D + \dfrac{1.28\,a}{S}}{C_L^{3/2}}$ is minimum.

Examining the horsepower-required curve, it will be seen that, as speed decreases from the maximum, the horsepower decreases to a minimum value, but if the speed decreases further the horsepower increases. The minimum horsepower is found by drawing a horizontal line tangent to the horsepower-required curve (see line AB, Fig. 59). The velocity at the point of tangency, which is the velocity of minimum power, corresponds to the angle of attack where $\dfrac{C_D + \dfrac{1.28\,a}{S}}{C_L^{3/2}}$ is the minimum.

For the Clark Y wing with aspect ratio of 6, it was shown that the maximum value of $C_L^{3/2}/C_D$ (or minimum value of $C_D/C_L^{3/2}$) will occur at 3° angle of attack, corresponding to 79.8 miles per hour. If a curve is plotted having as ordinates, $\dfrac{C_L^{3/2}}{C_D + \dfrac{1.28\,a}{S}}$; abscissas, angle of attack; the curve will resemble the curve for the wing alone, but the maximum ordinate will be at a higher angle of attack. For the airplane whose performance is shown in Fig. 59, the maximum value of $\dfrac{C_L^{3/2}}{C_D + \dfrac{1.28\,a}{S}}$ is at 11° angle of attack, corresponding to an airspeed of 56.6 miles per hour.

This velocity is the airspeed for minimum horsepower required, which means the least fuel consumption in gallons per hour. If a plane is to make an endurance record for time in the air, the fuel supply should be used as economically as possible. The airplane should then be flown so that the wing is at the angle of attack for maximum $\dfrac{C_L^{3/2}}{C_D + \dfrac{1.28\,a}{S}}$. In the example of the monoplane weighing 2,000 lb. with Clark Y wing, the pilot should regulate his throttle and stick so that the airplane maintains level flight at 56.6 miles per hour airspeed as long as the airplane's total weight is 2,000 lb. As time elapses, fuel consumed will make the airplane lighter. Always, during the endurance flight, the pilot will want to keep his wing at angle of attack of maximum $\dfrac{C_L^{3/2}}{C_D + \dfrac{1.28\,a}{S}}$ (11° for this Clark Y wing and ratio of parasite to wing area); but as the airplane's weight decreases, the velocity necessary for level flight at this angle of attack will decrease. With less weight, the horsepower required will be less, so that the pilot can throttle his engine more. With less throttle-opening, the fuel consumption (in gallons per hour) will be diminished, so the hourly decrease in weight will be less. The pilot should have a precalculated schedule of best airspeeds to fly at various hours during his flight.

Flying at the angle of maximum $\dfrac{C_L^{3/2}}{C_D + \dfrac{1.28\,a}{S}}$ of the wing will mean that the least gasoline is consumed per hour, so that an airplane can stay the longest time in the air on a given supply of fuel. Flying either faster or slower will involve a greater fuel consumption per hour. The foregoing is true whether in still air or in wind. Airspeed, not ground speed, is important.

If the object of the flight is not greatest duration, but to cover the greatest distance on a given gasoline supply or, what is more usual, to fly to a destination burning the least possible amount of fuel, the airplane should be flown faster. To cover a given distance, the gasoline consumption depends not only on the consumption per hour but on the total hours required for the flight. It is therefore advantageous to fly at greater speed, more specifically to fly so the wing is at the angle of attack of maximum L/D_{total}.

Drawing a line tangent to the horsepower-required curve from the origin (0 miles per hour, 0 hp.; see line CD, Fig. 59) determines the speed and horsepower for minimum fuel consumption for a given distance. The point of tangency will be the point where the ratio of horsepower to velocity is least. Neglecting any variation in fuselage drag, the horsepower required is

$$\text{H.P.}_{\text{req.}} = \frac{1}{375}\left(C_D \times 0.00256\,SV^3 + \frac{1.28\,a}{S} \times 0.00256\,SV^3\right)$$

To cover a given distance at a velocity V requires $\dfrac{\text{distance}}{V}$ hours. The gasoline consumption depends on horsepower-hours; therefore

$$\text{Total consumption} = \text{Constant} \times \text{Distance} \times \frac{\text{H.P.}_{\text{req.}}}{V}$$

$$\frac{\text{H.P.}_{\text{req.}}}{V} = \frac{1}{375}\left(C_D \times 0.00256\,SV^2 + \frac{1.28\,a}{S} \times 0.00256\,SV^2\right)$$

But
$$V^2 = \frac{W}{C_L \times 0.00256\,S}$$

Then

$$\frac{\text{H.P.}_{\text{req.}}}{V} = \frac{W}{375}\left(\frac{C_D + \dfrac{1.28\,a}{S}}{C_L}\right)$$

This has a minimum value when $\left[\dfrac{C_D + \left(\dfrac{1.28\,a}{S}\right)}{C_L}\right]$ is minimum. With $\dfrac{1.28\,a}{S}$ constant, for any airfoil section, there is only one angle of attack where $\dfrac{C_D + \left(\dfrac{1.28\,a}{S}\right)}{C_L}$ has a minimum value. This angle of attack will be slightly greater than the angle of attack for minimum C_D/C_L, i.e., maximum L/D, but as the parasite area is made smaller with respect to wing area, the difference between angle of minimum $\dfrac{C_D + \left(\dfrac{1.28\,a}{S}\right)}{C_L}$ and angle of minimum $\dfrac{C_D}{C_L}$ becomes less.

With the Clark Y wing alone; the angle of maximum L/D is 1°; with a Clark Y monoplane and the parasite of the illustrative example, the angle of maximum L/D_{total} is 5°, the corresponding velocity being 71.2 miles per hour. This checks with the point of tangency of line CD in Fig. 59.

The pilot maintains this speed as long as the weight remains the 2,000 lb. The angle for best L/D_{total} is constant for the airplane, since the wing area and parasite area are fixed. The speed for any given angle of attack varies as the square root of the wing loading. As fuel is consumed, and the airplane is lightened, the airspeed for least fuel consumption per distance decreases as the square root of the weights.

The foregoing discussion of most economical airspeed applies to still air. If a head wind holds back a plane, it will take a longer time to reach its destination; a tail wind means a quicker trip. The formula for total fuel consumption must therefore be modified so that the horsepower-hours contains the expression V_G, the ground speed, for divisor instead of V, the airspeed.

$$\text{Horsepower-hours} = \text{Distance} \times \frac{\text{H.P.}_{\text{req.}}}{V_G}$$

$$= \frac{\text{Distance}}{375} \frac{\left(C_D + \frac{1.28\,a}{S}\right)0.00256\,SV^3}{V_G}$$

$$= \frac{\text{Distance}}{375} \frac{\left(C_D + \frac{1.28\,a}{S}\right)\frac{W}{C_L}\sqrt{\frac{W}{C_L \times 0.00256\,S}}}{V_G}$$

The horsepower-hours or total fuel consumption will be a minimum when $\frac{C_{D\text{total}}}{C_L^{3/2}V_G}$ is minimum or when $\frac{C_L^{3/2}V_G}{C_D + \frac{1.28\,a}{S}}$ is a maximum.

A chart or table may be made out for a given airplane for various strength winds and for various angles of the wind to the airplane's heading. The ground speed, V_G, is found by trigonometry.

$$(V_G)^2 = V^2 + W^2 + 2\,VW\cos\theta$$

W is windspeed in miles per hour
θ is angle of wind from dead ahead

The desirable changes in airspeed on account of wind are surprisingly small. For the illustrative problem with a 10-mile head wind the optimum angle of attack is 5°, for a 10-mile tail wind the best angle is still 5°, so that the airspeed should be maintained as in still air at 71.2 miles per hour. For a 20-mile head wind the best angle becomes 4°, meaning an airspeed of 75 miles per hour; for a 20-mile tail wind it is best to fly at 6° with an airspeed of 68.1 miles per hour. With a 30-mile head wind an airspeed of 85.4 miles per hour is best; with a 30-mile tail wind an airspeed of 68 miles per hour is best.

Problems

1. For the monoplane of Clark Y wing, 36 ft. by 6 ft., with 3.8 sq. ft. equivalent flat plate area, what should be the airspeed for minimum horsepower when fuel has been burned so that total weight is 1,800 lb.?

2. For the monoplane of problem 1, weight 1,800 lb., what should be airspeed in still air to cover a given distance with least possible expenditure of fuel?

3. For the monoplane of problem 1, weight 1,800 lb., what should be the airspeed for minimum gas consumption, if there is a 35-mile-per-hour head wind?

4. What should be airspeed for problem 3 if it is a 35-mile-per-hour tail wind?

5. What should be airspeed for problem 3 if the weight were only 1,600 lb.?

Range. The calculation of the range or distance that can be flown non-stop is not simple. The airplane leaves the ground with a given weight, but, as fuel is consumed, the weight decreases. To achieve the greatest possible distance, the airplane should be flown constantly at the angle of maximum L/D_{total} but, as the weight decreases, both velocity and horsepower required for this angle decrease, for constant angle of attack V varies as $W^{1/2}$ and H.P. varies as $W^{3/2}$. With less power being used, the rate of fuel consumption is less, that is, the rate of change of weight is less. Also, with less power required, the throttle is closed more and more, which decreases the revolutions per minute of the engine. Since the revolutions per minute decrease at a different rate from the airspeed, the $V/(ND)$, and consequently the efficiency, of the propeller changes. A close approximation to the range may be obtained by assuming an average fuel consumption and an average propeller efficiency.

Let W = total weight of airplane at take-off.
W_t = total weight of airplane at the end of t seconds after take-off.
Q = total weight of fuel at take-off.
Q_t = weight of fuel consumed at the end of t seconds.
C = fuel consumption (lb. per hp. per hr.).
$(\text{B.H.P.})_t$ = brake horsepower available at time t.

In a short time interval dt, the weight of fuel consumed is

$$dQ_t = \frac{C \times (\text{B.H.P.})_t}{3{,}600}\,dt$$

The horsepower available ($\eta \times$ B.H.P.) must equal the horsepower required at any time t.

$$\eta(\text{B.H.P.})_t = \frac{DV}{550}$$

$$= \frac{W_t V}{\left(\frac{L}{D}\right) 550}$$

Then

$$(\text{B.H.P.})_t = \frac{W_t V}{\eta \left(\frac{L}{D}\right) 550}$$

Substituting in the initial equation gives

$$dQ_t = \frac{C W_t V}{3{,}600 \times 550\, \eta \left(\frac{L}{D}\right)} dt$$

$$= \frac{C W_t V}{1{,}980{,}000\, \eta \left(\frac{L}{D}\right)} dt$$

$$V dt = 1{,}980{,}000 \frac{\eta}{C} \times \frac{L}{D} \times \frac{dQ_t}{W_t}$$

Let R be the total number of miles traveled (i.e., the range). Then

$$dR = V dt$$

$$= 1{,}980{,}000 \frac{\eta}{C} \times \frac{L}{D} \times \frac{dQ_t}{W_t}$$

$$= 1{,}980{,}000 \frac{\eta}{C} \times \frac{L}{D} \times \frac{dQ_t}{W - Q_t}$$

Integrating and bearing in mind that at take-off the quantity consumed is zero and at the end of the flight $Q_t = Q$

$$R = 1{,}980{,}000 \frac{\eta}{C} \times \frac{L}{D} \int \frac{dQ_t}{W - Q_t}$$

$$= 1{,}980{,}000 \frac{\eta}{C} \times \frac{L}{D} \log_e \frac{W}{W - Q} \text{ feet}$$

$$= 1{,}980{,}000 \times 2.303 \frac{\eta}{C} \times \frac{L}{D} \times \log_{10} \frac{W}{W - Q} \text{ feet}$$

$$= \frac{1{,}980{,}000 \times 2.303}{5{,}280} \frac{\eta}{C} \times \frac{L}{D} \times \log_{10} \frac{W}{W - Q} \text{ miles}$$

$$= 863.5 \frac{\eta}{C} \times \frac{L}{D} \times \log_{10} \frac{W}{W - Q} \text{ miles}$$

The above equation is known as Breguet's formula for maximum range.

The range as calculated by Breguet's method is the ultimate that can be achieved. The speed is much less than the maximum.

$$\text{At high speed: } R_{vm} = \frac{\text{Amount of fuel} \times \text{high speed}}{\text{Rate of fuel consumption at high speed}}$$

Since it is rarely that an airplane will be flown at the low speed corresponding to the absolutely maximum range, the range is customarily considered to be between the maximum and the range at high speed. Common practice is to regard the maximum range as

$$R_{\text{max.}} = 0.75(R_B - R_{vm}) + R_{vm}$$

where R_B = range by Breguet formula.

While specific fuel consumption is slightly higher when engine is run at rated power than when the engine is throttled, only a slight error is involved by using average fuel consumption.

Example. An airplane weighs 4,000 lb. and takes off with 80 gallons of fuel. It has a Clark Y wing of 216 sq. ft. area and has 3.8 sq. ft. equivalent flat plate area of parasite. It has a 180-hp. engine. The maximum efficiency of the propeller is 78 per cent. Assume fuel consumption to be 0.55 lb. per B.H.P. per hr. and maximum velocity to be 135 miles per hour. Find the range.

Solution.

$$\begin{aligned}
\text{Weight of fuel } (Q) &= 80 \times 6 \\
&= 480 \text{ lb.} \\
R_B &= 863.5 \frac{\eta}{C} \times \frac{L}{D_{\text{max.}}} \times \log \frac{W}{W - Q} \\
&= 863.5 \frac{0.78}{0.55} \times 11.9 \log \frac{4{,}000}{3{,}520} \\
&= 808 \text{ miles} \\
R_{vm} &= \frac{480 \times 135}{180 \times 0.55} \\
&= 655 \text{ miles} \\
R &= 0.75(R_B - R_{vm}) + R_{vm} \\
&= 0.75(808 - 655) + 655 \\
&= 770 \text{ miles}
\end{aligned}$$

Problems

1. A Martin Clipper weighs 51,000 lb. and takes off with 4,000 gallons of fuel. Its top speed is 180 miles per hour. It has four twinrow Wasp engines of 800 hp. each. Assume propellers to have a maximum efficiency of 82 per cent, the maximum L/D to be 14.2. The fuel consumption is to be 0.48 lb. per hp. per hour. What is the range?

2. An Aeronca with a total weight of 1,000 lb. takes off with 8 gallons of fuel. Its top speed is 93 miles per hour. The engine is rated at 36 hp. Assume propeller to have a maximum efficiency of 75 per cent, the maximum L/D to be 9.5, and the fuel consumption to be 0.56 lb. per hp. per hour. What is the range?

3. A Curtiss Robin with total weight of 4,200 lb. takes off with 320 gallons of fuel. Its top speed is 118 miles per hour and its Wright Whirlwind engine is rated at 165 hp. Assume propeller to have a maximum efficiency of 75 per cent, the maximum L/D of the airplane to be 9.0, and fuel consumption to be 12.5 gallons per hour. What is the range?

4. A Bellanca Pacemaker with total weight of 5,600 lb. takes off with 200 gallons of fuel. Its maximum speed is 160 miles per hour, and it has a 420-hp. engine. Assume propeller to have 80 per cent maximum efficiency and maximum L/D to be 11.5. The fuel consumption is 0.55 lb. per hp. per hour. What is the range?

5. A Lockheed Electra, weighing 10,000 lb., takes off with 194 gallons of fuel. Its maximum speed is 210 miles per hour and it has two Wasp Junior engines rated at 400 hp. with fuel consumption 0.48 lb. per hp. per hour. Assume maximum propeller efficiency to be 82 per cent and maximum L/D of airplane to be 14.5. What is the range?

Rate of Climb at Sea-Level. Examining the curves of horsepower available and required versus velocity as shown in Fig. 59, it will be seen that for any velocity less than maximum there is greater horsepower available than is required for level flight. If level flight is desired, the engine may be throttled so that it is furnishing only the horsepower required for level flight. If it is desired to increase the altitude, the engine may be opened to full throttle when it will give the thrust horsepower shown on the horsepower-available curve. The extra power, which is the difference in ordinates for horsepower available and horsepower required at any velocity, represents the power available to do the work of raising the airplane.

Power is ability to do work in unit time. By definition, a horsepower is the power to do 33,000 ft.-lb. of work in 1 min. Then the rate of climb, in feet per minute, can be found by the formula

$$\text{R.C. (ft. per min.)} = \frac{(\text{H.P.}_{\text{avail.}} - \text{H.P.}_{\text{req.}}) \times 33{,}000}{W}$$
$$= \frac{\text{Excess H.P.} \times 33{,}000}{W}$$

Another conception of the conditions in climb is shown in Fig. 60. The airplane in the sketch is ascending along a path which makes an angle θ with the horizontal. The direction opposite to the flight path is the direction of the relative wind. The wing is at angle of attack α to the relative wind. Lift is perpendicular and total drag is parallel to the relative wind. For convenience, there is assumed to be zero angle of incidence so that the wing chord is parallel to the propeller axis. For equilibrium, the forces parallel to the flight path must balance as must the forces perpendicular to the flight path. Neglecting the small force on the tail which is ordinarily considered negligible,

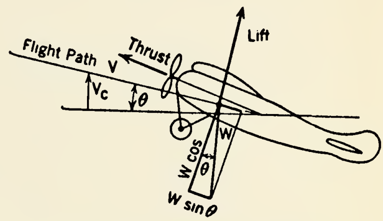


FIG. 60. Forces in climb.

$$T \cos \alpha = D + W \sin \theta$$
$$T \sin \alpha + L = W \cos \theta$$

T = thrust
D = total drag
W = weight
L = lift

Since α is small, $\cos \alpha$ may be considered unity and $T \sin \alpha$ may be neglected; the two equations become

$$T = D + W \sin \theta$$
$$L = W \cos \theta$$
$$\sin \theta = \frac{T - D}{W}$$

and, since θ is small,

$$L = W \text{ approximately}$$

But $$V_c = V \sin \theta$$

V_c = vertical velocity (miles per hour)
V = velocity along flight path (miles per hour)

Then $$\text{R.C. (ft. per min.)} = \frac{T - D}{W} \times V \times \frac{5{,}280}{60}$$

$$\text{Since propeller efficiency } \eta = \frac{\text{Useful power output}}{\text{Total power input}}$$

$$= \frac{\text{Thrust} \times \text{Velocity}}{\text{Brake horsepower} \times 375}$$

$$TV = \eta \times \text{B.H.P.} \times 375$$

$$\frac{TV}{375} = \eta \times \text{B.H.P.}$$

$$= \text{H.P.}_{\text{avail.}}$$

Referring to the equation for rate of climb

$$\text{R.C. (ft. per min.)} = \frac{T - D}{W} \times V \times 88$$

$$= \frac{(TV - DV) \times 88 \times 375}{375 \times W}$$

$$= (\text{H.P.}_{\text{avail.}} - \text{H.P.}_{\text{req.}}) \times \frac{33{,}000}{W}$$

At every velocity less than maximum there will be a different rate of climb. For the illustrative problem, this rate of climb is shown in Fig. 61. It will be seen that, for this particular combination of propeller, engine, and airplane, the maximum rate of climb is achieved, at sea-level, of 790 ft. per min. when the airspeed is 83 miles per hour.

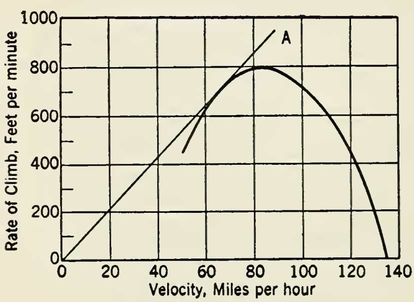


Fig. 61. Rate of climb at sea-level.

Climb may be made at any speed below maximum, but if the pilot desires the greatest rate of climb at sea-level, he should open the throttle wide and pull back on his stick till the airspeed indicator reads 83 miles per hour, the best climbing speed.

This rate of climb is for the propeller which was chosen for maximum efficiency at airspeed of 135 miles per hour. At other airspeeds the efficiency is much less so that the horsepower available is low.

The propeller instead of being chosen for 135-mile-per-hour airspeed might have been chosen for best performance at a lesser airspeed. In Fig. 62 are shown the available-horsepower curves for the 150-hp. engine equipped with propeller chosen for design airspeed of 110 miles per hour and 83 miles per hour. It will be noted that while the rate of climb is improved, as shown by the greater excess horsepower, the maximum airspeed is much decreased.

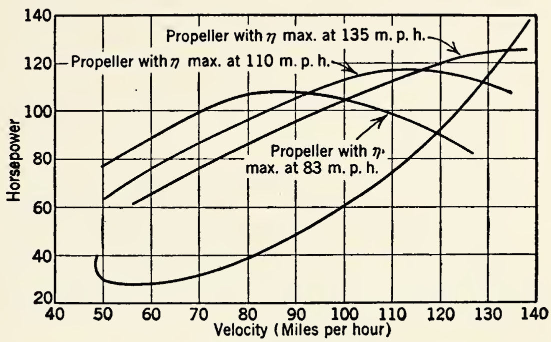


FIG. 62. Horsepower available with propellers of differing pitch.

Angle of Climb at Sea-Level. The rate of climb is another name for vertical velocity. In Fig. 61 it is shown for the illustrative example that the greatest rate of climb, 790 ft. per min. or 8.95 miles per hour, is obtained when the airspeed is 83 miles per hour. The sine of the angle of climb is then 8.95 ÷ 83, and the angle of climb is 6.2°. By flying slower, the actual rate of climb will be less, but the ratio of climb to forward speed may be greater. By drawing a line from the origin tangent to the rate of climb curve, line *OA* in Fig. 61, the point of tangency will give the maximum angle of climb.

For the airplane in the illustrative example, the point of tangency is 66-miles-per-hour airspeed, and 700-ft.-per-min. rate of climb. This gives an angle of climb of 6.9°.

In military maneuvers, rate of climb is very important. While "dog-fighting," the airplane that can gain altitude on its opponent will have an advantage even if a wide circle is necessary to attain this altitude.

In taking-off, an airplane goes straight, and it is the angle of climb, not the rate of climb, that is the decisive factor in clearing obstacles. In taking-off from a small field, it is better to fly at the best angle of climb rather than at the best rate of climb. For the illustrative example, with wide-open throttle, the stick should be pulled back until the airspeed indicator reads not 83 but 66 miles per hour.

On a calm day the angle of climb is as deduced from the graph. A pilot always takes-off into the wind, so if there is any wind the actual angle of climb will be greater than that given.

By diving the airplane, greater speed can be attained than in level flight. If, from a dive, the stick is suddenly pulled back, the airplane, owing to momentum, retains momentarily some of this excess speed, so that a climb can be made for a few seconds at a greater rate and a greater angle than from level flight. This maneuver is called a zoom.

Take-off Distance. There are three phases to the take-off of an airplane. First, there is a very short period during which the tail is being raised from the ground. Second, there is a comparatively long period during which the airplane is gaining speed with the tail up so that the wing is at a low angle of attack. In the final phase the stick is pulled back to put the wing at a big angle of attack so that the plane is lifted into the air. If the field is sufficiently long, the airplane may be kept in the second position until it has attained enough speed to lift itself while flying at the low angle of attack, thus eliminating the third phase described above.

In the following description, it is assumed that the take-off is at sea-level with no wind. If the airplane has retractable landing-gear, it is naturally not folded up till the plane is in the air so the drag on the ground is with the gear extended.

At the end of the run, when the wing is suddenly put at a high angle of attack, the wing drag will become very big, tending to slow down the airplane. Because of this and also because excess horsepower will be needed for climbing, the stick is not pulled back until a speed has been attained which is somewhat greater than the minimum or stalling speed. It is customary in calculations to use a speed which corresponds to 90 per cent of maximum C_L.

In starting, the throttle is opened wide, the brakes released, and the tail raised. The plane starts to roll along the ground with the wing at the angle of attack of maximum L/D_{total}. At first, the entire weight of the airplane rests on the ground, the lifting force on the wing being negligible. As the speed increases, more and more of the airplane weight is carried by the wing, so that the plane rests less heavily on the ground. Since the weight on the ground is less, the friction of the wheels with the ground becomes less and less engine power is needed to overcome this friction. When the airplane is just about ready to leave the ground, practically the entire weight is being borne by the wing.

The coefficient of friction varies with the surface of the runway, depending on whether it is the smooth deck of a ship or a soft gravel field. The accepted values of the coefficient of friction (μ) are as follows:

Concrete runway or wooden deck	$\mu = 0.02$
Hard turf, level field	$\mu = 0.04$
Average field, short grass	$\mu = 0.05$
Average field, long grass	$\mu = 0.10$
Soft ground	$\mu = 0.10 - 0.30$

With a fixed pitch propeller, the thrust is quite different when the airplane is stationary from the thrust when the airplane is moving at design speed. Use is made of a graph such as Fig. 63*a*, which is for the average two-bladed metal propeller, to find the static thrust coefficient (K_{T_0}) and the static thrust is found by the formula, for static thrust (T_0)

$$T_0 = \frac{K_{T_0} \times \text{B.H.P.}}{\text{R.P.M.} \times D}$$

The initial accelerating force (F_0) is the static thrust of the propeller minus the friction of the wheels with the ground.

$$F_0 = T_0 - W\mu$$

As the speed increases, the ground friction becomes less but the air resistance becomes greater. Just prior to take-off, ground friction has become negligible, the sole retarding force being the air drag. The propeller thrust has decreased from its static value to a new value which depends on the airspeed. The take-off is at the angle of attack where C_L is 90 per cent of maximum C_L. The velocity at take-off (V_t) will be

$$V_t = \sqrt{\frac{W}{0.90\, C_{L\text{max.}} \frac{\rho}{2} S}}$$

$$= V_{\text{min.}} \sqrt{\frac{1}{0.90}}$$

$$= 1.054\, V_{\text{min.}}$$

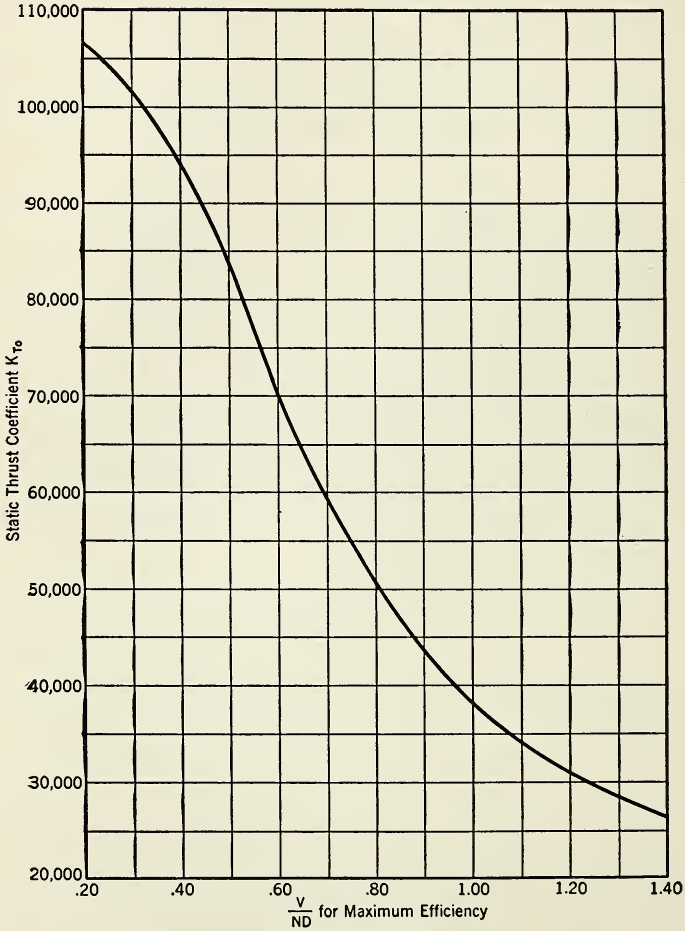


FIG. 63*a*. Variation of static thrust coefficient with design V/ND.

The thrust at take-off (T_t) is found by first obtaining the thrust horsepower at take-off speed by means of Fig. 63*b* and then making use of the relation

$$T_t = \frac{550 \times \text{T.H.P.}}{V_t} \qquad V \text{ in feet per second}$$

or

$$T_t = \frac{375 \times \text{T.H.P.}}{V_t} \qquad V \text{ in miles per hour}$$

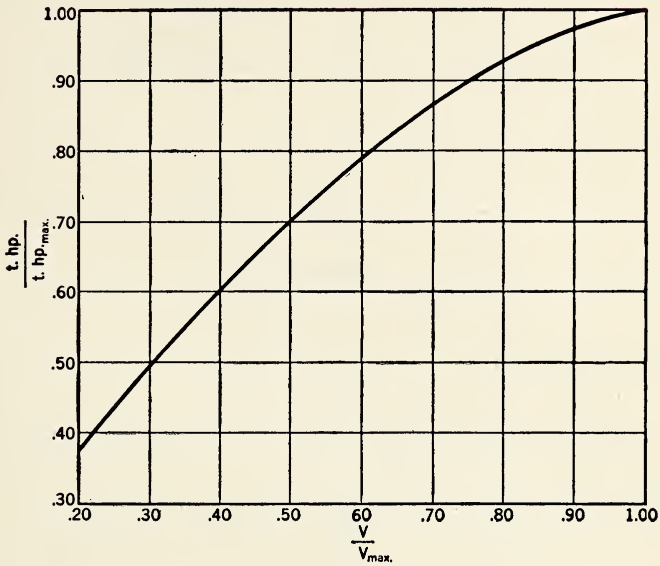


FIG. 63*b*. Variation of thrust horsepower with velocity.

The accelerating force at take-off (F_t) is the propeller thrust minus the air drag of the entire airplane.

$$F_t = T_t - W\left(\frac{D_{\text{total}}}{L}\right)_{\text{max.}}$$

It is assumed that the accelerating force varies linearly with the airspeed. Although this assumption is not exactly correct, the error involved is so slight as to be negligible. Then, if the force is F_0 when V is zero, and the force is F_t when the velocity is V_t, the force F at airspeed V will be

$$F = F_0 - \frac{(F_0 - F_t)V}{V_t}$$

$$= F_0\left(1 - \left[\frac{F_0 - F_t}{F_0}\right]\frac{V}{V_t}\right)$$

Letting
$$K = \frac{F_0 - F_t}{F_0}$$
$$F = F_0\left(1 - K\frac{V}{V_t}\right)$$

The acceleration produced by this varying force (F) is equal at any instant to the force divided by the mass (W/g) of the airplane.

$$\text{Acceleration} = \frac{dV}{dt} = \frac{dV}{ds} \times \frac{ds}{dt} = \frac{gF}{W}$$

but
$$\frac{ds}{dt} = V$$
$$\frac{V\,dV}{ds} = \frac{g}{W}F_0\left(1 - K\frac{V}{V_t}\right)$$
$$\frac{V\,dV}{1 - \frac{K}{V_t}V} = \frac{g}{W}F_0\,ds$$

This is in the form

$$\frac{x\,dx}{a + bx}; \int \frac{x\,dx}{a + bx} = \frac{1}{b^2}\,[a + bx - a\log_e(a + bx)].$$

Therefore integrating the above gives

$$s = \frac{W}{gF_0}\left[\frac{1}{\left(\frac{K}{V_t}\right)^2}\left(1 - \frac{K}{V_t}V - \log_e\left(1 - \frac{K}{V_t}V\right)\right)\right]$$

and between the limits of V and zero

$$s = \frac{W}{gF_0}\left[-\frac{VV_t}{K} - \left(\frac{V_t}{K}\right)^2\log_e\left(1 - K\frac{V}{V_t}\right)\right]$$

When $V = V_t$

$$s = \frac{V_t^2W}{gF_0}\left[\frac{1}{K}\left(-1 - \frac{1}{K}\log_e(1 - K)\right)\right]$$

V_t in feet per second
s in feet

Example. Find the take-off run for a monoplane, weighing 2,000 lb., having a Clark Y wing 216 sq. ft. in area, and 3.8 sq. ft. parasite, powered with an engine rated at 150 hp. at 1800 r.p.m. and a propeller 7.59 ft. diameter and blade angle of 23°. Runway is smooth concrete. Neglect preliminary distance before tail is up.

Solution.

From Fig. 63*a* $K_{T_0} = 46{,}000$

$$\text{Static thrust,} \quad T_0 = \frac{K_{T_0} \times \text{B.H.P.}}{\text{r.p.m.} \times D}$$
$$= \frac{46{,}000 \times 150}{1{,}800 \times 7.59}$$
$$= 505.0 \text{ lb.}$$

Initial accelerating force,

$$F_0 = T_0 - \mu W$$
$$= 505 - 2{,}000 \times 0.02$$
$$= 465 \text{ lb.}$$

Thrust horsepower at design conditions,

$$\text{T.H.P.} = \text{B.H.P.} \times \eta$$
$$= 150 \times 0.825 \qquad \text{(see p. 195)}$$
$$= 123.8$$

$$\text{Take-off speed,} \quad V_t = 1.054 \times V_{\text{min.}} \qquad \text{(see p. 186)}$$
$$= 1.054 \times 48.1$$
$$= 50.7 \text{ miles per hour}$$

Ratio, take-off speed to maximum speed,

$$\frac{V_t}{V_{\text{max.}}} = \frac{50.7}{135}$$
$$= 0.376$$
$$\text{T.H.P.}_{t}/\text{T.H.P.}_{\text{design}} = 0.580 \qquad \text{(from Fig. 57)}$$
$$\text{T.H.P.}_{t} = 123.8 \times 0.580$$
$$= 71.8 \text{ H.P.}$$

$$\text{Thrust at take-off,} \quad T_t = \frac{375 \times \text{T.H.P.}_{t}}{V_t}$$
$$= \frac{375 \times 71.8}{50.7}$$
$$= 531 \text{ lb.}$$

Accelerating force at take-off,

$$F_t = T_t - W\left(\frac{D_{\text{total}}}{L}\right)_{\text{max.}}$$
$$= 531 - \frac{2{,}000}{11.9}$$
$$= 363$$

$$K = \frac{F_0 - F_t}{F_0}$$
$$= \frac{465 - 363}{465}$$
$$= 0.219$$

$$s = \frac{(50.7 \times 1.47)^2}{32.2} \times \frac{2{,}000}{465}\left[\frac{1}{0.219}\left(-1 - \frac{1}{0.219}\log_e(1 - 0.219)\right)\right]$$
$$= 436 \text{ ft.}$$

Problems

1. Find the run in still air, after tail is up, to take-off an airplane weighing 4,000 lb., the maximum L/D_{total} being 12.5, the stalling speed being 55 miles per hour, the design top speed being 150 miles per hour; the engine giving 200 hp. at 1,850 r.p.m. and the 9-ft. diameter propeller having an efficiency of 80 per cent under design conditions. $\mu = 0.03$.

2. Find the distance in still air, after tail is up, to take-off an airplane weighing 7,000 lb., the maximum L/D_{total} being 10.8, $V_{\text{min.}}$ being 50 miles per hour, $V_{\text{max.}}$ being 140 miles per hour; the engine giving 300 hp. at 1,800 r.p.m., the 8-ft. propeller having 81 per cent efficiency under design conditions. Field is hard turf, μ estimated as 0.04.

3. Find the distance in still air, after tail is up, to take-off a DH weighing 4,300 lb., the maximum L/D_{total} being 6.4, $V_{\text{min.}}$ being 61 miles per hour, $V_{\text{max.}}$ being 124 miles per hour; the engine giving 425 hp. at 1,750 r.p.m., the 9.8-ft. propeller having 75 per cent efficiency under design conditions. Field is hard turf, μ estimated as 0.04.

4. Find the distance in still air, after tail is up, to take-off a Sperry Messenger, weighing 1,076 lb., the maximum L/D_{total} being 5.44, $V_{\text{min.}}$ being 27 miles per hour, $V_{\text{max.}}$ being 90 miles per hour; the engine being rated at 63 hp. at 1,830 r.p.m., the 6.5-ft. propeller having 75 per cent efficiency under design conditions. It is estimated μ is 0.04.

5. Find the distance in still air, after the tail is up, for a Grumman Fighter weighing 4,800 lb., to take-off if the maximum L/D_{total} is 9.3, $V_{\text{min.}}$ is 65 miles per hour, $V_{\text{max.}}$ is 216 miles per hour; the engine giving 770 hp. at 2,100 r.p.m.; the 9.5-ft. propeller having 82 per cent efficiency under design conditions. Take-off is from the deck of a naval vessel, which is stationary.

Gliding Angle at Sea-Level. There is no marked difference in the meaning of glide and dive. A very steep glide is called a dive. Ordinarily a glide is considered to be with power partly or completely off. A dive may be either with power on or off, although usually the word dive by itself means the maneuver with the power off, and with power on the term power-dive is used.

Gliding angle is the angle below the horizontal of the flight path when the airplane descends with the engine either completely throttled or "turning-over" so slowly that there is no appreciable thrust. The forces acting on the airplane, neglecting a small force on the tail, are weight, lift, and total drag. These forces are shown in Fig. 64. The component of weight parallel to the flight path is the force that pulls the airplane along the flight path. When the airplane attains a steady speed along the flight path, the weight multiplied by the sine of the glide angle just equals the total drag.

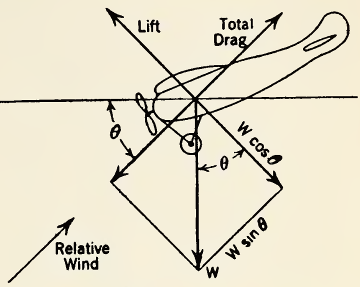


FIG. 64. Forces in glide.

The lift is equal and opposite to the component of the weight perpendicular to the flight path. If the lift is less than $W \cos \theta$, the airplane will "squash" or settle, and the flight path will be steeper. If the lift is greater than $W \cos \theta$, the airplane will not descend on that angle of glide but will tend to level out, so that the glide angle will be flatter. For any airplane there will be an angle of glide associated with each angle of attack determined by the following equations.

$$W \cos \theta = L$$
$$= C_L \times 0.00256\, SV^2$$
$$V^2 = \frac{W \cos \theta}{C_L \times 0.00256\, S}$$

and $$W \sin \theta = D_{\text{total}}$$
$$= C_D \times 0.00256\, SV^2 + 1.28\, a \times 0.00256\, V^2$$
$$= \left(C_D + \frac{1.28\, a}{S}\right) \times 0.00256\, SV^2$$

Then $$\tan \theta = \frac{D_{\text{total}}}{L}$$
$$= \frac{C_D + \dfrac{1.28\, a}{S}}{C_L}$$

A table may be calculated as follows, which is for the illustrative example of a monoplane with Clark Y wing, 36 ft. by 6 ft., and parasite of 3.8 sq. ft. E.F.P.A. weighing 2,000 lb.

TABLE XIII

α	C_L	C_D	$C_D + \frac{1.28a}{S}$	$\tan\theta$	θ	$\cos\theta$	$\frac{W\cos\theta}{0.00256\,S}$	V^2	V
−4	0.07	0.010	0.032	0.456	24.5	0.910	3 300	47 100	217
−3	0.14	.010	.032	.229	12.9	.975	3 540	25 300	159
−2	0.215	.012	.034	.158	9.0	.988	3 580	16 650	129
−1	0.285	.014	.036	.126	7.2	.992	3 600	12 620	112
0	0.36	.017	.039	.108	6.2	.994	3 610	10 000	100
1	0.43	.020	.042	.098	5.6	.995	3 615	8 400	92
2	0.50	.024	.046	.092	5.3	.996	3 620	7 240	85
3	0.57	.028	.050	.088	5.0	.996	3 620	6 350	80
4	0.645	.033	.055	.085	4.9	.996	3 620	5 610	75
5	0.715	.038	.060	.084	4.8	.996	3 620	5 060	71
6	0.785	.045	.067	.085	4.9	.996	3 620	4 610	68
7	0.857	.052	.074	.086	4.9	.996	3 620	4 230	65
8	0.93	.060	.082	.087	5.0	.996	3 620	3 890	62
12	1.19	.095	.117	.099	5.6	.995	3 615	3 040	55
16	1.435	.139	.161	.105	6.0	.994	3 610	2 520	50

In Fig. 65*a* are plotted angle of glide versus angle of attack and velocity versus angle of attack, for this example. It is to be noted that the angle of glide is big for negative angles of attack, that as angle of attack is increased the angle of glide decreases until at one angle of attack, in this case 5°, the angle of glide is a minimum of 4.8°. Further increase of angle of attack increases the angle of glide.

In the illustrative example, with a dead engine, the pilot pushes forward on the stick. The airspeed will increase. When it reaches 71 miles an hour, the pilot will manipulate the stick to maintain that speed. The angle of attack will then be 5° and the angle of glide will be the flattest possible, namely, 4.8°.

By maintaining the flattest possible glide, the greatest horizontal distance can be traveled in descending. The pilot can thus reach an emergency field and land, even though the field is at a considerable distance from the spot where his engine quit. The actual horizontal distance that may be achieved is a function of the altitude of the airplane when the engine quits. If h is the original altitude,

$$\text{Horizontal gliding distance} = \frac{h}{\tan\theta}$$

The minimum gliding angle, as found from a graph similar to that of Fig. 65*a*, will give the flattest possible glide. It will be noted that this angle of glide is a function of the lift and drag

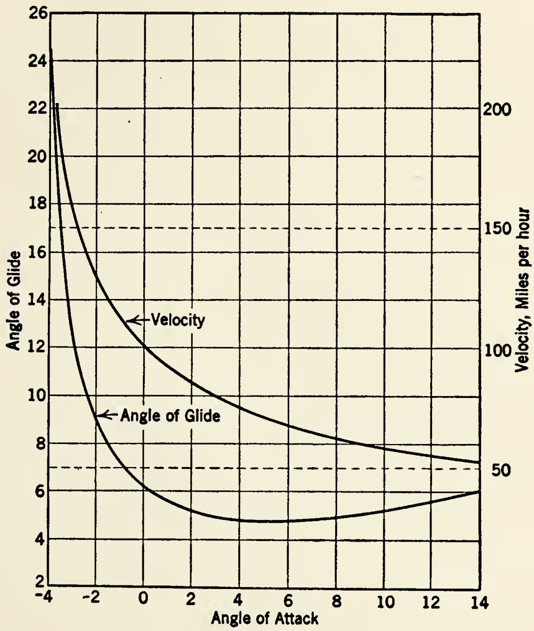


FIG. 65*a*. Angles and velocities in glide.

coefficients of the airfoil section and of the ratio of parasite area to wing area. It is independent of weight. That is, an airplane will have the same optimum gliding angle whether empty, partly loaded, or fully loaded.

The velocity plotted in Fig. 65*a* is velocity along the flight path. This velocity varies as the square root of the weight. A fully loaded airplane will therefore glide faster than a partly loaded plane even though it glides at the same angle. The slowest descent along the flight path would be at a large angle of attack. The danger of getting out of control would mean that some angle less than that of maximum lift should be used.

Vertical speed of descent is the sine of the angle of glide multiplied by the velocity along the flight path.

$$V_v = V \sin \theta$$

For the illustrative example at the flattest angle of glide, 4.8°, the airspeed is 71 miles per hour. The rate of vertical descent is, then,

$$\begin{aligned} 71 \times \sin 4.8° &= 71 \times 0.084 \\ &= 5.9 \text{ miles per hour} \end{aligned}$$

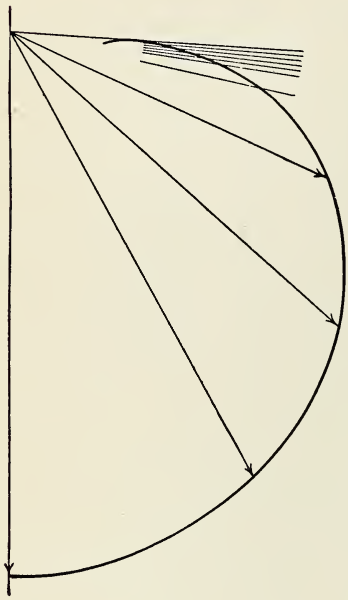


Fig. 65*b*. Polar diagram of velocities in dives.

At a greater angle of attack, the vertical descent may be slightly slower; for example, at a 16° angle of attack, while the angle of glide is 6°, the airspeed is only 50 miles per hour and the rate of vertical descent is

$$\begin{aligned} 50 \times \sin 6° &= 50 \times 0.104 \\ &= 5.2 \text{ miles per hour} \end{aligned}$$

Decreasing the angle of attack below that for minimum gliding angle will make the angle of glide increase and the airspeed increase. At the angle of zero lift, tangent θ will be infinite, that is, θ will be 90° and the airplane will be in a vertical dive. For this case, the weight pulls the airplane downward faster and faster, until the total drag equals the weight, when the airplane will not go down any faster. This speed is called the terminal velocity.

$$W = \left(C_D + \frac{1.28\,a}{S}\right) \times 0.00256\,S\,(V_{\text{terminal}})^2$$

$$V_{\text{terminal}} = \sqrt{\frac{W}{\left(C_D + \frac{1.28\,a}{S}\right) \times 0.00256\,S}}$$

For the illustrative example

$$\begin{aligned} V_{\text{terminal}} &= \sqrt{\frac{2{,}000}{(0.032) \times 0.00256 \times 216}} \\ &= 338 \text{ miles per hour} \end{aligned}$$

The data on glides and dives are frequently presented in the form of a polar diagram, as shown in Fig. 65*b*. The radius is the velocity to scale of the airplane when gliding or diving in the direction from the horizontal of the radius.

Problems

1. Plot angle of attack versus angle of glide for an airplane with a Clark Y wing 36 ft. by 6 ft., having 2 sq. ft. equivalent flat plate area of parasite.

2. If the airplane in problem 1 above weighs 2,500 lb., what is the airspeed for flattest gliding angle?

3. What horizontal distance can be traveled if the airplane in problem 1 glides from 5,000 ft.?

4. If the airplane in problem 1 weighs 3,000 lb., what is the airspeed for flattest gliding angle?

5. What is the airspeed of the airplane in problem 4 when gliding down on a 45° path?

6. What is the terminal velocity of the airplane in problem 4?

7. Plot a polar diagram of angle of glide and airspeed for an airplane weighing 2,500 lb. and having a Clark Y wing 36 ft. by 6 ft. and 5 sq. ft. equivalent flat plate area of parasite.

Glide Tests. After an airplane has actually been constructed, quite often tests are made to determine the full-scale lift and total drag coefficients. These determinations are useful for reference in designing future airplanes. They are also of primary importance if minor changes are being made in streamlining, engine-cowling, etc.

The airplane is tested in flight and the airspeed in various glides noted. The angle of glide cannot be measured directly; the rate of descent is found from a calibrated barograph, or by timing the readings on an altimeter. The airspeed is held constant in any one glide. The difficulty encountered in obtaining useful results from this sort of test is that of eliminating or making proper allowance for thrust. With the engine throttled, thrust may be positive, zero, or negative. With the engine stopped, the propeller produces additional drag ordinarily absent in flight.

Example. An airplane weighs 4,650 lb. Its wing area is 460 sq. ft. The wing section is Clark Y, aspect ratio 6. With airspeed indicator constant at 118 miles per hour, the airplane glides from 1,000-ft. to 500-ft. altitude in 26 sec. Neglect propeller thrust and drag. Find equivalent flat plate area of parasite.

Solution.

$$V_v = 500 \text{ ft. in } 26 \text{ sec.}$$
$$= 19.2 \text{ ft. per sec.}$$
$$= 13.1 \text{ miles per hour}$$

Angle of glide,
$$\theta = \sin^{-1}\frac{13.1}{118}$$
$$= \sin^{-1} 0.1115$$
$$= 6.4°$$
$$C_L = \frac{W \cos \theta}{0.00256\, S V^2}$$
$$= \frac{4{,}650 \times \cos 6.4°}{0.00256 \times 460 \times (118)^2}$$
$$= 0.282$$
$$C_D + \frac{1.28\, a}{S} = C_L \tan \theta$$
$$= 0.282 \times 0.112$$
$$= 0.0316$$

From Fig. 17, for Clark Y airfoil, aspect ratio 6, when C_L is 0.282, $C_{D\text{wing}}$ is 0.014

$$C_D + \frac{1.28\, a}{S} = 0.0316$$
$$0.014 + \frac{1.28\, a}{460} = 0.0316$$
$$a = 6.25 \text{ sq. ft.}$$

Problems

1. A monoplane weighs 5,000 lb.; its wing area is 400 sq. ft., aspect ratio of 6, airfoil section U.S.A. 35A. The parasite has an equivalent flat plate area of 7 sq. ft. Plot a polar diagram of velocity at various gliding angles.

2. An observation airplane weighs 4,225 lb., airfoil section Clark Y, total wing area 365 sq. ft. Equivalent flat plate area of parasite 8.4 sq. ft. Find terminal velocity in a vertical dive. (Since dive is at zero lift, there is no induced drag correction.)

3. A pursuit airplane weighs 2,932 lb.; airfoil section Clark Y; total wing area 264 sq. ft.; equivalent flat plate area of parasite 7.1 sq. ft. Find terminal velocity in a vertical dive.

4. A pursuit airplane weighs 2,548 lb.; airfoil section Clark Y, total wing area 247 sq. ft.; equivalent flat plate area of parasite 7.7 sq. ft. Find terminal velocity in a vertical dive.

5. A monoplane with Clark Y airfoil has an aspect ratio of 5.23; it weighs 3,500 lb.; its total wing area is 300 sq. ft. At an airspeed of 125 miles per hour, it glides at an angle of 8°. What is equivalent flat plate area of parasite?

Effect of Changing Weight. By changing weight, wing loading and power loading are changed.

$$V = \sqrt{\frac{W}{C_L \times 0.00256\,S}}$$

$$375\,\text{H.P.}_{\text{req.}} = (C_D \times 0.00256\,S + 1.28 \times 0.00256\,a)V^3$$

$$= (0.00256\,C_D S + 1.28 \times 0.00256\,a)\left(\frac{W}{C_L \times 0.00256\,S}\right)^{3/2}$$

At any one angle of attack, V varies as the square root of the weight and required horsepower as the cube of the square root of the weight.

$$V \text{ varies as } W^{1/2}$$
$$\text{H.P.}_{\text{req.}} \text{ varies as } W^{3/2}$$

If the horsepower-required curve has been found for an airplane as in Figs. 56, 57, and 58, any point on that curve gives the velocity for level flight at some one angle of attack (the abscissa) and the horsepower needed for that same angle of attack (the ordinate). If weight is added to the airplane, all other dimensions remaining the same, for each point on the original curve there will be a point on a new curve with the relations that the abscissas are to each other as the square root of the relative weights, and the ordinates are to each other as the three-halves power of the relative weights.

Putting the matter in another way, at the same velocity, for the same airplane, the lift coefficients must vary directly as the weights. Therefore, if load is added to an airplane, at any given speed the airplane must fly at a higher angle of attack. A higher angle of attack always means a greater drag coefficient throughout the flying range. An airplane will require more horsepower when more heavily loaded, since horsepower required at constant speed depends on drag coefficient, wing and parasite area. This means that if horsepower curves are drawn for a lightly loaded plane and the same plane heavily loaded, at any value of velocity, the horsepower needed for the heavier plane will always be greater than for the lighter airplane, therefore the two curves will never cross.

In detail, if an airplane is loaded more heavily, the landing speed will be increased. As the maximum lift coefficient and wing area are the same in both instances, the landing speeds will vary as the square root of the weights. With same engine and propeller, the maximum speed will be decreased slightly. The maximum speed is the intersection of the horsepower-required curve and thrust-horsepower curve, and no exact relation can be expressed for change in top speed for change in weight.

The excess horsepower at any speed will be decreased with increased weight. The rate of climb, being the excess power divided by the weight, will be decreased with increased weight. The speed of best climb will be increased as will the speed of maximum angle of climb.

The flattest gliding angle will be always the same for any airplane regardless of weight. The velocity at any angle of glide, including terminal velocity, will vary as the square root of the weight.

Figure 66 shows the effect of adding weight in changing the horsepower required. With a total weight of 4,500 lb., the horsepower-required curve is tangent to the horsepower-available

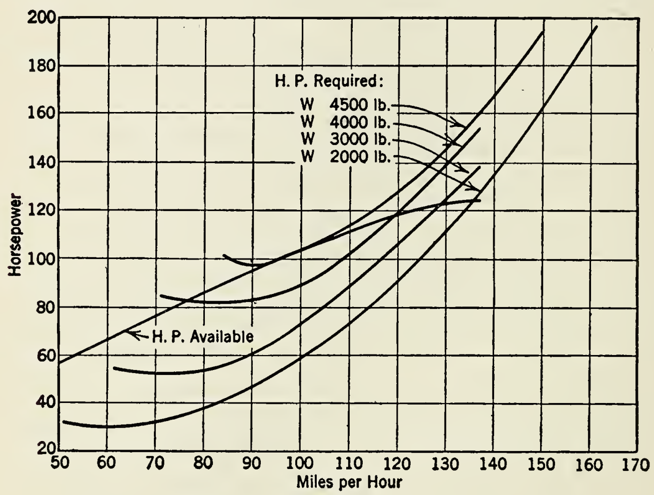


FIG. 66. Effect of weight on horsepower required.

curve; this is the limiting weight; for this weight, the ceiling is at sea-level. It is to be noted that, as weight is increased, minimum speed is no longer determined by $C_{L\max.}$ but by lower intersection of horsepower curves.

Example. An airplane weighs 3,000 lb. and has a landing speed of 50 miles per hour. What is landing speed with 500 lb. additional load?

Solution.

$$\frac{V_{\text{min.}}}{V_{\text{min.}}'} = \sqrt{\frac{W}{W'}}$$

$$V_{\text{min.}} = 50\sqrt{\frac{3{,}500}{3{,}000}}$$

$$= 53 \text{ miles per hour}$$

Example. An airplane, weighing 3,000 lb., requires the least horsepower to fly level, 40 hp. when flying at 80 miles per hour. If 500 lb. load are added, what are velocity and power for minimum horsepower in level flight?

Solution. At same angle of attack

$$\frac{V}{V_1} = \sqrt{\frac{W}{W_1}}$$

$$V = 80\sqrt{\frac{3{,}500}{3{,}000}}$$

$$= 86.5 \text{ miles per hour}$$

$$\frac{\text{H.P.}}{\text{H.P.}_1} = \left(\frac{W}{W_1}\right)^{3/2}$$

$$\text{H.P.} = 40 \times \left(\frac{3{,}500}{3{,}000}\right)^{3/2}$$

$$= 50.4 \text{ hp.}$$

Problems

1. An airplane weighs 2,485 lb. Its landing speed is 45 miles per hour. What is landing speed when 300 lb. extra load are added?

2. An airplane weighs 4,500 lb. Its landing speed is 48 miles per hour. What is landing speed when 250 lb. of fuel have been burned?

3. An airplane weighing 3,900 lb. uses minimum horsepower for level flight at an airspeed of 90 miles per hour. What is velocity for minimum horsepower with 600 lb. of load removed?

4. The Barling Bomber, fully loaded, had a gross weight of 42,500 lb. and a landing speed of 52 miles per hour. What is landing speed with 3,000 lb. less load?

5. A basic training plane weighing 4,060 lb. has a landing speed of 55 miles per hour. What is landing speed after 250 lb. of fuel have been burned?

Effect of Change in Wing Area. With constant weight, decreasing the wing area has the effect of increasing the wing loading. Airspeed at any angle of attack varies inversely as the square root of the wing area. Horsepower required at any angle of attack varies inversely as the square root of the area.

$$V \text{ varies as } \frac{1}{\sqrt{S}}$$

$$\text{H.P.}_{\text{req.}} \text{ varies as } \frac{1}{\sqrt{S}}$$

For a given velocity, an airplane with a smaller wing area will have to fly at a larger angle of attack. A larger angle of attack means a greater drag coefficient, but drag coefficient does not vary lineally with angle of attack. At high speeds (small angles of attack) the drag coefficient varies very little with angle of attack. At slow speeds, the slope of the drag coefficient curve versus angle of attack is very steep. At constant airspeed, wing drag varies as drag coefficient multiplied by wing area. At high speed, since lift coefficient varies lineally with angle of attack, angle of attack must vary inversely with wing area. At angles

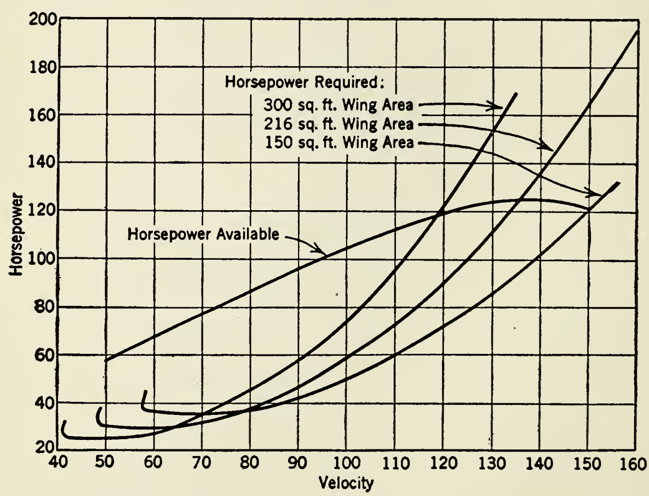


Fig. 67. Effect of wing area on horsepower required.

near the angle of minimum drag, an airplane having the same weight as another airplane but less wing area will at the same airspeed have to fly at a larger angle of attack. This increased angle will not mean a proportionate increase in drag coefficient. The airplane with smaller wing area will have less drag and less horsepower required.

At lower speeds, the increase in drag coefficient with increased angle of attack is much greater. Near stalling speeds, drag coefficient varies approximately as the square of the angle of attack. An airplane having the same weight as another plane but a smaller wing will have a greater drag and greater horsepower required at high angles of attack.

The horsepower curves will therefore cross as shown in Fig. 67. This means that an airplane's top speed can be increased by clipping the wings, provided that there is ample engine power.

Decreasing wing area increases landing speed.

Decreasing wing area increases the minimum horsepower and the velocity for minimum horsepower.

Decreasing wing area increases the minimum gliding angle, decreases the horizontal gliding distance, and increases the terminal dive velocity.

All the above effects are assuming that the parasite resistance remains the same.

Example. An airplane with 300 sq. ft. wing area has a landing speed of 40 miles per hour. If wing area is reduced to 250 sq. ft., what is the landing speed?

Solution.

$$\frac{V_{\text{min.}}}{V'_{\text{min.}}} = \sqrt{\frac{S'}{S}}$$

$$V_{\text{min.}} = 40\sqrt{\frac{300}{250}}$$

$$= 43.8 \text{ miles per hour}$$

Problems

1. An airplane with 340 sq. ft. of wing area lands at 40 miles per hour. It is desired to reduce landing speed to 35 miles per hour; how much area should be added to the wing?

2. A certain airplane with wing area of 400 sq. ft. is flying at angle of best L/D, when airspeed is 70 miles per hour. What airspeed corresponds to angle of best L/D when wings have been clipped to 350 sq. ft.?

3. A certain airplane with 450 sq. ft. of wing area flies with least horsepower of 40 hp. at an airspeed of 90 miles per hour. After wings have been clipped to 410 sq. ft., what is least horsepower and what is corresponding velocity?

4. An airplane with 450 sq. ft. of wing area lands at 50 miles per hour; what will be landing speed if wings are clipped to 400 sq. ft.?

5. An airplane with 630 sq. ft. of wing area lands at 45 miles per hour; what will be landing speed if 50 sq. ft. are added to the wing area?

Effect of Change in Engine. A more powerful engine means a larger and heavier engine. To consider the effect of increased power alone, it must be assumed that weight and parasite are unchanged, that is, any increase in engine weight is offset by a decrease in payload.

A change in power-available curve makes no change in the power-required curve. Provided there is no added weight or added drag, a more powerful engine will not affect the landing speed, nor will it affect the speed of minimum horsepower required, minimum gliding angle, or terminal velocity. It will increase maximum speed, rate of climb at any speed, and angle of climb at any speed.

When an airplane is at its top speed, it is flying at a low angle of attack. At small angles of attack the change in C_D with change of angle is quite small, and the change in parasite drag coefficient is negligible. When an airplane is flying level, the rate of climb is zero and thrust is equal to total drag.

$$\begin{aligned}
\eta \times \text{B.H.P.} \times 550 &= TV \\
&= DV \\
&= \left(C_D + \frac{1.28\,a}{S}\right)\frac{\rho}{2} S V^3 \\
V^3 &= \frac{\eta \times \text{B.H.P.} \times 550}{\left(C_D + \frac{1.28\,a}{S}\right)\frac{\rho}{2} S} \\
&= \frac{K_1 \times \eta \times \text{B.H.P.}}{\left(C_D + \frac{1.28\,a}{S}\right) S} \quad \text{where } K_1 = \frac{550}{\rho/2}
\end{aligned}$$

Since most propellers have approximately the same design efficiency, and assuming the drag coefficients as constant, the above expression may be reduced to

$$V = K_2 \sqrt[3]{\frac{\text{B.H.P.}}{S}}$$

The maximum airspeed may therefore be said to vary approximately as the cube root of the engine horsepower, provided that the change in horsepower is not excessive.

Example. An airplane with a 200-hp. engine has a maximum speed of 120 miles per hour. If total weight is unchanged, what is maximum speed with a 250-hp. engine?

Solution.

$$\frac{V}{V'} = \sqrt[3]{\frac{\text{B.H.P.}}{\text{B.H.P.}'}}$$

$$V = 120\sqrt[3]{\frac{250}{200}}$$

$$= 129 \text{ miles per hour}$$

Problems

1. An airplane with a 200-hp. engine has a maximum speed of 130 miles per hour. If a 165-hp. engine is substituted without change in total weight, what is maximum speed?

2. An airplane with a 400-hp. engine has a top speed of 150 miles per hour. Substituting a 500-hp. engine, what is the maximum speed?

3. An airplane with a 220-hp. engine has a top speed of 140 miles per hour. With total weight unchanged, what horsepower engine is needed to attain 150 miles per hour top speed?

4. An airplane with a 300-hp. engine has a top speed of 180 miles per hour. With total weight unchanged, what will be maximum speed with a 350-hp. engine?

5. An airplane with a 300-hp. engine has a top speed of 180 miles per hour. With total weight unchanged, a 250-hp. engine is substituted. What is maximum speed?

Power Loading and Wing Loading. Power loading is total weight per brake horsepower. Wing loading is the total weight per square foot of wing area. In the preceding section, the maximum speed was found to be

$$V = K_2\sqrt[3]{\frac{\text{B.H.P.}}{S}}$$

This may be rewritten as

$$V = K_2\sqrt[3]{\frac{W/S}{W/\text{B.H.P.}}}$$

A high wing loading accompanied by a low power loading will give a high maximum speed. A high wing loading, however, means a high landing speed. If landing speed is fixed, high speed depends on $\frac{1}{W/P}$, the reciprocal of power loading.

The rate of climb is given by

$$\text{R.C. (ft. per min.)} = \frac{33{,}000\ (\text{H.P.}_a - \text{H.P.}_{\text{req.}})}{W}$$

$$= \frac{33{,}000\ \eta\ (\text{B.H.P.})}{W} - \frac{60\ DV}{W}$$

$$= \frac{33{,}000\ \eta}{W/\text{B.H.P.}} - \frac{60\ V}{L/D}$$

$$= \frac{33{,}000\ \eta}{W/\text{B.H.P.}} - \frac{60\sqrt{W/C_L\frac{\rho}{2}S}}{\dfrac{C_L}{C_D + \dfrac{1.28\ a}{S}}}$$

$$= \frac{33{,}000\ \eta}{W/\text{B.H.P.}} - \frac{60}{\sqrt{\frac{\rho}{2}}} \times \frac{C_D + \dfrac{1.28\ a}{S}}{C_L^{3/2}} \times \sqrt{\frac{W}{S}}$$

The first term contains the reciprocal of the power loading; the second term, the square root of the wing loading. The rate of climb depends on the difference between the first and second terms. For good rate of climb, an airplane should have a small power loading in order that the first term should be large and a small wing loading in order that the second term be small.

Therefore a high wing loading helps top speed; a low wing loading helps climb. A low power loading helps both top speed and climb.

Span Loading and Aspect Ratio. The total drag is the force which is overcome by the propeller thrust. Assuming that fuselage, landing gear have been " cleaned up " so that parasite drag is reduced to a minimum, since the profile drag of most wings is about the same, the only other way in which drag may be decreased is by reducing induced drag.

It will be recalled that both parasite and profile drag vary as the square of the velocity, and the horsepower required to overcome these drags varies as the cube of the velocity. The induced drag varies inversely as the square of the velocity, and the horsepower inversely as the velocity. The induced drag therefore becomes less and less important as airspeed gets higher. The aspect ratio or span loading is related to the induced drag only.

The profile drag depends on the thickness and camber of the wing. The wing should be as thin as possible consistent with structural considerations.

At high speeds parasite drag is responsible for as much as 70 per cent of the total drag. Good streamlining is essential for high speeds, and it is of paramount importance for racing planes.

At speed less than the maximum, the induced drag becomes more important, and a small span loading or high aspect ratio becomes essential. This is true at climbing speeds and, as will be shown in the next chapter, is partly true for high speeds at high altitude.

Because a high-aspect ratio and a low-wing loading are both desirable for performance at low speeds, in England it is customary to find the ratio of these two characteristics and refer to this ratio as the span loading.

$$\frac{\text{Wing loading}}{\text{Aspect ratio}} = \frac{W/S}{b^2/S} = \frac{W}{b^2}$$

It is to be noted that in English textbooks span loading refers to weight divided by the span squared whereas in American textbooks span loading refers to weight divided by the span.

CHAPTER XI

PERFORMANCE AT ALTITUDE

Effect of Altitude on Horsepower Required. The performance of an airplane is affected by the density of the air. Lift, drag, and horsepower required are all functions of air density. The thrust and power absorbed by the propeller are functions of air density. The brake horsepower of an internal-combustion engine depends on density.

The fundamental equation for lift is

$$L = C_L \frac{\rho}{2} S V^2$$

For level flight lift equals weight. The wing area, S, is fixed. Then if air density (ρ) decreases, either C_L or V or both must increase. A given airplane flying at a higher altitude must either fly at a bigger angle of attack or it must fly faster or both.

Flying at the same angle of attack, C_L constant:

At sea-level: $W = C_L \frac{\rho_0}{2} S V_0^2$

At altitude a: $W = C_L \frac{\rho_a}{2} S V_a^2$

ρ_0 = sea-level mass density
ρ_a = air density at altitude a
V_0 = velocity, feet per second, at sea-level
V_a = velocity, feet per second, at altitude a

Then

$$V_a^2 = \frac{\rho_0}{\rho_a} V_0^2$$

$$V_a = \sqrt{\frac{\rho_0}{\rho_a}}\, V_0$$

Since the density is always greatest at sea-level, $\sqrt{\rho_0/\rho_a}$ is always greater than unity, so that, at the same angle of attack, the velocity at altitude must always be greater than at sea-level.

$$D_0 = \left(C_D + \frac{1.28\,a}{S}\right)\frac{\rho_0}{2} S V_0^2$$

$$D_a = \left(C_D + \frac{1.28\,a}{S}\right)\frac{\rho_a}{2} S V_a^2$$

D_0 = drag in pounds at sea-level
D_a = drag in pounds at altitude a

But

$$V_a^2 = \frac{\rho_0}{\rho_a} V_0^2$$

Therefore

$$\begin{aligned} D_a &= \left(C_D + \frac{1.28\,a}{S}\right)\frac{\rho_a}{2} S \frac{\rho_0}{\rho_a} V_0^2 \\ &= \left(C_D + \frac{1.28\,a}{S}\right)\frac{\rho_0}{2} S V_0^2 \\ &= D_0 \end{aligned}$$

That is, whatever the altitude, at the same angle of attack, the drag or thrust required for level flight is the same. While the density is less, the airspeed must be greater and the product remains constant.

$$\text{H.P.}_{\cdot\text{req.}0} = \frac{D_0 V_0}{375}$$

$$\begin{aligned} \text{H.P.}_{\cdot\text{req.}a} &= \frac{D_a V_a}{375} \\ &= \frac{D_0 V_0 \sqrt{\dfrac{\rho_0}{\rho_a}}}{375} \\ &= \text{H.P.}_{\cdot\text{req.}0} \times \sqrt{\frac{\rho_0}{\rho_a}} \end{aligned}$$

$\text{H.P.}_{\cdot\text{req.}0}$ = horsepower required at sea-level

$\text{H.P.}_{\cdot\text{req.}a}$ = horsepower required at altitude a

That is, at the same angle of attack, the horsepower varies inversely as the square root of the density.

The minimum speed, V_s, increases with altitude, since

$$V_{s0} = \sqrt{\frac{W}{C_{L\max.} \frac{\rho_0}{2} S}}$$

$$\begin{aligned} V_{sa} &= \sqrt{\frac{W}{C_{L\max.} \frac{\rho_a}{2} S}} \\ &= V_{s0} \sqrt{\frac{\rho_0}{\rho_a}} \end{aligned}$$

V_{s0} = stalling speed at sea-level

V_{sa} = stalling speed at altitude a

The square root of the reciprocal of the densities is tabulated in Table I. If the power requirements are known for sea-level conditions, the requirements for any altitude are found by making use of the factor from Table I. For any point on the total horsepower versus velocity curve for sea-level, a point may be found, for the same angle of attack, for any altitude by multiplying the abscissa by $\sqrt{\rho_0/\rho_a}$ and the ordinate by $\sqrt{\rho_0/\rho_a}$ to give coordinates of the horsepower required versus velocity curve at the altitude a.

Example. For the airplane described in the first example in the preceding chapter, find the horsepower-required curves for 10,000-ft. altitude and 15,000-ft. altitude.

Solution. From Table I:

$$\text{for } a = 10{,}000 \text{ ft.}; \quad \sqrt{\rho_0/\rho_a} = 1.16$$
$$\text{for } a = 15{,}000 \text{ ft.}; \quad \sqrt{\rho_0/\rho_a} = 1.26$$

TABLE XIV

From preceding chapter: For sea-level			For 10,000-ft. altitude		For 15,000-ft. altitude	
α	V	H.P.$_{\text{req.}}$	V	H.P.$_{\text{req.}}$	V	H.P.$_{\text{req.}}$
−4	228	566	264	656	288	715
−3	161	199	187	231	203	251
−2	130	111	151	129	164	140
−1	112	76	130	88	141	96
0	100	60	116	70	126	76
4	74.9	34	87.0	39	94.4	43
8	62.3	29	72.3	34	78.5	37
12	55.2	29	64.0	34	69.5	37
16	50.2	30	58.2	35	63.4	38
18	48.1	31	55.8	36	60.6	39
19	48.1	33	55.8	38	60.6	42

These curves are plotted in Fig. 68. It is to be noted that, since V and H.P. are always in the same proportion for one angle of attack, a line drawn from the point 0 velocity, 0 hp., through a point on the horsepower-required curve for sea-level, if prolonged, will pass through the point corresponding to that same angle of attack on the horsepower-required curve for all altitudes.

Horsepower Available at Altitude. At altitudes, the brake horsepower of the engine drops off. The variation in brake horsepower is approximately as the 1.3 power of the density.

Owing to the decrease in density, the propeller tends to turn over faster; but since the engine power drops faster than the density, the propeller will actually run slower.

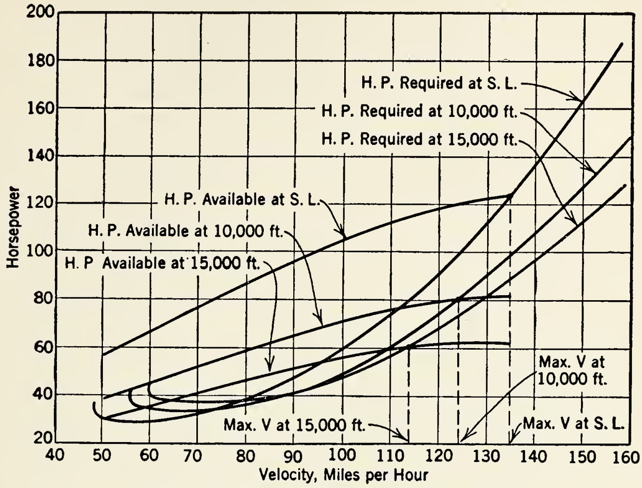


FIG. 68. Performance at altitude.

A quick way to find the horsepower available at altitude is as follows. In Fig. 55*b* is shown the decrease in revolutions per minute with altitude at the same airspeed as at sea-level. Since for most engines the brake horsepower varies directly with revolutions per minute, the same factor which is used in reducing the revolutions per minute can be used in reducing the brake horsepower. That is, on account of the fewer revolutions per minute less power will be developed. In addition, owing to the reduced density of air entering the carburetor, etc., the engine will develop less power even if running at the same revolutions per minute. This correction factor is found in Fig. 44*b*.

Because the propeller is turning over at a different speed, its $V/(ND)$ for various airspeeds will be different from the $V/(ND)$ for those airspeeds at sea-level. The propeller efficiencies will consequently differ at altitude from the efficiencies at sea-level for the same airspeeds.

Example. Find the horsepower available at 10,000-ft. and 15,000-ft. altitude for the 150-hp. engine used on the monoplane of the first illustrative example in the preceding chapter.

TABLE XV

Calculations for 10,000-ft. Altitude

Sea-level		10,000 ft.	Sea-level	10,000 ft.					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
V	N	N	B.H.P at reduced N	B.H.P	V/ND	% design V/ND	% design eff.	% eff.	H.P.avail.
50	1 565	1 522	128	87	0.381	43.8	53.3	44.0	38.2
60	1 565	1 522	128	87	.456	52.5	62.8	51.8	45.1
70	1 570	1 530	128	87	.530	61.0	72.0	59.5	51.8
80	1 583	1 540	129	87	.602	69.2	80.5	66.5	58.0
90	1 600	1 550	130	88	.670	77.0	88.0	72.5	63.8
100	1 633	1 590	132	89	.728	83.8	94.0	77.5	69.0
110	1 673	1 630	135	91	.778	89.5	97.0	80.0	72.8
120	1 751	1 705	142	96	.815	93.7	99.0	81.6	78.5
130	1 781	1 735	145	98	.870	100.0	100.0	82.5	81.0
135	1 800	1 755	146	99	.890	102.2	100.0	82.5	81.7

Calculations for 15,000-ft. Altitude

Sea-level		15,000 ft.	Sea-level	15,000 ft.					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
V	N	N	B.H.P at reduced N	B.H.P	V/ND	% design V/ND	% design eff.	% eff.	H.P.avail.
50	1 565	1 490	124	67	0.390	45.0	55.0	45.4	30.4
60	1 565	1 490	124	67	.468	53.8	64.0	52.8	35.4
70	1 570	1 495	125	67	.542	62.3	73.5	60.6	40.6
80	1 583	1 510	126	68	.616	71.0	82.3	68.0	46.2
90	1 600	1 525	127	68	.685	78.7	89.3	73.7	50.1
100	1 633	1 555	130	70	.745	85.5	94.5	78.0	54.6
110	1 673	1 590	133	71	.797	91.5	98.0	80.9	57.5
120	1 751	1 670	139	75	.835	96.0	99.5	82.1	61.6
130	1 781	1 700	142	76	.890	102.2	100.0	82.5	62.8
135	1 800	1 715	143	77	.914	105.0	99.0	81.7	62.8

Explanation of Table

Columns 1 and 2, V and N from sea-level calculations, last chapter.

Column 3, items in column 2 multiplied by altitude factor from Fig. 55*b*: 0.975 for 10,000-ft. altitude; 0.952 for 15,000-ft. altitude.

Column 4, 150 multiplied by ratio of items in column 3 to 1,800 r.p.m.

Column 5, items in column 4 multiplied by altitude factor from Fig. 44*b*: 0.676 for 10,000-ft. altitude; 0.537 for 15,000-ft. altitude.

Column 7, items in column 6 divided by design $V/(ND)$ (= 0.870).

Column 8, from Fig. 55*a*, using items in column 7 as entrants.

Column 9, items in column 8 multiplied by design efficiency (82.5 per cent).

Column 10, items in column 5 multiplied by items in column 9.

Maximum Speed at Altitude. The maximum speed at any altitude is found by the intersection of the total horsepower required at that altitude with the total horsepower available at that altitude. It is quite possible with propeller designed for altitude work, or other special conditions, that the maximum speed may be slightly higher at moderate altitudes than at sea-level. Ordinarily the maximum speed is less at altitude than at the ground.

Minimum Speed at Altitude. The stalling speed increases with altitude, that is, an airplane must fly faster to keep from stalling. The stalling speed or minimum speed at any altitude is to the minimum speed at sea-level as the reciprocal of the square root of the relative density.

The ordinary pitot static airspeed indicator measures impact pressure. The scale of the airspeed indicator gives the airspeed corresponding to the impact pressure at the ground of that airspeed. At altitude, the impact pressure will be less for any airspeed than the pressure for that airspeed at sea-level so that the airspeed indicator will read low. The indicated airspeed at altitude will be the true airspeed multiplied by the square root of the relative density.

This " error " of the airspeed indicator is rather fortunate from a pilot's viewpoint. The minimum speed at sea-level being known, whenever the airspeed indicator reads that airspeed, the airplane is flying at minimum velocity. At altitude, the airplane is flying faster than the airspeed indicator shows but it is necessary for the airplane to fly faster to keep from stalling.

Minimum Horsepower at Altitude. For any given airplane, there is one angle of attack at which $C_L^{3/2}/C_{D\text{total}}$ is maximum, and it is at this angle of attack that the least horsepower will be required to maintain level flight. At any altitude, this angle of attack for minimum horsepower will be the same, but the velocity corresponding to this angle of attack will be greater at higher altitudes. The velocity will be to the velocity at sea-level for minimum horsepower as the reciprocal of the square root of the relative density.

It is also true that the angle of attack for maximum L/D will be the same at all altitudes, but at higher altitudes greater velocity must be maintained for this same angle of attack.

As described for stalling speeds, the readings of the airspeed indicator will be the same at all altitudes when the airplane is flying at the angle of attack of minimum horsepower required. The airspeed indicator has the same reading at all altitudes when the airplane is flying at the angle of attack of maximum L/D.

Rate of Climb at Altitude. The ability to climb is determined by the excess of the horsepower available at any speed to the horsepower required at that speed. At altitudes above sea-level, the ordinates of the total horsepower-required curve become greater and the ordinates of the horsepower-available curve become less. The difference between these ordinates at the same speed represents the power for climbing.

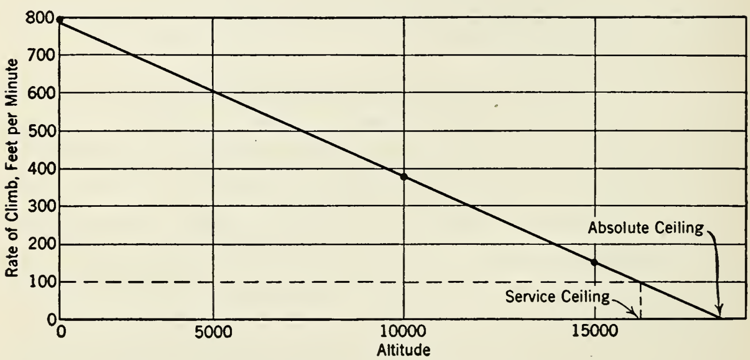


FIG. 69. Variation of rate of climb with altitude.

The greatest difference in ordinates gives the maximum climbing power at that altitude. As the altitude is increased, the power decreases. The airspeed for maximum climb is greater with increased altitude and is very close to the airspeed for minimum horsepower required, that is, the angle of attack for best climb is almost the same as the angle of maximum $C_L^{3/2}/C_{D\text{total}}$. Therefore with increasing altitude the speed for maximum climb increases.

If the maximum rate of climb is plotted against altitude as in Fig. 69, it will be seen that the resulting curve is practically a straight line. Actually the slope of the rate of climb curve varies lineally with altitude.

It is customary to assume that the rate of climb varies lineally with altitude.

Absolute and Service Ceilings. If the rate-of-climb versus altitude curve is continued to intersect the base line, this intersection marks the absolute ceiling or altitude where the rate of climb is zero. This is the highest altitude that it is possible for the airplane to reach.

At this altitude, the horsepower-available curve is tangent to the horsepower-required curve. The airplane can be flown at only one speed. At either greater or less velocity, there will not be sufficient available horsepower to maintain level flight. This one velocity of flight will be very close to the best climbing speeds at altitude near the ceiling.

At the absolute ceiling control is very sluggish. The rate of climb has been steadily decreasing with altitude, so that as one climbs very near to the absolute ceiling the rate of climb becomes infinitesimally small. It will therefore require an infinite time to reach the absolute ceiling, and no airplane ever reaches there unless one conceives of an infinite fuel supply.

More practical is the service ceiling which is the altitude where the rate of climb is 100 ft. per min.

If the rate of climb is assumed to change lineally with altitude

$$H_s = \frac{H[(\text{R.C.})_0 - 100]}{(\text{R.C.})_0}$$

H = absolute ceiling, feet
H_s = service ceiling, feet
$(\text{R.C.})_0$ = rate of climb at sea-level, feet per minute

Example. For an airplane weighing 4,000 lb., the excess horsepower at sea-level is 60 hp.; at 10,000-ft. altitude there is 17 excess horsepower. What is service ceiling?

Solution.

$$(\text{R.C.})_0 = \frac{60 \times 33{,}000}{4{,}000}$$
$$= 495 \text{ ft. per min.}$$

$$\begin{aligned}
(\text{R.C.})_{10{,}000} &= \frac{17 \times 33{,}000}{4{,}000} \\
&= 140 \text{ ft. per min.} \\
\frac{H - 10{,}000}{H} &= \frac{140}{495} \\
495H - 4{,}950{,}000 &= 140\ H \\
355\ H &= 4{,}950{,}000 \\
\text{Absolute ceiling} &= H = 13{,}970 \text{ ft.} \\
H_s &= 13{,}970\left(\frac{395}{495}\right) \\
&= 11{,}150 \text{ ft.}
\end{aligned}$$

Problems

(Assume that rate of climb varies lineally with altitude.)

1. An airplane weighs 3,500 lb.; its rate of climb at sea-level is 1,000 ft. per min.; its absolute ceiling is 16,000 ft. What is its service ceiling?

2. The service ceiling of an airplane is 14,000 ft. Its rate of climb at sea-level is 950 ft. per min. What is the absolute ceiling?

3. An airplane weighs 2,500 lb. What is the excess horsepower at its service ceiling?

4. The absolute ceiling of an airplane is 17,000 ft. The rate of climb at sea-level is 1,150 ft. per min. What is the rate of climb at 10,000-ft. altitude?

5. The service ceiling of an airplane is 18,000 ft. The rate of climb at sea-level is 1,000 ft. per min. What is the rate of climb at 10,000-ft. altitude?

Time to Climb to Altitude. Assuming rate of climb to vary lineal with altitude, the slope of the rate-of-climb versus altitude curve is constant, and is equal to $-(\text{R.C.})_0/H$. The rate of climb at any altitude, h, is

$$\begin{aligned}
\text{R.C.}_h &= (\text{R.C.})_0 - \frac{h(\text{R.C.})_0}{H} \\
&= (\text{R.C.})_0\left[\frac{H - h}{H}\right]
\end{aligned}$$

If rate of climb is expressed in differential form as dh/dt

$$\begin{aligned}
\frac{dh}{dt} &= (\text{R.C.})_0\left[\frac{H - h}{H}\right] \\
dt &= \frac{H\,dh}{(\text{R.C.})_0(H - h)}
\end{aligned}$$

Integration gives

$$
\begin{aligned}
t &= \int_h^0 \frac{H\,dh}{(\text{R.C.})_0(H - h)} \\
&= \frac{H}{(\text{R.C.})_0}\int_h^0 \frac{dh}{H - h} \\
&= \frac{H}{(\text{R.C.})_0}\log_e(H - h)\Big|_h^0 \\
&= \frac{H}{(\text{R.C.})_0}[\log_e H - \log_e(H - h)] \\
&= \frac{H}{(\text{R.C.})_0}\log_e \frac{H}{H - h} \\
&= 2.303\frac{H}{(\text{R.C.})_0}\log_{10}\left[\frac{H}{H - h}\right]
\end{aligned}
$$

t = time to climb to altitude, h, in minutes
H = absolute ceiling in feet
$(\text{R.C.})_0$ = rate of climb at sea-level, in feet per minute

Example. At sea-level an airplane's rate of climb is 1,000 ft. per min. Its absolute ceiling is 15,000 ft. How long will it take to climb to 7,000-ft. altitude?

Solution.

$$
\begin{aligned}
t &= 2.303 \times \frac{15{,}000}{1{,}000} \times \log\left(\frac{15{,}000}{15{,}000 - 7{,}000}\right) \\
&= 9.5 \text{ min.}
\end{aligned}
$$

Problems

1. At sea-level, an airplane weighing 4,000 lb. has 120 excess horsepower. Its absolute ceiling is 9,000 ft. (*a*) How long will it take to climb from sea-level to 5,000 ft.? (*b*) How long will it take to climb from sea-level to 6,000-ft. altitude?

2. A Heath monoplane has an absolute ceiling of 12,000 ft. Its rate of climb at sea-level is 450 ft. per min. How long will it take to climb from sea-level to 10,000-ft. altitude?

3. A Waco airplane, whose ceiling is 16,000 ft., climbs 1,050 ft. per min. at sea-level. How long will it require to climb from sea-level to 10,000 ft.?

4. A Bellanca airplane, whose ceiling is 17,000 ft., climbs 900 ft. per min. at sea-level. How long will it require to climb from sea-level to 12,000 ft.?

5. A Douglas airliner has a ceiling of 22,000 ft. It climbs 1,100 ft. per min. at sea-level. How much time will it take to climb from 10,000- to 15,000-ft. altitude?

Finding Ceiling by Time-to-Climb Formula. Since the time to climb is a function of ceiling, it is possible to use this relation to find the ceiling of an airplane by noting the altitudes attained at two times in a continuous climb. The simplest procedure is to use two equal time intervals, so that the time interval counted from instant of take-off to the time of reading the second altitude is twice the time interval from take-off to the time of taking the first altimeter reading. Let t_1 be the time in minutes from take-off to the attainment of altitude of h_1 feet, and t_2 be the time in minutes from take-off till altitude of h_2 feet is reached, then

$$t_1 = 2.303 \frac{H}{(RC)_0} \log_{10} \left[\frac{H}{H - h_1}\right]$$

$$t_2 = 2.303 \frac{H}{(RC)_0} \log_{10} \left[\frac{H}{H - h_2}\right]$$

$$\frac{t_2}{t_1} = \frac{\log_{10} \left[\frac{H}{H - h_2}\right]}{\log_{10} \left[\frac{H}{H - h_1}\right]}$$

$$\frac{t_2}{t_1} \log \frac{H}{H - h_1} = \log \frac{H}{H - h_2}$$

$$\left(\frac{H}{H - h_1}\right)^{t_2/t_1} = \frac{H}{H - h_2}$$

If t_2 is twice t_1, $t_2/t_1 = 2$

$$\left(\frac{H}{H - h_1}\right)^2 = \frac{H}{H - h_2}$$

$$H^2 - Hh_2 = H^2 - 2\,Hh_1 + h_1^2$$

$$2\,Hh_1 - Hh_2 = h_1^2$$

$$H = \frac{h_1^2}{2\,h_1 - h_2}$$

Example. An airplane takes 7 min. 30 sec. to reach 8,000 ft. altitude. In that same time interval (i.e., 15 min. from sea-level) it reaches 13,600 ft. altitude. What is the ceiling?

$$H = \frac{(8{,}000)^2}{2 \times 8{,}000 - 13{,}600}$$

$$= 26{,}667 \text{ ft. ceiling}$$

Problems

1. An airplane climbs in 10 min. from sea-level to 7,780 ft. altitude; continuing the climb 10 min. later the altitude is 12,500 ft. What is the ceiling?

2. An airplane climbs in a certain time to 9,000 ft. altitude; in double that time counted from sea-level the airplane reaches 14,000 ft. altitude. What is the ceiling?

3. An airplane, with a ceiling of 20,000 ft., climbs to a height of 6,000 ft. in a certain time. What height will it attain in twice the time?

4. An airplane, with a ceiling of 16,000 ft., can climb to 10,000 ft. in 20 min. What height will it have reached in 10 minutes?

5. An airplane climbs in a certain time to 2,000 ft.; in twice that time it has climbed to 3,500 ft. What is the ceiling?

Stratosphere Flying. An airplane which flies at a certain speed at the ground must, at altitude, either fly faster or fly at greater angle of attack; since weight and wing area are constant, either C_L or V must increase to compensate for decrease in density. If the airplane is flown at the same angle of attack at altitude as at sea-level, the velocity at altitude must be to the velocity at the ground inversely as the square root of the relative densities. The horsepower required at altitude will be to the horsepower required at the ground in that same ratio, inversely as the square root of the relative densities.

When flying at ground level (except at angles greater than that of maximum $C_L^{3/2}/C_{D\,\text{total}}$), an increase in velocity means an increase in horsepower required. At low angles of attack, there is very little change in C_D with angle of attack; therefore the change in horsepower is very closely as the cube of the velocity.

Therefore if it is desired to increase the speed of flying, one may either stay at the same altitude and decrease the angle of attack or climb to a higher altitude. In each case more horsepower will be required, but in the former the horsepower must be increased approximately as the cube of the airspeed and in the latter it must be increased directly as the airspeed.

In Fig. 70 are shown the horsepower-required curves for the airplane used as example 1 in the preceding chapter, for sea-level; for 10,000-ft., 15,000-ft., 30,000-ft., 40,000-ft., and 50,000-ft. altitude. It will be noted that to fly this airplane at 160 miles per hour at sea-level requires 196 hp.; at 10,000-ft. altitude, 148 hp.; 15,000-ft. altitude, 133 hp.; 30,000-ft. altitude, 94 hp.; 40,000-ft. altitude, 73 hp.; and 50,000-ft. altitude, 74 hp.

Viewed in another aspect, with, say, 100 hp. available, at sea-level, 124 miles per hour can be flown; at 30,000-ft. altitude, 164 miles per hour; at 50,000-ft. altitude, 210 miles per hour. At any altitude the least horsepower required is that corresponding to the angle of attack of maximum $C_L^{3/2}/C_{D\,\text{total}}$. For the specimen example this means a speed of 59 miles an hour at sea-level, but 154 miles per hour at 50,000-ft. altitude. The increase in speed, 154 ÷ 59 or 2.6 times, means an increase of 2.6 times

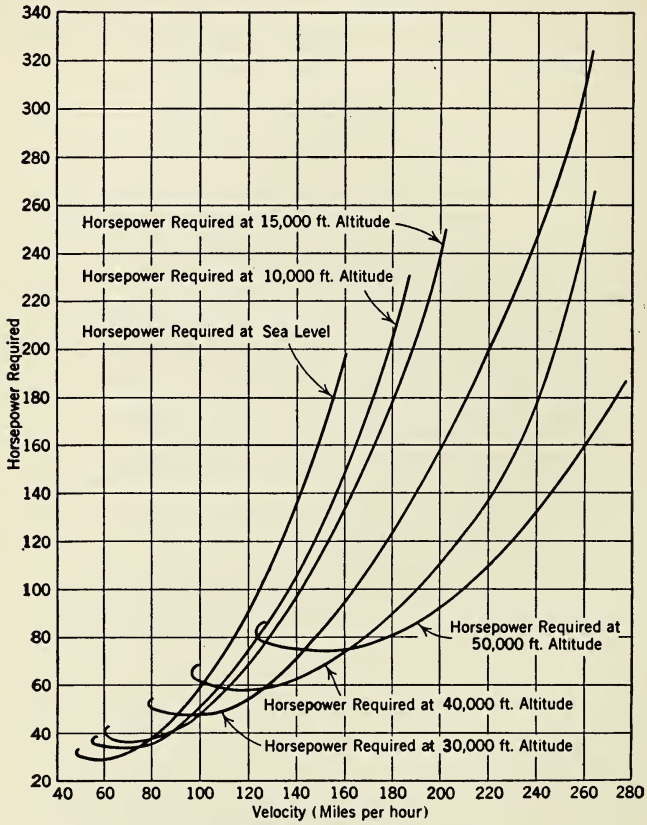


FIG. 70. Horsepower required at high altitude.

(74 ÷ 29) the horsepower. If 154 miles an hour were flown at sea-level, it would require 176 hp. or 6 times the horsepower.

Flying at high altitude is conditional on having the sufficient horsepower available. Unsupercharged engines drop off in horsepower with altitude. Propellers which are proper for sea-level density are unsuitable for high altitude. Superchargers can provide air at sea-level density at high altitudes; one proposed German airplane is to be provided with three superchargers, one which is to be put into service at 25,000-ft. altitude, a second which is to be added at 35,000 ft., and the third to be added at 45,000 ft. Variable-pitch propellers will of necessity be used on planes designed for stratospheric flying.

Rapid Estimation of Performance. For a clear understanding of the factors affecting airplane performance, the performance curves described in Chapters X and XI should be drawn and studied. For rapid estimation, however, various methods have been evolved. The method of W. Bailey Oswald is described below; for a fuller explanation the reader is referred to N.A.C.A. Report 408, from which this material was extracted.

Dr. Oswald assumes that the profile drag coefficient (C_{D_0}) and the parasite drag are each independent of the angle of attack. While this is not true at high angles of attack, it is substantially correct at high speeds (low angles of attack) with which this method is chiefly concerned. By combining the profile drag of the wing ($C_{D_0}\rho/2\ SV^2$) and the parasite drag of the remainder of the airplane ($1.28\ \rho/2\ aV^2$), the total drag exclusive of the induced drag of the wing is

$$C_{D_0}\frac{\rho}{2}SV^2 + 1.28\frac{\rho}{2}aV^2 = (C_{D_0}S + 1.28\ a)\frac{\rho}{2}V^2.$$

Dr. Oswald divides the airplane weight by the term in parentheses above, and terms the quotient, the parasite loading (L_p).

$$L_p = \frac{W}{C_{D_0}S + 1.28\ a}$$

The weight divided by the square of the effective span is called the " span loading " (L_s) by Dr. Oswald. This does not conform with the accepted meaning of the term, see Chapters V and VI, that span loading is the weight divided by the effective span. For a rectangular monoplane, the " span loading " is

$$L_s = \frac{W}{b^2}$$

For a biplane it is (see Chapter VI)

$$L_s = \frac{W}{(Kb_1)^2}$$

The weight divided by the design thrust horsepower is termed the thrust-power loading (L_t) in the Oswald method. The design thrust horsepower is the brake horsepower at rated revolutions per minute, multiplied by the propeller efficiency under design conditions.

$$L_t = \frac{W}{\text{B.H.P.}_{\text{rated}} \times \eta_{\text{design}}} = \frac{W}{\text{T.H.P.}_{\text{design}}}$$

Oswald also uses a factor (T_v) to represent the ratio of thrust horsepower at any speed (V) to the thrust horsepower at design (i.e., maximum) speed (V_m).

$$T_v = \frac{\text{T.H.P. at velocity } V}{\text{T.H.P. at velocity } V_m} \quad \text{(at sea-level)}$$

The factor T_a is the ratio of thrust horsepower at some altitude (a) to the thrust horsepower at sea-level, both at the same velocity.

$$T_a = \frac{\text{T.H.P.}_a \text{ at altitude}}{\text{T.H.P. at sea-level}} \quad \text{(at same velocity } V\text{)}$$

A large number of cases have been examined of modern planes with unsupercharged engines, and it has been found that

$$T_v = R_v^m \qquad \text{where } R_v = \frac{V}{V_m}$$

$m = 0.65$ for prop with design $C_s = 0.9$

$m = 0.61$ for prop with design $C_s = 1.2$

$m = 0.55$ for prop with design $C_s = 1.6$

$m = 0.55$ for props with peak efficiency

and

$$T_a = \frac{\sigma - 0.165}{0.835}$$

$$= 1.198(\sigma - 0.165)$$

The rate of climb at any speed and at any altitude is the excess power at that speed and altitude divided by the weight.

$$\text{R.C. (ft. per min.)} = \frac{(\text{H.P.}_{\text{avail.}} - \text{H.P.}_{\text{req.}}) \times 33{,}000}{W}$$

$$\text{R.C.} = 33{,}000\left(\frac{\text{H.P.}_{\text{avail.}}}{W} - \frac{\text{H.P.}_{\text{req.}}}{W}\right)$$

$$= 33{,}000\left[\frac{\text{B.H.P.}_{\text{rated}} \times \eta_{\text{des.}} \times T_aT_v}{W} - \frac{1}{550}\frac{C_{D_0}S + 1.28\,a}{W}\frac{\rho}{2}V^3 - \frac{1}{550\,W}\left(\frac{W^2}{\pi\frac{\rho}{2}b^2V}\right)\right]$$

$$= 33{,}000\left(\frac{T_aT_v}{L_t} - \frac{\frac{\rho}{2}V^3}{550\,L_p} - \frac{L_s}{550\,\pi\frac{\rho}{2}V}\right)$$

substituting $R = \dfrac{V}{V_m}$, $\sigma = \dfrac{\rho}{\rho_0}$

$$= 33{,}000\left(\frac{T_aT_v}{L_t} - \frac{\sigma\frac{\rho_0}{2}R^3V_m^{\,3}}{550\,L_p} - \frac{L_s}{550\,\pi\sigma\frac{\rho_0}{2}RV_m}\right)$$

At sea-level, when flying at maximum velocity, the rate of climb is zero and

$$\sigma = R = T_a = T_v = 1.$$

$$0 = \frac{1}{L_t} - \frac{\frac{\rho_0}{2}V_m^{\,3}}{550\,L_p} - \frac{L_s}{550\,\pi\frac{\rho_0}{2}V_m}$$

$$\frac{\frac{\rho_0V_m^{\,3}}{2}}{550\,\text{L}_p} = \frac{1}{L_t} - \frac{L_s}{550\,\pi\frac{\rho_0}{2}V_m}$$

$$\frac{1}{L_p} = \frac{550}{\frac{\rho_0}{2}V_m^{\,3}L_t}\left(1 - \frac{L_sL_t}{550\,\pi\frac{\rho_0}{2}V_m}\right)$$

$$\frac{L_s^{\,3}L_t^{\,4}}{L_p} = \frac{550}{\frac{\rho_0}{2}}\frac{L_s^{\,3}L_t^{\,3}}{V_m^{\,3}}\left(1 - \frac{L_sL_t}{550\,\pi\frac{\rho_0}{2}V_m}\right)$$

$$\frac{L_sL_t^{\,4/3}}{L_p^{\,1/3}} = \left(\frac{550}{\frac{\rho_0}{2}}\right)^{1/3}\frac{L_sL_t}{V_m}\left(1 - \frac{L_sL_t}{550\,\pi\frac{\rho_0}{2}V_m}\right)^{1/3}$$

substituting

$$\Lambda = \frac{L_s L_t^{4/3}}{L_p^{1/3}}$$

$$\Lambda = 77.3 \frac{L_s L_t}{V_m}\left(1 - 0.487 \frac{L_s L_t}{V_m}\right)^{1/3} \qquad V \text{ in feet per second}$$

$$\Lambda = 52.8 \frac{L_s L_t}{V_m}\left(1 - 0.332 \frac{L_s L_t}{V_m}\right)^{1/3} \qquad V \text{ in miles per hour}$$

In Fig. 71*a*, Λ is plotted against $V_m/L\,L_t$, and from this graph the maximum velocity at sea-level (V_m) may be found if the other parameters are known.

From one of the above equations, an expression can be found for V_m in terms of the three "loadings" (L_p, L_s, and L_t), which if substituted in the equation for rate of climb will permit a simplification.

Since

$$\frac{\frac{\rho_0}{2} V_m^3}{550\,L_p} = \frac{1}{L_t} - \frac{L_s}{550\,\pi \frac{\rho_0}{2} V_m}$$

$$V_m^3 = \frac{550\,L_p}{\frac{\rho_0}{2} L_t}\left(1 - \frac{L_s L_t}{550\,\pi \frac{\rho_0}{2} V_m}\right)$$

Substituting this value for V_m^3 in the equation

$$\text{R.C.} = 33{,}000\left(\frac{T_a T_v}{L_t} - \frac{\sigma \frac{\rho_0}{2} R^3 V_m^3}{550\,L_p} - \frac{L_s}{550\,\pi \frac{\rho_0}{2} \sigma R V_m}\right)$$

$$= 33{,}000\left(\frac{T_a T_v}{L_t} - \frac{\sigma \frac{\rho_0}{2} R^3}{550\,L_p}\left(\frac{550\,L_p}{\frac{\rho_0}{2} L_t}\right)\left(1 - \frac{L_s L_t}{550\,\pi \frac{\rho_0}{2} V_m}\right) - \frac{L_s}{550\,\pi \frac{\rho_0}{2} \sigma R V_m}\right)$$

$$= \frac{33{,}000}{L_t}\left(T_a T_v - \sigma R^3 + \frac{\sigma R^3 L_s L_t}{550\,\pi \frac{\rho_0}{2} V_m} - \frac{L\,L_t}{550\,\pi \frac{\rho_0}{2} \sigma R V_m}\right)$$

$$= \frac{33{,}000}{\sigma R L_t}\left(T_a T_v \sigma R - \sigma^2 R^4 - \frac{0.332 L_s L_t}{V_m}[1 - \rho^2 R^4]\right)$$

V in miles per hour

At altitudes above sea-level, when flying at maximum velocity, the rate of climb is zero.

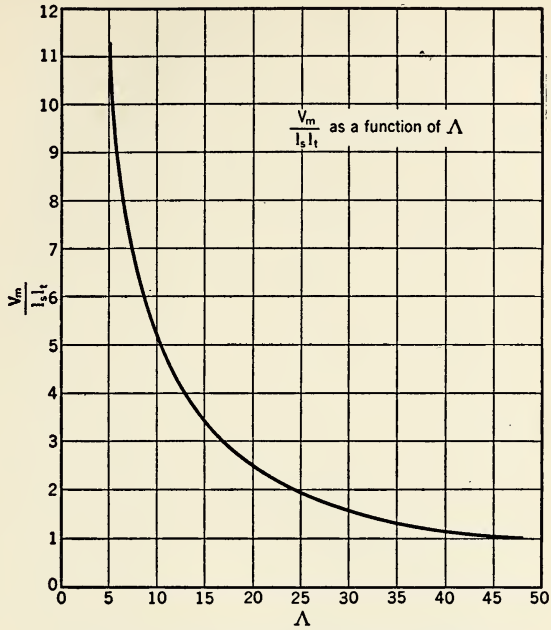


FIG. 71*a*. Performance prediction curves.

$$\frac{0.332\, L_sL_t(1 - \sigma^2R^4)}{V_m} = T_aT_v\sigma R_{vm} - \rho^2 R_{vm}{}^4$$

$$\frac{L_sL_t}{V_m} = 3.014\,\frac{T_aT_v\sigma R_{vm} - \sigma^2 R_{vm}{}^4}{1 - \sigma^2 R_{vm}{}^4}$$

where
$$R_{vm} = \frac{V_{\text{max.}}\text{ at altitude}}{V_{\text{max.}}\text{ at sea-level}}$$

but
$$\Lambda = 52.8\,\frac{L_sL_t}{V_m}\left(1 - 0.332\,\frac{L_sL_t}{V}\right)^{1/3}$$

Substituting the value of L_sL_t/V_m found above gives

$$\Lambda = 52.8\left(3.014\,\frac{T_aT_v\sigma R_{vm} - \sigma^2R_{vm}{}^4}{1 - \sigma^2R_{vm}{}^4}\right)\left(1 - 0.332\,\frac{L_sL_t}{V_m}\right)^{1/3}$$

$$= 159.1\,\frac{1.198(\sigma - 0.165)\sigma R_{vm}{}^{m+1} - \sigma^2R_{vm}{}^4}{1 - \sigma^2R_{vm}{}^4}\left(1 - 0.322\,\frac{L_sL_t}{V_{vm}}\right)^{1/3}$$

In the above equation, Λ is expressed in terms of R_{vm}, the relative density σ and certain parameters of the airplane. In Fig. 71*b*, R_{vm} is plotted against Λ for various altitudes.

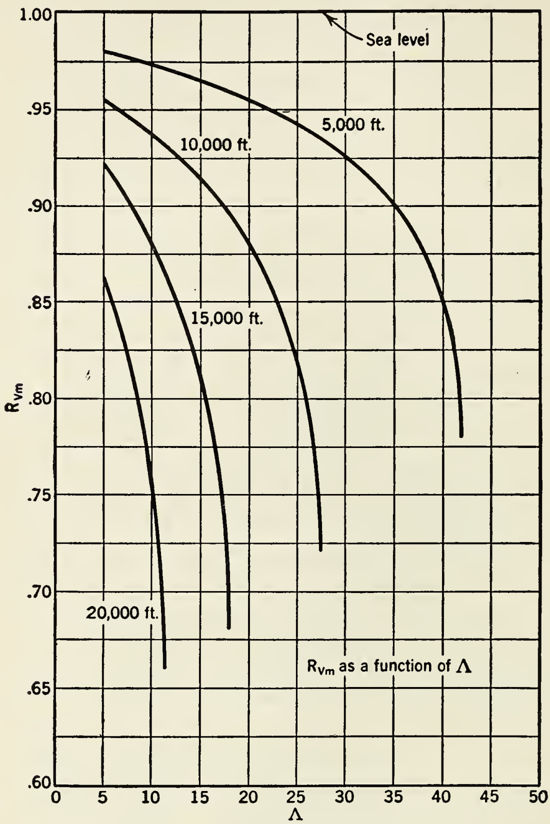


FIG. 71*b*. Performance prediction curves.

In a similar manner, equations may be deduced for other items of performance, and curves plotted. In Fig. 71*c*, $L_t(\text{R.C.})_h$ is plotted against Λ, so that by the aid of this graph, the rate of climb at any altitude h may be found.

As an illustration of the Oswald method, the performance of an airplane is partially worked out below. For comparison, the airplane of the first example in Chapter X is used.

Example. Find the maximum velocity and rate of climb at sea-level, 5,000 feet, and 10,000 feet altitude, for a monoplane weighing

2,000 lb. and having a Clark Y rectangular wing 36 ft. by 6 ft. The parasite drag has an equivalent flat plate area of 3.8 sq. ft. The engine is rated at 150 hp. at 1,800 r.p.m., and the propeller under design conditions has $C_s = 1.57$ and efficiency of 82 per cent.

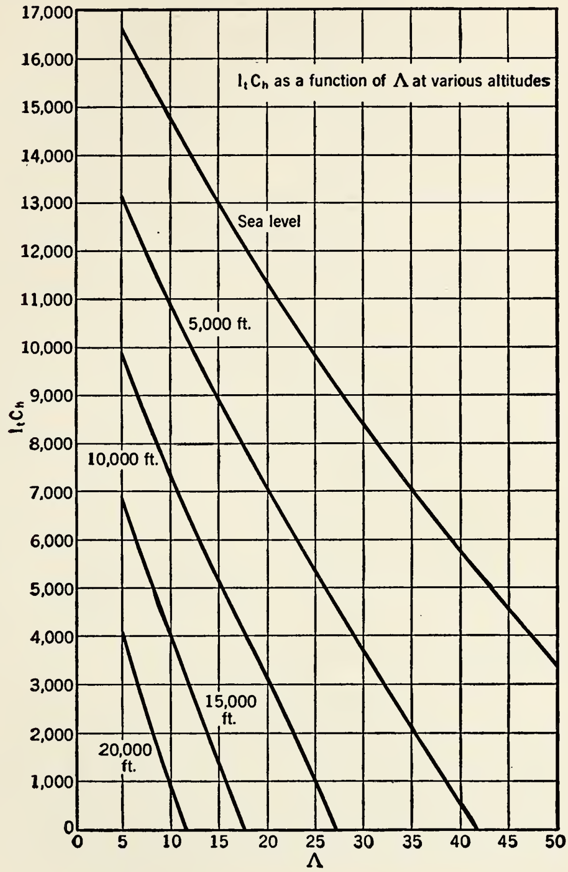


FIG. 71*c*. Performance prediction curves.

Solution.

For Clark Y airfoil $C_{D_0} = 0.01$ (see Fig. 38)

$$L_p = \frac{W}{C_{D_0}S + 1.28\,a}$$

$$= \frac{2{,}000}{0.01 \times 216 + 1.28 \times 3.8}$$

$$= 284.7$$

$$
\begin{aligned}
L_s &= \frac{W}{b^2} \\
&= \frac{2{,}000}{(36)^2} \\
&= 1.543 \\
L_t &= \frac{W}{\text{B.H.P.} \times \eta} \\
&= \frac{2{,}000}{150 \times 0.82} \\
&= 16.26 \\
\Lambda &= \frac{L_s(L_t)^{4/3}}{L_p^{1/3}} \\
&= \frac{1.543 \times (16.26)^{4/3}}{(284.7)^{1/3}} \\
&= 9.66 \\
L_sL_t &= 1.543 \times 16.26 \\
&= 25.09
\end{aligned}
$$

Altitude	Level flight			Climb	
	V_m	R_{vm}	V_{mh}	L_t(R.C.)	R.C.
0	133.0			14,900	916.3
5,000		0.970	129.0	11,250	691.8
10,000		0.935	124.4	7,550	464.3
15,000		0.884	117.6	4,250	261.3

Explanation of Table

Column 2 is obtained from Fig. 71*a* (for $\Lambda = 9.66$, $V_m/L_sL_t = 5.3$).
Column 3 is obtained from Fig. 71*b*.
Column 4 is obtained by multiplying items in column 3 by $V_m (= 133.0)$.
Column 5 is obtained from Fig. 71*c*.
Column 6 is obtained by dividing items in column 5 by $L_t (= 16.26)$.

CHAPTER XII

TURNS

Centrifugal Force. Heretofore the flight of an airplane has been considered in one direction. If direction is being changed, centrifugal force while the turn is being accomplished must be considered. Before the turn starts and immediately after the turn ceases, centrifugal force is not acting.

Acceleration is the change of velocity. Velocity has not only magnitude but also direction. Even if magnitude is unchanged but there is a change in direction of a velocity, that change is acceleration. That acceleration is always radially inward towards the center about which the object is circling at that instant. The force which causes the body to accelerate inward in a turn is measured by the mass times the acceleration.

In constant circular motion, the acceleration is $\omega^2 R$ where ω is the angular velocity, or the acceleration is V^2/R where V is the linear velocity. The centrifugal force of an airplane in a turn is equal in magnitude and opposite in direction to the accelerating inward (centripetal) force

$$\text{C.F.} = \frac{W}{g}\frac{V^2}{R}$$

g = acceleration of gravity in feet per second^2
V = airspeed in feet per second
R = radius of turn in feet

Banking. In straight flight, the force of gravity, called weight, is the only exterior force acting on the plane. It acts vertically downward, i.e., towards the center of the earth.

In a turn centrifugal force acts also on the airplane, and the exterior force acting on the airplane is the resultant of centrifugal force and weight. In turning in a horizontal plane, centrifugal force is outward, so the resultant will be outward and downward. Lift must balance this resultant force, so lift must act inward and upward and must be equal in magnitude to this resultant force.

In turns it is customary to bank or depress the inner wing. If a flat turn is made, that is, without banking, centrifugal force will cause the airplane to skid outwards. Except in flying in formation, this is not especially objectionable but is considered poor flying technique.

The proper angle of bank depends on the airspeed and the sharpness of the turn. If the angle of bank is insufficient the airplane will skid, that is, move outward. If the angle of bank is too much, the airplane will slip, that is, move inward and downward.

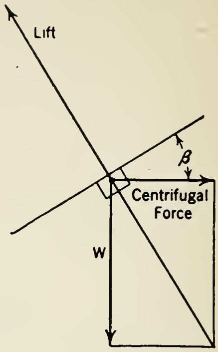


FIG. 72*a*. Forces in a banked turn.

Lift must equal in magnitude the resultant of weight and centrifugal force. If lift is not as great as this resultant, the airplane will squash or settle down in the direction of the resultant force. If lift is greater than the resultant, the airplane will execute a climbing turn or spiral.

From examination of Fig. 72*a*, if β is the angle of bank,

$$\tan \beta = \frac{\text{C.F.}}{W}$$

$$= \frac{\frac{W}{g}\frac{V^2}{R}}{W} \qquad \begin{array}{l} V \text{ in feet per second} \\ R \text{ in feet} \end{array}$$

$$= \frac{V^2}{gR}$$

It will be noted that the angle of bank β is independent of weight. A big, heavy bomber and a light sport-plane, if they have the same airspeed, require the same angle of bank for the same radius of turn. Angle of bank is also independent of wing area, airfoil section, etc.

If the angle of bank is correct, lift will act in the opposite direction to the resultant. The magnitude of the lift force must equal the magnitude of the resultant force, or

$$L = \frac{W}{\cos \beta}$$

$$= \frac{\frac{W}{g}\frac{V^2}{R}}{\sin \beta}$$

For any angle greater than zero, the cosine is less than unity; therefore, if the weight is divided by a number less than 1, the quotient is greater than the weight. For straight, level flight, lift equals weight. For a turn, lift must be greater than the weight. The lift may be increased either by increasing the airspeed or by increasing the angle of attack or both.

In low-powered airplanes or in gliders, it is quite often necessary to push forward on the stick in order by diving to get the added velocity for the turn.

Whether the added lift is gained by increasing the airspeed or by increasing the angle of attack, more power is required in a turn than in level flight.

Example. An airplane is flying straight at 150 miles per hour. Its weight is 6,000 lb. Its wing area is 200 sq. ft. (A.R.6), and Clark Y airfoil is used. The parasite has an equivalent flat plate area of 2 sq. ft. (*a*) What angle of attack is needed? (*b*) What horsepower is required? The airplane turns at 150 miles per hour airspeed with a 30° angle of bank. (*c*) What is the centrifugal force? (*d*) What lift is needed? (*e*) What is the proper radius of turn? (*f*) What angle of attack is needed? (*g*) What horsepower is required?

Solution.

(*a*)
$$\begin{aligned} C_L &= \frac{W}{0.00256\,SV^2} \\ &= \frac{6{,}000}{0.00256 \times 200 \times \overline{150}^2} \\ &= 0.522 \end{aligned}$$

From Fig. 17, $\alpha = 2.1°$

(*b*) From Fig. 17,

$$\begin{aligned} C_D &= 0.025 \\ \text{H.P.} &= \left(C_D + \frac{1.28\,a}{S}\right)\frac{0.00256\,SV^3}{375} \\ &= \left(0.025 + \frac{1.28 \times 2}{200}\right)\frac{0.00256 \times 200 \times \overline{150}^3}{375} \\ &= 175 \text{ hp.} \end{aligned}$$

(*c*)
$$\begin{aligned} \text{C.F.} &= W \tan\beta \\ &= 6{,}000 \times 0.577 \\ &= 3{,}460 \text{ lb.} \end{aligned}$$

(*d*)
$$\begin{aligned} \text{Lift} &= \frac{W}{\cos\beta} \\ &= \frac{6{,}000}{0.866} \\ &= 6{,}930 \text{ lb.} \end{aligned}$$

(e)
$$\tan\beta = \frac{V^2}{gR}$$
$$R = \frac{V^2}{g\tan\beta}$$
$$= \frac{(150 \times 1.47)^2}{32.2 \times 0.577}$$
$$= 2{,}600 \text{ ft.}$$

(f)
$$C_L = \frac{\text{Lift}}{0.00256 \times 200 \times \overline{150}^2}$$
$$= 0.603$$

From Fig. 17,
$$\alpha = 3.4°$$

(g) From Fig. 17,
$$C_D = 0.03$$
$$\text{H.P.} = \left(C_D + \frac{1.28\,a}{S}\right)\frac{0.00256\,SV^3}{375}$$
$$= \left(0.030 + \frac{1.28 \times 2}{200}\right)\frac{0.00256 \times 200 \times \overline{150}^3}{375}$$
$$= 203 \text{ hp.}$$

Problems

1. A plane of 2,000 lb. gross weight is turning at 100 miles per hour with an angle of bank of 45°. (*a*) What is the centrifugal force? (*b*) What is the lift? (*c*) What should be the radius of turn?

2. An airplane is making a 40° banked turn of 800-ft. radius. What should be the airspeed?

3. An airplane weighing 2,500 lb. has a Clark Y wing 250 sq. ft. in area (A.R.6) and parasite with equivalent flat plate area of 1.5 sq. ft. What horsepower is required in straight flight at 120 miles an hour and in a banked turn of a quarter-mile radius at 120 miles per hour?

4. A racing plane weighing 1,800 lb. is rounding a pylon at 340 miles per hour. The radius of the turn is 100 ft. What should be angle of bank, and what is centrifugal force?

5. An airplane weighing 5,000 lb. has a Clark Y wing of 300 sq. ft. area (A.R.6) and parasite with equivalent flat plate area of 3.0 sq. ft. (*a*) What horsepower is required to fly straight at 100 miles per hour? (*b*) What horsepower to make a 35° banked turn if the same angle of attack is maintained in the turn as in the straight flight?

Limiting Radius. In turns more horsepower is required than in straight flight because more lift is needed, and this greater lift is attained by increasing either velocity or angle of attack, both of which increases drag. Since maximum speed in straight flight entails using all the available horsepower, it is impossible at top speed to make a perfect turn, i.e., without slipping or squashing.

At speeds less than maximum, the excess horsepower can be used to accomplish turns. This extra horsepower is a fixed quantity for each airspeed, so that for each airspeed there is a limit to the horsepower that can be used on turns. The smaller the radius of turn, the greater horsepower is required; therefore, if the horsepower available is fixed, there is a definite limit on the sharpness of the turn.

The horsepower available at any velocity being known, the drag coefficient may be computed. The angle of attack and lift coefficient are then known. With lift coefficient and velocity known, the lift may be found. The greatest angle of bank may be then computed, and from that, the corresponding minimum radius of turn for a series of different velocities; it will be discovered that at one airspeed, which is the same airspeed as that for best climb, the radius of turn is smallest. With the assigned power plant this will be the sharpest possible turn, though with increased power a turn of even smaller radius can be made.

Example. Find the minimum radius of turn for an airplane weighing 2,000 lb., having a Clark Y wing 36 ft. by 6 ft. and 3.8 sq. ft. equivalent flat plate area of parasite and a 150-hp. engine rated at 1,800 r.p.m.

Solution. This is the same airplane as in the first illustrative example in the chapter on airplane performance for which the horsepower-available curve is plotted in Fig. 59.

Calculations are made for various airspeeds of which the following is a sample.

$$V = 110 \text{ miles per hour}$$

$$= 162 \text{ ft. per sec.}$$

$$\text{H.P.}_{\text{avail.}} = 110, \text{ from Fig. 59}$$

$$\text{H.P.} = \left(C_D + \frac{1.28\,a}{S}\right)\frac{0.00256 \times S \times V^3}{375}$$

$$110 = \left(C_D + \frac{1.28 \times 3.8}{216}\right)\frac{0.00256 \times 216 \times 110^3}{375}$$

$$C_D = 0.0335$$

From Fig. 17,

$$\alpha = 4.1° \text{ and } C_L = 0.65$$

$$\begin{aligned} \text{Lift} &= C_L \times 0.00256\,SV^2 \\ &= 0.65 \times 0.00256 \times 216 \times \overline{110}^2 \\ &= 4{,}330 \text{ lb.} \end{aligned}$$

$$\begin{aligned} \cos\beta &= \frac{W}{\text{Lift}} \\ &= \frac{2{,}000}{4{,}330} \\ &= 0.461 \\ \beta &= 62°\,33' \\ \tan\beta &= 1.925 \\ \tan\beta &= \frac{V^2}{gR} \\ R &= \frac{\overline{162}^2}{32.2 \times 1.925} \\ &= 424 \text{ ft.} \end{aligned}$$

The results of these computations are shown in Fig. 72*b*. The minimum permissible radius of turn has its smallest value when the airspeed is 65 miles per hour.

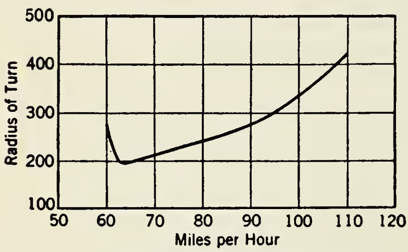


FIG. 72*b*. Minimum radius of properly banked turn.

At any airspeed, a wider turn can be made than shown in Fig. 72*b*, which, since it will require less horsepower, will permit throttling the engine.

Spiral Glide. In discussing a turn without loss of altitude, it was shown that more power is needed in turns than in level flight. In case of engine failure, the force that causes motion along the flight path is a component of the weight of the airplane itself. If a turn is desired while descending in a glide, for example, in order to land up-wind, greater lift must be attained or squashing will result. In general, lift can be gained either by increasing the angle of attack or by increasing the velocity. With a dead engine, velocity can be increased only by diving more steeply, i.e., increasing the angle of glide. If without the turn the airplane was descending at minimum angle of glide, in turning the angle of glide will be increased.

In a spiral glide, the airplane is descending on a helical path, this helix being on the surface of an imaginary circular cylinder whose radius is r. At any instant, the tangent to the flight path is at an angle θ to the horizontal tangent to the cylinder at that point. The component of the weight in the direction of the flight path which causes the motion along that path is $W \sin \theta$, and this equals the total drag as in a straight glide. Because the wing is banked, the direction of lift is inward and it is the component of lift in the vertical plane through the longitudinal axis of the airplane ($L \cos \beta$) that is equal and opposite to the component of weight ($W \cos \theta$) perpendicular to the drag.

While centrifugal force is actually acting in a direction slightly below the horizontal, it is usual to consider only its horizontal component ($WV^2 \cos^2 \theta/gr$), which is balanced by the horizontal component of lift ($L \sin \beta$).

Summing up the above statements, using D to represent total drag of the airplane,

$$D = W \sin \theta$$

$$L \cos \beta = W \cos \theta$$

$$L \sin \beta = \frac{WV^2 \cos^2 \theta}{gr}$$

Dividing the first by the second equation gives

$$\tan \theta = \frac{D}{L \cos \beta}$$

And from the third equation

$$r = \frac{WV^2 \cos^2 \theta}{gL \sin \beta}$$

Since $2 \pi r$ is the horizontal distance traveled in a complete turn, the altitude lost in a complete turn is $2 \pi r \tan \theta$. Then the minimum altitude will be lost per turn when conditions are such that $r \tan \theta$ is minimum.

$$r \tan \theta = \frac{WV^2 \cos^2 \theta}{gL \sin \beta} \times \frac{C_D}{C_L \cos \beta}$$

$$= \frac{W \dfrac{L}{C_L(\rho/2)S} \cos^2 \theta C_D}{gL \sin \beta \cos \beta C_L}$$

$$= \frac{2\,W\cos^2\theta}{g\frac{\rho}{2}S\left(\frac{C_L{}^2}{C_D}\right)2\sin\beta\cos\beta}$$

$$= \frac{2}{g\frac{\rho}{2}}\left(\frac{W}{S}\right)\frac{\cos^2\theta}{\left(\frac{C_L{}^2}{C_D}\right)\sin 2\,\beta}$$

Since in all cases the airplane would be flown at the flattest possible angle of glide, θ will be small and $\cos\theta$ will be nearly equal to one. The slowest rate of descent will be when both $C_L{}^2/C_D$ and $\sin 2\,\beta$ have their largest values. An investigation of standard airfoils shows that $C_L{}^2/C_D$ has a maximum value close to the burble and $\sin 2\,\beta$ will have its greatest value when β is 45°. The above expression shows that with a bigger wing loading (W/S), the minimum rate of descent is decreased.

Comparison may be made of the angle of glide θ in a simple glide, where $\theta = \cot^{-1} L/D$, with the angle of glide in turning where $\theta = \cot^{-1} \cos\beta\,(L/D)$. Whereas in the straight glide, the angle of attack is chosen for maximum L/D, in the turning glide the angle of attack is chosen to be near the stalling angle and the L/D is for that angle. When $\beta = 45°$, cosine $\beta = 0.707$. For the example of straight glide given earlier in this book, it was shown that the minimum gliding angle would be 4.8°. For the same airplane $C_L{}^2/C_D$ is maximum at 18°, where C_L/C_D is 8.3. The cotangent of θ is $8.3 \times 0.71 = 5.9$ and θ is 9.6°. The velocity during the glide will be $\sqrt{W\sin\theta/(C_D\,[\rho/2]\,S)}$ (where $W/S = 2000/216$), so that for this example the velocity is 83.6 ft. per sec. (57.0 miles per hour). The radius of turn corresponding to a 45° bank and a glide of 9.6° is $V^2\cos\theta/(g\tan\beta)$ or 214 ft. The loss of altitude in one complete turn $(2\,\pi r\tan\theta)$ is 227 ft.

Loading in Turns. In straight level flight, the lift on an airplane is equal to the total weight of the airplane plus or minus a small tail load. Practically the lift is considered equal to the weight. This is called unit load, or since the force is equal to the mass of the airplane multiplied by the normal acceleration due to gravity, it is called a force of one g (where g is 32.2 ft. per sec.^{2}).

During a turn, the lift must equal the weight divided by the cosine of the angle of bank, so that the greater the angle of bank, the greater must be the lift force. The ratio of lift or load during any maneuver to the unit load is called the load factor. Since the mass of the airplane does not change, it may be stated that in a maneuver an airplane is stressed to 3 g, 4 g, etc., meaning that the force or loading is the mass of the plane multiplied by $3g$, $4g$, etc.

In turns, the load factor is $\frac{L}{W} = \frac{1}{\cos \beta}$.

Example. In a 45° banked turn, what is the load factor?

Solution.

$$\frac{1}{\cos 45^\circ} = \frac{1}{0.7071} = 1.41$$

Load factor is 1.4 or plane is stressed to $1.4g$.

Problems

1. Airplane is making a turn of $\frac{1}{4}$-mile radius at a speed of 150 miles per hour. What is the load factor?
2. Airplane is making a turn of 150-yd. radius at a speed of 180 miles per hour. What is the load factor?
3. Airplane is making a turn of 100-yd. radius at a speed of 175 miles per hour. What is the load factor?
4. Airplane is making a turn of 200-ft. radius at a speed of 190 miles per hour. What is the load factor?
5. Airplane is making a turn of 150-ft. radius at a speed of 200 miles an hour. What is the load factor?

Loading in Vertical Banks. In a turn wherein the wings are banked up vertically, lift acts horizontally. At high speeds undoubtedly some small amount of lift is derived from the sides of the fuselage, but this cannot be sufficient to sustain the weight of the airplane, so that sideslip will take place. In a turn of this character, centrifugal force becomes very great.

In flying into such a turn, no matter if the immediately previous flying was at high speed and low angle of attack, the airplane would probably " squash " or settle outward. This movement would change the angle of attack to a very high angle and the increasing drag would cut down on the airspeed; an accurate computation of the loading in a vertical bank becomes exceedingly complicated. Some idea of the loading may be obtained by the following considerations.

In a vertical bank, lift acts inward; centrifugal force acts outward. Neglecting " squashing," lift should equal centrifugal force.

$$L = C_L \frac{\rho}{2} S V^2 = \frac{W V^2}{g R}$$

In straight level flight, slowest speed, i.e., stalling speed, is flown when the angle of attack is that of maximum lift coefficient and

$$W = C_{L\text{max.}}\frac{\rho}{2}SV_s^2$$

Substituting this value in the above equation gives

$$C_L\frac{\rho}{2}SV^2 = \frac{\left(C_{L\text{max.}}\frac{\rho}{2}SV_s^2\right)V^2}{gR}$$

Then

$$C_L = \frac{C_{L\text{max.}}V_s^2}{gR}$$

and

$$R = \frac{V_s^2}{g} \times \frac{C_{L\text{max.}}}{C_L}$$

The radius of turn will be minimum when C_L is equal to $C_{L\text{max.}}$, that is, when the angle of attack is that of maximum lift. If, owing to excessive centrifugal force, the airplane squashes outward, the horizontal curve described will be of greater radius than if the stick is pushed forward somewhat, so that the angle between the flight path and the wing is the angle of maximum lift.

This kind of turn is not to be confused with that described in the previous section; as in a vertical bank, the drag will be excessive and the speed will decrease during the turn.

Since the minimum radius of turn will be when $C_L = C_{L\text{max.}}$, the turning radius will be

$$R_{\text{min.}} = \frac{V_s^2}{g}$$

The load factor in a vertical bank will be

$$\frac{L}{W} = \frac{C_{L\text{max.}}\frac{\rho}{2}SV^2}{C_{L\text{max.}}\frac{\rho}{2}SV_s^2}$$

$$= \frac{V^2}{V_s^2}$$

It is to be noted that minimum speed varies with altitude, so at altitude the minimum radius will be greater.

Example. An airplane, whose stalling speed is 40 miles per hour, makes a vertically banked turn of minimum radius. The initial speed entering the turn is 125 miles per hour. What is the radius, and what is the load factor?

Solution.

$$
\begin{aligned}
R_{\text{min.}} &= \frac{V_s^2}{g} \\
&= \frac{(40 \times 1.47)^2}{32.2} \\
&= 107 \text{ ft.} \\
\text{Load factor} &= \frac{V^2}{V_s^2} \\
&= \left(\frac{125}{40}\right)^2 \\
&= 9.8
\end{aligned}
$$

Problems

1. A Curtiss pursuit plane whose stalling speed is 58 miles per hour goes into a vertically banked turn of minimum radius at a speed of 170 miles per hour. What is the radius, and what is the load factor?

2. A Gee-Bee Supersportster whose stalling speed is 91 miles per hour goes into a vertically banked turn of minimum radius at a speed of 270 miles per hour. What are the radius and the load factor?

3. A Supermarine racing plane whose landing speed is 107 miles per hour goes into a vertically banked turn of minimum radius at 409 miles per hour. What are the radius and load factor?

4. An airplane whose stalling speed is 38 miles per hour goes into a vertically banked turn of minimum radius at 80 miles per hour. What are the radius and load factor?

5. A Curtiss Condor whose landing speed is 55 miles per hour goes into a vertically banked turn of minimum radius at 140 miles per hour. What are the radius and load factor?

Dives. When a plane is flying horizontally and the pilot pushes the stick forward moving the elevator down, the first action is for the tail to move up. If the stick movement is fairly quick so that the tail is thrown up suddenly, momentum carries the airplane onward in a horizontal path. The relative wind strikes the upper side of the wing, that is, momentarily there is a negative angle of attack, with a consequent negative lift. During that instant, momentum takes the place of weight and acting horizontally forward, with respect to the axis of the plane it acts in a direction which in normal flight would be forward and upward. This loading situation is called inverted flight condition since it essentially duplicates flying upside down. When actually flying upside down, with the conventional unsymmetrical airfoil, flight must be at a high negative angle of attack with consequent big drag, so that the airspeed will be comparatively low. The load factors obtained in the inverted flight condition at the beginning of a dive are much greater than those obtained in actual inverted flight, so the former load factors are those used in stress analysis of the airplane structure.

As the airplane goes into the dive the pilot keeps the nose of the airplane depressed, or else with increased velocity the increased lift will pull the airplane out of the dive.

In coming out of the dive, the pilot pulls back on the stick which moves the elevator up tending to throw the tail downward with respect to the flight path. Momentum tends to make the plane settle or squash so that momentarily, at least, the airplane is at high angle of attack while still retaining the high speed attained in the dive. If V is the velocity when the airplane is pulled out of the dive into the high angle of attack position, then

$$L = C_{L\text{max.}} \frac{\rho}{2} S V^2$$

but

$$W = C_{L\text{max.}} \frac{\rho}{2} S V_s^2$$

and

$$\text{Load factor} = \frac{L}{W} = \frac{C_{L\text{max.}} \frac{\rho}{2} S V^2}{C_{L\text{max.}} \frac{\rho}{2} S V_s^2}$$

$$= \frac{V^2}{V_s^2}$$

The expression for load factor is the same as in a vertical bank, but an airplane usually goes into a bank from level flight whereas in a dive the velocity may be much greater than maximum level flight velocity. Coming out of a dive causes one of the most severe stresses put on airplanes, and unless care is taken the wings may be pulled off. By coming out of a dive gradually, the stresses are much less.

Problems

1. A Waco plane whose landing speed is 49 miles per hour is pulled out of a dive at 175 miles per hour. What is the load factor?

2. A Lockheed plane whose landing speed is 64 miles per hour is pulled out of a dive at 250 miles per hour. What is the load factor?

3. An Aeronca whose landing speed is 36 miles per hour is pulled out of a 120-mile-per-hour dive. What is the load factor?

4. A Supermarine racer whose stalling speed is 107 miles per hour is pulled out of a dive at 450 miles per hour. What is the load factor?

5. A Northrop airplane whose stalling speed is 58 miles per hour is pulled out of a dive at 220 miles per hour. What is the load factor?

Loops. In looping, a turn is executed in a vertical plane. Since, at the start of this maneuver, the pilot pulls back on the stick putting the airplane at a high angle of attack, with big drag which decreases the airspeed, it is important that the velocity be high in beginning the loop. Unless there is ample power, the pilot starts his loop by diving to gain speed.

The loop is rarely a perfect circle, for the airspeed will decrease as the plane climbs up preparatory to going over on its back, and after the top of the loop has been passed, the airspeed will increase again.

The exact load factors in a loop cannot be calculated unless the exact pattern of the loop is known, but an approximate idea may be gained if it is assumed that the path of the loop is a vertical circle. If R is the radius of this circle, V_1 the velocity and L_1 the lift at the bottom of the loop, and V_2 the velocity and L_2 the lift at the top of the loop, then

$$L_1 = \frac{W}{g}\frac{V_1^2}{R} + W$$

$$L_2 = \frac{W}{g}\frac{V_2^2}{R} - W$$

The work done in bringing the airplane from the bottom of the loop to the top of the loop is the weight times the vertical distance or $2RW$. In climbing, the energy lost is the difference in kinetic energy.

$$2RW = \frac{W}{g}\left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right)$$

Therefore

$$4Rg = V_1^2 - V_2^2$$

Assuming that at the top of the loop the lift is zero, which is approximately true in most cases,

$$L_2 = 0 = \frac{W}{g}\frac{V_2^2}{R} - W$$
$$W = \frac{W}{g}\frac{V_2^2}{R}$$
$$V_2^2 = Rg$$

Then since

$$V_1^2 - V_2^2 = 4Rg$$
$$V_1^2 - Rg = 4Rg$$
$$V_1^2 = 5Rg$$

But

$$L_1 = \frac{W}{g}\frac{V_1^2}{R} + W$$
$$= \frac{W}{g}\frac{5Rg}{R} + W$$
$$= 6W$$

or the load factor is $6g$.

CHAPTER XIII

THE CONTROL SURFACES

Axes. An airplane may rotate about three axes. The longitudinal or fore-and-aft axis is an imaginary line through the center of gravity of the airplane parallel to the thrust line of the propeller. It is called the X axis. The lateral axis is an imaginary line through the center of gravity of the airplane, perpendicular to the X axis, and horizontal when the airplane is on an even keel. The lateral axis is called the Y axis. The vertical or Z axis passes through the center of gravity perpendicular to the X and Y axes and is vertical when the airplane is on even keel; see Fig. 73.

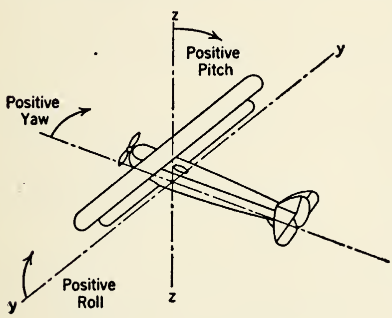


FIG. 73. Axes of airplane.

Rotational Motions of the Airplane. Rotation about the X axis is called *roll.* By tacit agreement among designers, it is termed positive roll if, when viewed from the rear, the airplane rotates in a clockwise direction. A rising left wing is the beginning of a positive roll.

Rotation about the Y axis is called *pitch.* If, when viewed from the left wing tip, the airplane rotates clockwise, it is positive pitch. Rotations are caused by moments. The moment which causes the airplane to rotate in a positive pitch is called a stalling moment; that which causes it to rotate in a negative pitch is called a diving moment.

Rotation about the Z axis is called *turn* or *yaw.* If when viewed from above the airplane rotates clockwise, it is positive yaw. A moment tending to cause a right turn is a positive yawing moment; that tending to cause a left turn is a negative yawing moment.

Angle of Attack of Airplane. Earlier in this book, the angle of attack of the wing was stated to be the angle between the direction of the relative wind and the wing chord. The wing chord is not usually parallel to the longitudinal axis of the airplane. The angle of attack of the airplane is the angle between the direction of the relative wind and the longitudinal axis of the airplane.

The angle between the chord of the wing and the longitudinal axis of the airplane is the angle of incidence. The angle of incidence is therefore the numerical difference between the angle of attack of the wing and the angle of attack of the airplane. Usually airplanes are so rigged that the angle of incidence is equal to the angle of maximum L/D of the airfoil section used.

Control Surfaces. It is necessary that means be provided to cause or stop rotation about any of the three axes, in order either to maintain the airplane in straight level flight or to execute various maneuvers. It is universal practice nowadays to make each control surface movable with respect to a fixed surface.

In the Canard type the horizontal and vertical rudders are in front of the main wing; in the conventional type, they are at the rear of the airplane.

The control surfaces for producing or regulating turns are the vertical tail surfaces. The fixed part is the vertical fin; the movable part is the rudder. Movement of the rudder is by the feet of the pilot. Pushing forward with the right foot on the rudder bar moves the rudder to the right and causes a right turn.

The control surfaces for producing or regulating pitch are the horizontal tail surfaces. The movable part is the elevator or "flipper"; the "fixed" part is the stabilizer. Though termed "fixed," the stabilizer except on small light planes can be adjusted during flight from the cockpit. Movement of the elevator is made by means of a control stick or post. Pushing forward on the stick causes the nose to go down; pulling back on it causes the nose to rise.

The control surfaces for producing or regulating roll are the ailerons. They are located at the rear of each wing tip. They are controlled by the control stick on a small plane and by a wheel (Deperdessin or "Dep" control) on large planes; moving the stick to the left makes the left aileron go up and the right aileron go down, causing negative roll. Moving the stick to the right causes the left aileron to go down and the right aileron to go up, producing positive roll.

Ailerons. Earlier attempts at flying were thwarted because the experimenters did not realize the need for lateral control.

The Wright brothers attained complete control by providing lateral control. Their first wings had simply upper surfaces, the ribs being strips of hard wood which had been steamed and bent to the proper curvature. Lateral control was effected by warping; that is, wires connected to the rear corners of the rectangular wings could be pulled, forcing the wing to be bent to a greater curvature. Upon release of the wire, the natural springiness of the wooden rib would cause it to return to its former shape.

Putting a lower surface on the wing precluded warping. The Wrights then used flat surfaces fastened onto the outer forward struts of their biplane. To produce a positive roll, the surface at the left side was tilted so as to slope backward and downward, on the right side the surface sloped backward and upward. It was direct impact pressure on these surfaces that caused the wing to move up or down.

The conventional type of lateral control is the aileron. The outer rear portion of each wing is movable, the hinged edge being more or less parallel to the leading edge. In normal flight these ailerons form part of the wing surface. Rarely they are rigged to droop slightly to give greater lateral stability.

With both ailerons in normal position, the lift on the left wing is the same as the lift on the right wing. When an aileron is moved down, it changes the airfoil section of the outer portion of the wing to an airfoil section of greater curvature and consequently greater lift. Moving an aileron upward changes the airfoil section to one of less curvature or of reverse curvature causing less lift.

Ailerons are designed so that moving an aileron down on one side causes the wing on that side to increase in lift by the same amount that the lift of the other wing is decreased by its aileron being moved up. Moving the control stick should change, not the total lift, but merely the proportion carried by each side. Unfortunately with the simplest ailerons this movement produces more drag on the side of the " down " aileron than on the side with the " up " one.

Moving an aileron down not only changes the airfoil to one of greater camber but also changes the chord so that the angle of attack is greater.

All control surfaces, whether they be ailerons, elevators, or rudders, are more effective at high speeds, since the forces acting on the surfaces vary as the square of the air velocity. The controls are said to be sluggish at low airspeeds. At high speeds considerable force is required to move a control surface away from its neutral position. At low airspeeds there is little resistance to the movement of a control surface. Even with slight experience a pilot knows when he is getting up near a stalled position by the looseness (or ease of movement) of his controls and by lack of response of the airplane to movements of the control surfaces.

Ailerons are very ineffective at high angles of attack. Examination of most of the common airfoil sections shows that the angle of maximum lift is about the same for them all. Then changing the shape of the airfoil by dropping an aileron will not increase lift; the change in chord puts the angle of attack beyond the burble point. The yawing effect of ailerons at high angles of attack is very pronounced, and it has been facetiously stated that, " at high angles, ailerons are good rudders."

To remedy the lack of effectiveness of ailerons for lateral control, slots, spoilers, or floating ailerons may be used.

Slots are primarily devices for giving higher lift, but since at high angles of attack they direct the airflow so that it follows directly the contour of the wing instead of burbling, slots add to the effectiveness of the ailerons.

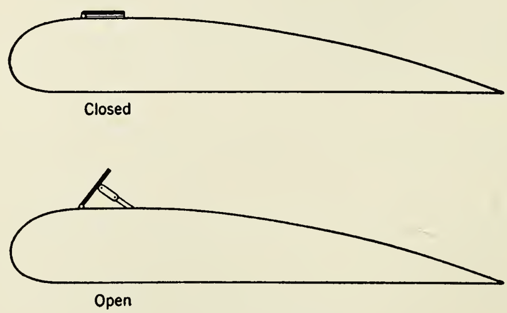


Fig. 74. Spoilers.

Spoilers. Though used very little, spoilers are quite effective for lateral control at high angles of attack. A spoiler consists of a long metal strip located on the top of the wing above the front spar. The strip can be quite narrow (see Fig. 74).

It is hinged at the front edge. In normal flight, this strip lies flat against the upper surface of the wing. By moving the stick to one side, the spoiler on the wing on that side stands erect.

The action of the spoiler is to destroy the smooth flow of air over the wing so that burbling takes place. The lift is consequently greatly reduced.

The outstanding objection to spoilers is that, unlike ailerons, their use reduces the total lift. When the spoiler is raised on one wing, the lift on the other wing remains as before. The reduction of lift on the wing with the raised spoiler means the same reduction in the total lift, and the airplane loses altitude. Though immaterial at moderate altitudes, this is exceedingly dangerous close to the ground as a crash might result.

Floating Ailerons. The floating aileron was installed on the Curtiss Tanager which won the Guggenheim Safety Contest in 1929. The floating aileron is placed outside the tip of each wing. The aileron is a symmetrical airfoil. Its axis is slightly behind the leading edge. Weight is added to the leading edge so that the aileron is balanced statically.

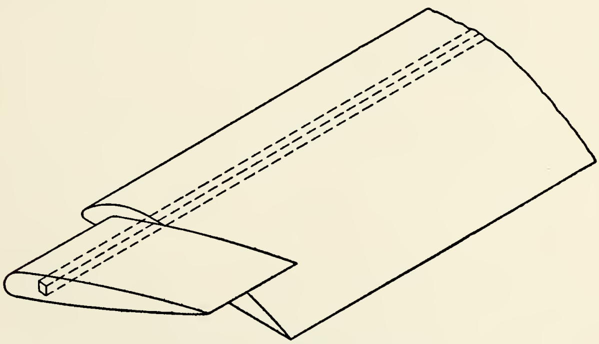


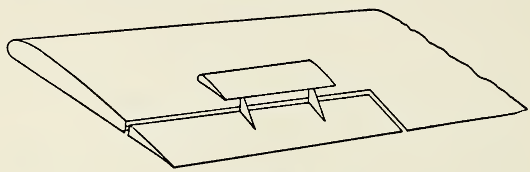
FIG. 75. Floating ailerons.

The ailerons are so rigged that they turn freely up and down provided that the ailerons on opposite wing tips rotate together. They will set themselves in the plane of the relative wind, much as a weather vane points into the wind. The pilot has no control over this action.

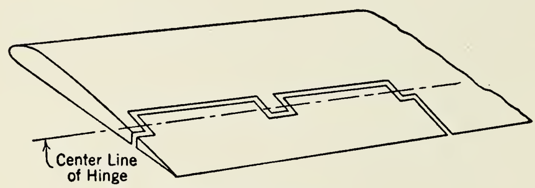
The pilot can displace one aileron with respect to the other. No matter at what angle the floating ailerons may have adjusted themselves with respect to the wing chord, the pilot by his control can move one aileron up and the other down by the same angle.

Since the airfoils are symmetrical, and the positive angle on one side the same as the negative angle on the other, the plus lift equals the minus lift and the total lift remains the same. In addition, the drag caused by the up aileron is the same as the drag of the down one, so that there is no yawing or turning tendency.

Balanced Control Surfaces. Since moving a surface against the pressure of a high wind requires considerable force, control sur-



Paddle Type



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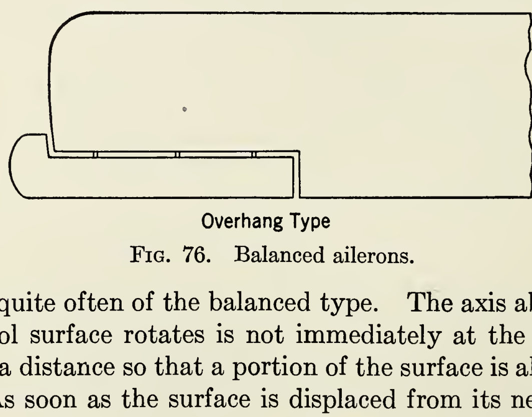


Overhang Type

Fig. 76. Balanced ailerons.

faces are quite often of the balanced type. The axis about which the control surface rotates is not immediately at the front edge but back a distance so that a portion of the surface is ahead of the hinge. As soon as the surface is displaced from its neutral position, the air strikes the front portion and reduces the effort the pilot must exert.

Although balanced controls enable the pilot to move large control surfaces easily, care must always be observed that they are not too well balanced. A pilot might then with little effort move the surface quickly to an extreme position, thus changing the motion of the plane suddenly, which causes high dynamic loads.

Differential Ailerons. Because the angular movement of an aileron downward causes more drag than the same angular movement upward, ailerons are sometimes rigged so that, when the stick is moved to one side, the aileron that moves upward has approximately twice the angular movement of the aileron that moves downward. This is done so that the drag on each wing tip shall be the same in order to eliminate yawing moment.

Frise Aileron. Another method of reducing the yawing moment is by use of the Frise type of aileron. Bearing a resemblance to the balanced type, the aileron is hinged about 20 per cent of the chord back from the leading edge and so designed that, when in its

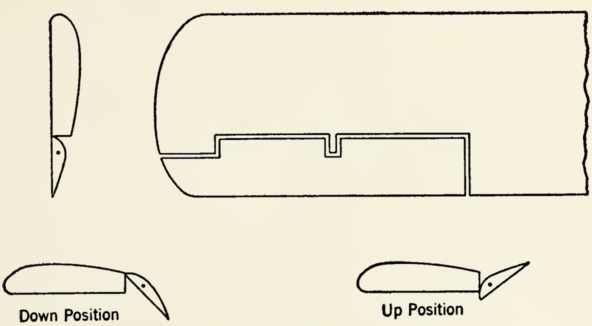


FIG. 77. Frise aileron.

" down " position, the upper surface of the aileron is a smooth prolongation of the curved upper surface of the wing. In its " up " position, the leading edge of the aileron projects below the continuation of the curved lower surface of the wing, so that additional drag is arbitrarily introduced at this point. When properly designed, the drag of both ailerons is the same, and there is no yawing moment.

Sizes of Ailerons. Ailerons are for the production of a rolling moment. This rolling moment, in foot-pounds, is the difference in lift between the two wings multiplied by the distance in feet between the centers of pressure on the two wings. This moment-arm is a function of the span. The rolling moment is usually expressed as

$$\text{Rolling moment} = L_e \frac{\rho}{2} S_w V^2 b$$

L_e = coefficient of rolling moment, dimensionless (N.B.: not to be confused with lift coefficient)
S_w = wing area, square feet
V = airspeed, feet per second
b = span, feet

In practice, it has been found that the coefficient L_e in the above equation should be at least 0.03 for all angles of attack.

To achieve this rolling moment it has been found that the total area of the ailerons should be about 10 per cent of the total wing area; 3 per cent below or above this value is permissible. The chord of the aileron is usually decided by the position of the rear spar of the wing, since the aileron hinges cannot be forward of the rear spar. This means that the aileron chord may be from 20 to 33 per cent of the wing chord. The spans of ailerons are between 40 and 65 per cent of the semi-spans of the wing.

Horizontal Tail Surfaces. The horizontal tail surface is very important as with it the airplane is " trimmed " so that with varying loads the plane may be flown at any angle of attack up to the maximum. It is common to use thin symmetrical or nearly symmetrical airfoil contours for tail surfaces. The size of the total horizontal surface and the distance behind the wing will be discussed in the following chapter on stability. The elevator is usually about 45 per cent of the total area although variations of plus or minus 10 per cent from this value have worked satisfactorily in practice. The aspect ratio of the entire tail surface is usually in the neighborhood of 3.

A large sudden movement of the elevator will cause a violent maneuver, putting severe strains on the airplane structure. For this reason, the elevators on commercial airplanes are not usually permitted to move more than 20° above or below their neutral position. On military planes greater movement is permitted, but it rarely exceeds 45°.

Occasionally an airplane is advertised as incapable of spinning. Spins usually start from stalls, and it is sometimes true that these airplanes have insufficient tail area to force them into a stalled position. This improper amount of tail surface is usually indicative of lack of controllability.

The stabilizer or " fixed " part of the horizontal tail surface has its rear edge fastened to a cross-member; the front edge may be moved up or down to change the stabilizer setting. Usually a vertical screw is used to adjust the height of the front of the stabilizer; turning the screw feeds the leading edge up or down, an arrangement being provided so that the screw may be rotated by a crank or handwheel in the pilot's cockpit. The screw by its leverage action permits easy adjustment by the pilot while it resists any changes of setting which might be caused by air forces acting on the tail.

The rear end of a stabilizer can be braced and thus made quite rigid, but the front end of an adjustable stabilizer cannot be braced and is held only at its central part. Even with a very sturdy structure the outer forward ends are apt to flutter in the slipstream. There appears to be a tendency at the present time to make the stabilizer fixed and non-adjustable. Necessary trim of the airplane longitudinally is attained by the use of tabs or bungees.

A tab is an auxiliary control surface, hinged to the rear edge of the elevator. When set at a definite angle to the plane of the elevator by means of a control wheel in the pilot's cockpit, the tab will remain at that angle with respect to the elevator regardless of any movement of the elevator. Stability can be attained by use of tabs in much the same way as with an adjustable stabilizer. They have the further advantage that their drag is much smaller than that of an adjustable stabilizer when the latter is meeting air at a large angle of attack.

Vertical Tail Surface. The vertical tail surfaces are to aid in maintaining directional stability and to cause turns. The area of the fin and rudder combined is usually between 5 and 6 per cent of the total wing area and between 40 and 45 per cent of the horizontal tail area. The movable rudder is usually 60 or 70 per cent of the total vertical area. The average maximum permissible movement of the rudder to either side of the center line of the airplane is 30°.

Effect of Propeller Action. If an ideal propeller could be devised, the air, after being acted on by this propeller, would pass directly to the rear. In practice, however, air is dragged around slightly with the propeller so that the slipstream has a helical motion; the rotation is clockwise viewed from the rear. The vertical tail surfaces receive therefore slightly more pressure on their left than on their right side, this tending to turn the airplane to the left.

For every action there is an equal and opposite reaction. Since a rotation is imparted to the propeller, the reaction of the propeller on the engine tends to turn the engine in the opposite direction. This causes greater pressure on the engine mount on the left side than the right side. As the flat surface of the wings would offer great resistance to a rapid rotation of the airplane in the opposite direction of rotation to that of the propeller, it is quite common to give more lift to the left wing than to the right wing to oppose this action. The left wing may be set at a very slightly greater angle of incidence, or in a biplane by tightening the rigging the left wing may be warped or twisted to have slightly greater camber. The latter is called " wash in." As a result of giving the left wing greater lift, the left wing has greater drag tending to turn the airplane to the left.

This left-turning tendency can be corrected by the pilot's using a small amount of right rudder, or the vertical fin can be set at a slight angle to the plane of symmetry.

The slipstream action and torque reaction are of course evident only when the engine is turning over at normal speed. If the engine stops and the airplane is in gliding flight, the slipstream and torque effects disappear, and if the fin has been offset the pilot will have to apply left rudder to prevent turning to the right.

It is to be noted that with twin engines, one mounted on each wing, the torque reaction is doubled, not neutralized.

CHAPTER XIV

STABILITY

Definition of Stability. Stability is the property of a body which, when the body is disturbed from a condition of equilibrium, causes forces or moments which act to restore it to its original condition. The greater the disturbance or change from its equilibrium position, the greater will be the magnitude of the forces or moments tending to return the body to its original attitude.

Stability of an airplane means that the airplane tends to remain at the same attitude with respect to the relative wind. It does not imply that the airplane is steady or that the airplane does not wobble with respect to the ground or to a fixed reference point in space. If the air is rough, the airplane may be constantly changing its attitude with respect to the ground.

The factors which make for a stable airplane are factors which preclude maneuverability. For stability, whenever attempt is made to change the attitude of the plane, forces resist this change. In maneuvering, these forces oppose any alteration in the flight path. Racing or pursuit planes should have little or no stability.

A plane is statically in equilibrium, if, when in flight, the sum of all forces acting in all directions equals zero and the sum of all moments about any point equals zero. The first part of the foregoing statement may be expressed as: the body is in equilibrium when the sum of the vertical forces is zero and the sum of the horizontal forces is zero.

$$\Sigma V = 0$$
$$\Sigma H = 0$$
$$\Sigma M = 0$$

If the body is disturbed from its equilibrium position, for stability there must be a restoring moment, and this moment must be larger for larger displacements from equilibrium position.

To be statically stable, the airplane must have the characteristic that a restoring moment or force acts in a direction to move the airplane back to the attitude from which it was disturbed. In discussing static stability, no thought is given to the magnitude of the restoring moment. This moment which acts to return the airplane to its equilibrium position may cause the airplane to acquire angular momentum so that it will swing past that position. Owing to the stability characteristic, another restoring moment will then act in the opposite direction, so that even with static stability there may be oscillation.

For dynamic stability, in addition to the requirement for static stability that there shall be a restoring moment, there is the further requirement that the moments created shall be of such character that the amplitudes of any displacement shall be of decreasing size so that the airplane will cease to oscillate and come to rest in its equilibrium position. Dynamic instability would mean that the restoring moment is so strong that each successive oscillation would have a bigger amplitude; such an action would mean impossibility of control, and disaster. The calculation of dynamic stability is an involved process and will not be treated in this book.

As there are three axes of rotation, so there are three classes of stability—longitudinal or fore-and-aft stability, lateral stability, and directional stability. They are interrelated, as rolling may produce turning, and vice versa.

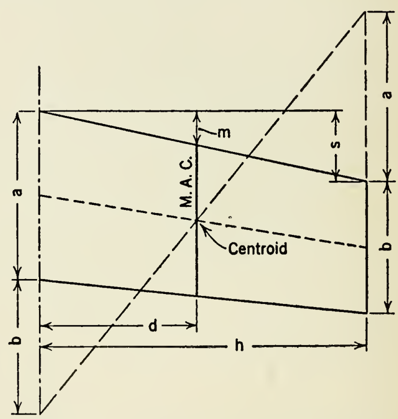


Fig. 78. Mean aerodynamic chord of tapered wing panel.

Mean Aerodynamic Chord. Any discussion of longitudinal stability involves a consideration of the mean aerodynamic chord of the wing or wings, usually abbreviated to M.A.C. The mean aerodynamic chord of a wing cellule is the chord of an imaginary airfoil which throughout the flight range will have the same force vectors as those of the wing cellule. This imaginary airfoil does not need to resemble any known airfoil. When the mean aerodynamic chord is obtained from wind-tunnel data of tests on a model of the airplane itself its location and length are definitely known. When such tests have not been made the following rules apply.

For a rectangular monoplane wing, the mean aerodynamic chord is identical with the chord of the wing section.

For a tapered monoplane, the mean aerodynamic chord of each wing panel passes through the centroid of the plan view of the wing panel. The leading edge of the M.A.C. is on a line connecting the leading edges of the root and tip sections, and the trailing edge of the M.A.C. is on a line connecting the trailing edges of the root and tip sections (see Fig. 78), which shows a swept-back, tapered wing. If d is the perpendicular distance from the root section to the M.A.C.

$$d = \frac{h(a + 2b)}{3(a + b)}$$

h = perpendicular distance root section to tip section
a = length of root chord
b = length of tip chord

The length of the mean aerodynamic chord is

$$\text{M.A.C.} = \tfrac{2}{3}\left(a + b - \frac{ab}{a + b}\right)$$

The distance, m, in a swept-back wing, of the leading edge of the M.A.C. to the rear of the leading edge of the root chord, is

$$m = \frac{s(a + 2b)}{3(a + b)} \qquad s = \text{total sweepback}$$

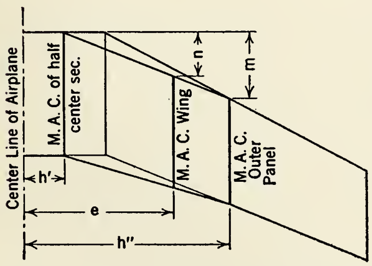


Fig. 79. Mean aerodynamic chord of wing with straight center section and swept-back outer panel.

In Fig. 78 is shown a graphical method of finding the M.A.C. of a wing panel.

For a monoplane with a straight center section and swept-back wing panels, the M.A.C. of the center section and the M.A.C. of the outer panel are found separately. The distance (e) of the M.A.C. of the entire semi-wing from the center line of the airplane (see Fig. 79) will be

$$e = \frac{h'S_c + h''S_p}{S_c + S_p}$$

h' = distance from center line of airplane to M.A.C. of semi-center-section
h'' = distance from center line of airplane to M.A.C. of wing panel
S_c = total area of center section
S_p = total area of both wing panels

The length of the mean aerodynamic chord is

$$\text{M.A.C.} = C_p + \frac{(C_c - C_p)(h'' - e)}{h'' - h'}$$

C_p = length of M.A.C. wing panel
C_c = length of M.A.C. center section

The rearward distance (n) of the leading edge of the M.A.C. behind the leading edge of the center section is

$$n = \frac{m(e - h')}{h'' - h'}$$

m = distance of L.E. of M.A.C. of wing panel behind L.E. of center section

For a monoplane with dihedral, the M.A.C. of the center section and the M.A.C. of the outer panel are found separately. The distance (n') (see Fig. 80) by which the M.A.C. is elevated is

$$n' = m' \frac{(e - h')}{h'' - h'}$$

m' is distance that M.A.C. of outer panel is raised

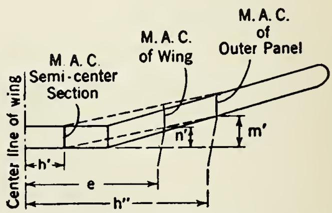


FIG. 80. Mean aerodynamic chord of wing with dihedral.

For biplanes, the M.A.C.'s for the upper and lower wings are found separately. The length of the M.A.C. of the biplane is

$$\text{M.A.C.} = \frac{eC_U S_U + C_L S_L}{eS_U + S_L}$$

S_U = area upper wing
S_L = area lower wing
C_U = M.A.C. upper wing
C_L = M.A.C. lower wing
e = relative efficiency of upper wing (see Fig. 82)

The relative loading e of the upper wing as given in Fig. 82 is only approximate but may be used for all angles of attack. For accurate work N.A.C.A. report 458 should be used.

The vertical distance of the leading edge of the M.A.C. of the biplane above the leading edge of the M.A.C. of the lower wing is g, where

$$g = G \frac{eS_U}{eS_U + S_L}$$

G = gap

The horizontal distance of the leading edge of the M.A.C. of the biplane ahead of the leading edge of the lower M.A.C. is s', where

$$s' = \frac{sg}{G}$$

s = horizontal distance between upper and lower leading edges

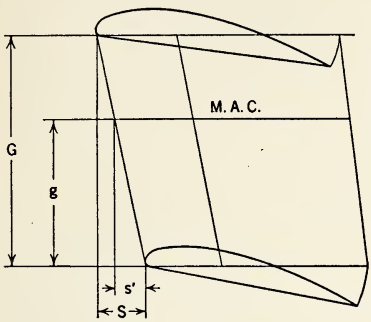


Fig. 81. Mean aerodynamic chord of biplane.

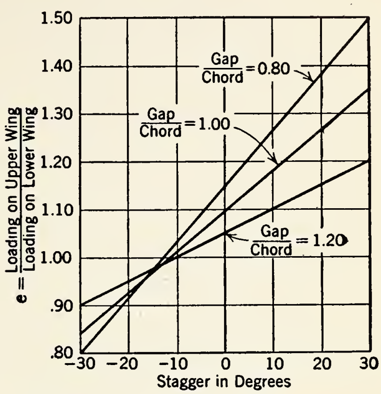


Fig. 82. Relative wing loading in a biplane.

Problems

1. A certain monoplane had a rectangular wing with 5-ft. chord and 32-ft. span, the maximum forward position of the center of pressure being at 30 per cent of the chord from the leading edge. It was found that the center of gravity of the airplane was to the rear of the maximum forward position of the center of pressure. To remedy this situation, it was proposed to leave unchanged the center section of the wing, the width of which was 4 ft., while the outer portions of the wing were to remain parallelograms of the same area but were to be given a sweepback of 2 ft. at each tip. (*a*) Find distance from longitudinal center line of airplane to M.A.C. of the altered wing. (*b*) Find distance that leading edge of M.A.C. has been moved back. (*c*) Find distance that maximum forward position of the C.P. of the M.A.C. of the wing has been moved back.

2. Solve problem 1 when sweepback at each tip is 1 ft., wing area being the same.

3. Solve problem 1 when trailing edge is left perpendicular to the longitudinal axis of the airplane while leading edge is given a sweepback beginning at the center so that the sweepback at each tip is 2 ft., the span being lengthened so that the wing area remains unchanged.

4. Solve problem 3 when sweepback at each tip is 1 ft., the span being of such a length that the wing area remains unchanged.

5. Solve problem 1 when the center section of 4-ft. width remains unchanged, the trailing edge remains perpendicular to the longitudinal axis of the airplane, the leading edge outside of the center section is given a sweepback so that the sweepback at each tip is 2 ft., the length of the span being such that the wing area remains unchanged.

6. Solve problem 5 when the sweepback at each tip is 1 ft., the span being of such a length that the wing area remains unchanged.

7. The center of gravity of a monoplane with rectangular wing of 4-ft. chord and 30-ft. span was not far enough below the wing. The proposed remedy was to leave unchanged the center section, the width of which was 4 ft., while the outer portions of the wing were given a dihedral of 2 ft. at each tip. (*a*) Find the horizontal distance from longitudinal center line of airplane to M.A.C. of altered wing. (*b*) Find the distance that M.A.C. of the wing has been moved up.

8. Solve problem 7 when the dihedral at each tip is 1 ft.

9. Solve problem 7 when the center section of the wing is left unchanged, the outer portions being given a dihedral of 2 ft. at each tip, the leading edge outside of the center section being given a sweepback of 2 ft., while the trailing edge remains perpendicular to the longitudinal axis of the airplane, the span being of such a length that the wing area remains unchanged.

10. Solve problem 9 when the dihedral at each tip is 1 ft. and the sweepback at each tip is also 1 ft., the span being of such a length that the wing area remains unchanged.

Longitudinal Balance. By using the convention of a mean aerodynamic chord, the forces resulting from biplane wings or from a monoplane wing having taper, dihedral, or sweepback can be considered as the forces acting on a rectangular monoplane wing. In the following discussion, though a simple wing will be alluded to, it may be considered as the (imaginary) rectangular monoplane wing whose chord is the M.A.C. of the actual wing or wings.

In level flight, the forces which must be considered are the weight, acting downward; the propeller thrust, acting forward; the lift, acting upward; the total drag, acting backward; and the tail load, which may be either upward or downward.

In the conventional high-wing monoplane shown in Fig. 83, the thrust and drag both act to produce stalling or positive pitching moment. The lift must then produce a negative or diving moment. The moment of the tail load must be such as to be equal in magnitude to the difference of the plus and minus moments and of the same sign as the smaller. In order to ensure that the lift always produces a diving moment, the center of gravity must be ahead of the most forward position of the center of pressure of the airplane. At high angles of attack, for some airfoils the center of pressure moves forward to a position 25 per cent of the chord back of the leading edge, so that the center of gravity must be in front of that. The tail is a symmetrical airfoil with constant center of pressure position.

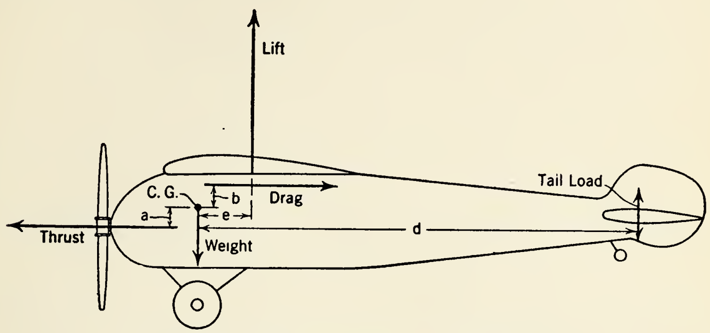


Fig. 83. Forces on an airplane in flight.

In Fig. 83, the distance of the center of gravity above the thrust line is a, its distance below the line of action of the total drag is b, its distance ahead of the line of action of lift is e, and the distance from the center of gravity to the center of pressure of the tail is d. Then,

$$T \times a + D \times b - L \times e \pm \text{Tail load} \times d = 0$$

Example. A monoplane, weighing 3,000 lb., having rectangular Clark Y wing of 48-ft. span and 8-ft. chord and $5\frac{1}{2}$ sq. ft. of equivalent flat plate area, is flying at 100 miles per hour. The center of gravity of the airplane is 20 in. back of the leading edge of the wing, 12 in. above the thrust line, and 8 in. below line of action of total drag. It is 25 ft. from the center of gravity of the airplane to the center of pressure of the tail. What should be the tail load?

Solution.

$$\begin{aligned} C_L &= \frac{W}{0.00256\, SV^2} \\ &= \frac{3{,}000}{0.00256 \times 384 \times \overline{100}^2} \\ &= 0.306 \end{aligned}$$

From Fig. 17 when $C_L = 0.306$, $C_D = 0.015$

$$D_{\text{total}} = \left(C_D + \frac{1.28\,a}{S}\right)0.00256 \times SV^2$$
$$= \left(0.015 + \frac{1.28 \times 5.5}{384}\right)0.00256 \times 384 \times \overline{100}^2$$
$$= 322 \text{ lb.}$$

Since $\Sigma H = 0$, Drag = Thrust

From Fig. 17, when $C_L = 0.306$, C.P. is at 46 per cent of chord, that is, 0.46×96 in. = 44 in. back of leading edge.

Moment arm of lift force = e = 44 in. − 20 in. = 24 in.

$$T \times a + D \times b - L \times e \pm \text{Tail load} \times 25 \times 12 = 0$$

$$\text{Tail load} = \frac{72{,}000 - 6{,}440}{300}$$
$$= 218 \text{ lb.}$$

Tail load must act to give positive moment, i.e., must be down.

Assumptions for Balance Computations. It is desirable to make measurements of pitching moments in the wind tunnel using a model of the airplane built exactly to scale. It is impracticable to have the propeller turning over in the miniature airplane, so that the airspeed is the same over the tail as over the wings. This is the condition of gliding flight. When engine failure occurs it should be possible to keep the airplane in balance, and if there is sufficient tail surface to balance under this condition, there will be ample surface for balancing under ordinary flight conditions.

In computing stability, because of the difficulty of predicting where the resultant of all the drag forces will be acting it is customary to neglect this moment also. Making the tail moment of such size as to be able to balance the lift moment is in the nature of a safety factor, for using this as criterion the designer will know that there will be sufficient horizontal tail area.

Tail Angle. Even if the adjustable stabilizer is set parallel to the wing, the angle of attack of the tail will not be the same as the angle of attack of the main wing because of downwash. The air flowing around the wing is given a downward velocity. As the air moves backward towards the tail this downward deflection becomes less. The direction of the air is different at different points above and below the plane of the wing.

The tail is usually located about three chord lengths back of the wing, but there is considerable variation in different airplanes.

In seaplanes, the horizontal tail surfaces are high above the plane of the wing. Several elaborate formulas have been devised giving the angle of downwash as a function of the distance back of the wing and of the distance above or below the wing. For very accurate work, these should be investigated and the most desirable one applied.

The Army assumes that the angle of downwash at the tail is one-half the angle of attack of the wing measured from the angle of zero lift. This appears to be a good representative value of downwash.

It should be borne in mind that the angle of downwash is the angle between the relative direction of the undisturbed air in front of the wing and the direction of the air after it has passed the wing. The angle of attack at the tail surface is called the tail angle. The tail angle will depend on the stabilizer setting as well as the angle of downwash, which in turn depends on the angle of attack of the wing.

Example. What is the tail angle when stabilizer is set at $+10°$ to chord of main wing, if angle of attack of airplane is $+5°$, angle of incidence is $+1\frac{1}{2}°$, and downwash is $-4°$?

Solution. (Assume relative wind as horizontal.)

Angle of attack of wing $= 5° + 1\frac{1}{2}° = 6\frac{1}{2}°$
Angle of tail with horizontal $= 6\frac{1}{2} + 10 = 16\frac{1}{2}°$
Angle of tail with downwashed air $= 16\frac{1}{2} - 4 = +12\frac{1}{2}°$

Example. Airplane has a Clark Y wing, aspect ratio 6, set at $+1\frac{1}{2}°$ incidence. Airplane is at $+2°$ angle of attack. Stabilizer set at $-5°$ to chord of wing. What is tail angle?

Solution.

Angle of attack of wing $= 2° + 1\frac{1}{2}° = 3\frac{1}{2}°$
Same measured from angle of zero lift $= 8\frac{1}{2}°$
Angle of downwash $8\frac{1}{2}° \times \frac{1}{2} = 4\frac{1}{4}°$
Angle of tail with horizontal $= 3\frac{1}{2}° - 5° = -1\frac{1}{2}°$
Angle of tail with downwashed air $- 1\frac{1}{2}° - 4\frac{1}{4} = -5\frac{3}{4}°$

Problems

1. Airplane has a Göttingen 398 wing, aspect ratio 6, set at $+\frac{1}{2}°$ incidence. Angle of attack of airplane is $+5°$. What is tail angle (*a*) if stabilizer is set at $-10°$ to wing chord? (*b*) if stabilizer is set at $+10°$ to wing chord?

2. Airplane has a C-80 wing, aspect ratio 6, set at $+3°$ incidence. Angle of attack of airplane is $-1°$. What is tail angle (*a*) if stabilizer is parallel to main wing chord? (*b*) if set at $-10°$ to wing?

3. Airplane has a Clark Y wing, aspect ratio of 6, with $1\frac{1}{2}°$ incidence. Airplane is at $-4°$ angle of attack. What is tail angle if stabilizer is set at 5° to wing?

4. Airplane has an M-6 wing, aspect ratio of 6, with 0° incidence. Airplane is at 8° angle of attack. What is tail angle if stabilizer is set at $-10°$ to wing?

5. Airplane has a Göttingen 398 wing, aspect ratio of 6 with 1° incidence. Airplane is at $-4°$ angle of attack. What is tail angle if stabilizer is set at $-5°$ to wing?

When the angle of attack is not given but the lift coefficient is known, for the part of the lift coefficient against angle of attack curve which is straight, the slope is constant. Assuming that this slope is 0.0718, which is approximately true for all airfoils having an aspect ratio of 6, then

$$\alpha_{\text{Z.L.}} = \frac{C_L}{0.0718}$$

If the wing, fictitious or actual, does not have an aspect ratio of 6, the angle of attack used in calculating downwash needs to be corrected. From Chapter V

$$\alpha'_{\text{Z.L.}} = \frac{\alpha_{\text{Z.L.}}}{F_{\text{A.R.}}}$$

$\alpha_{\text{Z.L.}}$ = angle of attack measured from zero lift chord for aspect ratio of 6

$\alpha'_{\text{Z.L.}}$ = angle of attack measured from zero lift chord for aspect ratio of problem

$F_{\text{A.R.}}$ = aspect ratio factor from Fig. 39

Then angle of downwash,

$$\epsilon = \tfrac{1}{2}\alpha'_{\text{Z.L.}}$$

$$= \frac{C_L}{2 \times 0.0718 \times F_{\text{A.R.}}}$$

Example. Airplane, weighing 2,000 lb. has 250 sq. ft. of wing area, aspect ratio of 8, and is flying at 100 miles per hour. Angle of attack of wing is $+7°$. Stabilizer set at $-10°$ to wing chord. What is tail angle?

$$C_L = \frac{W}{0.00256\,SV^2}$$

$$= \frac{2{,}000}{0.00256 \times 250 \times \overline{100}^2}$$

$$= 0.314$$

From Fig. 39, for aspect ratio of 8, $F_{A.R.} = 1.06$

$$\epsilon = \frac{C_L}{2 \times 0.0718 \times F_{A.R.}}$$
$$= \frac{0.314}{2 \times 0.0718 \times 1.06}$$
$$= 2.08°$$

Angle of tail with horizontal $= +7 - 10° = -3°$
Angle of tail with downwashed air $= -3 - 2.1 = -5.1°$

Problems

1. Airplane, weighing 4,000 lb., has 300 sq. ft. of wing area, aspect ratio of 9, and is flying at 125 miles per hour. Angle of attack of airplane is +2°. Angle of incidence is 1°. Stabilizer is set at +5°. What is tail angle?

2. A Lockheed airplane, weighing 4,900 lb., has 293 sq. ft. of wing area, aspect ratio is 6.24, and it is flying at 150 miles per hour. If stabilizer is set at −10° to wing chord and angle of attack of wing is +2°, what is tail angle?

3. A Fokker monoplane weighing 11,000 lb. has 748 sq. ft. of wing area with aspect ratio of 9.1, and is flying at 90 miles per hour. If angle of attack of wing is +4° and stabilizer is set at −5° to wing chord, what is tail angle?

4. A Douglas observation plane weighs 4,800 lb., has 376 sq. ft. of wing area with E.M.A.R. of 5.2, and is flying at 120 miles per hour. If angle of attack of wings is +5° and stabilizer is set at +5° to wing chord, what is tail angle?

5. A training biplane weighs 2,400 lb., has 280 sq. ft. of wing area with E.M.A.R. of 4.6, and is flying at 85 miles per hour. If angle of attack of wing is +7° and stabilizer is set at −5°, what is tail angle?

Tail Moment. To find the moment about the center of gravity due to the tail load, the lift (either positive or negative) acting on the tail must be multiplied by the distance from the center of gravity to the center of pressure of the tail surface. The tail surface, being usually a symmetrical airfoil, has a constant center of pressure position.

The lift force on the tail is found in the usual way. The tail surface is usually of a smaller aspect ratio than 6. To correct for this, the angle of attack may be corrected for the smaller aspect ratio, or what is the exact equivalent the tail area may be multiplied by the aspect ratio factor, $F_{\text{A.R.}}$, to give an effective horizontal surface area, S'_t.

$$S'_t = S_t \times F_{\text{A.R.}}$$

The airfoil characteristics of the tail airfoil surface are usually different from those for the main wing, but the slope of all airfoil lift curves is approximately the same and equal to 0.0718. If α_t is the tail angle measured from the zero lift chord, the lift coefficient of the tail surface is $0.0718 \times \alpha_t$.

Example. Find tail moment, when horizontal surface is 40 sq. ft. in area, span of tail is 11 ft., symmetrical airfoil section stabilizer is set at $-10°$ to main wing. Distance from center of gravity of airplane to center of pressure of stabilizer is 14 ft. Main wing is Clark Y, angle of attack of wing is 4°, airspeed is 100 miles per hour.

Solution. For Clark Y wing, angle of zero lift is $-5°$.

$$\begin{aligned} \text{Downwash angle } \epsilon &= \frac{4° + 5°}{2} \\ &= 4.5° \\ \text{Tail angle} &= -10.5° \\ \text{Aspect ratio of tail} &= \frac{\overline{11}^2}{40} \\ &= 3.07 \end{aligned}$$

From Fig. 39, for aspect ratio of 3.07, $F_{\text{A.R.}} = 0.828$

Lift force on tail (down)

$$\begin{aligned} &= 0.0718 \times \alpha_t \times \rho/2 \times S_t \times F_{\text{A.R.tail}} \times V^2 \\ &= 0.0718 \times 10.5 \times 0.00119 \times 40 \times 0.828 \times \overline{146.7}^2 \\ &= 639 \text{ lb.} \\ M_t &= 639 \times 14 \\ &= +8940 \text{ ft-lb.} \end{aligned}$$

Problems

1. A tail surface, 30 sq. ft. in area, has a span of 11 ft.; its airfoil section is symmetrical. Stabilizer is set at $-5°$ to main wing which is Clark Y section and is at 2° angle of attack. Distance from center of gravity of airplane to center of pressure of stabilizer is 15 ft. What is tail moment if airspeed is 120 miles per hour?

2. A tail surface 45 sq. ft. in area has a span of 13 ft., and its airfoil section is symmetrical. Stabilizer is set at 0° to main wing which is Clark Y section and is at 1° angle of attack. Distance from center of gravity to center of pressure of stabilizer is 16 ft. What is tail moment if airspeed is 90 miles per hour?

3. A tail surface 47 sq. ft. in area has a span of 15 ft., and its airfoil section is symmetrical. Stabilizer is set at $-10°$ to main wing which is Clark Y section and is at 6° angle of attack. Distance from center of gravity of airplane to center of pressure of stabilizer is 17 ft. What is tail moment if airspeed is 110 miles per hour?

The sum of the moment due to the wing and the moment due to the tail at various angles of attack is the criterion of the airplane's longitudinal stability. Not only must it be possible for the airplane to balance at all normal flying conditions, but for stability, when the angle of attack is increased there must be a diving moment, and vice versa.

For stability, the slope of the curve of moments plotted against angle of attack must be negative, so that with larger angle of attack there must be less positive moment. If the slope of the moment curve is positive the airplane will be unstable.

Moment Curves. In a first approximation, the velocities over the wing and over the tail may be considered the same. If the airplane is balanced for a given angle of attack for one velocity, it will be balanced at any other velocity. If it is unbalanced at one velocity, it will be unbalanced for all velocities. For simplicity of calculation, therefore, one velocity may be assumed for all angles of attack.

As the horizontal tail surface is a symmetrical airfoil, its lift coefficient is assumed to vary directly with angle of attack. In the following computation, the elevator is in line with the stabilizer in all conditions; this is termed "fixed elevator condition."

The moment of the wing, drag being neglected, is the product of the lift, which varies with angle of attack, times its moment arm, which also varies with angle of attack. The moment of the tail is the product of its lifting force, which varies with its angle of attack and may be up or down, times its moment arm. Since the tail surface is a symmetrical airfoil, its center of pressure does not move and the moment arm of the tail, which is the distance from the center of gravity of the airplane to the center of pressure of the tail, is constant. Upward forces are positive as are stalling moments.

Example. A monoplane has a rectangular Clark Y wing, 36 ft. by 6 ft. The rectangular horizontal tail surface has 25 sq. ft. area with an aspect ratio of 6. The center of gravity of the airplane lies on the chord of the wing and is 27 per cent of the chord back of the leading edge. The distance from the center of gravity to the center of pressure of the tail is 18 ft. Consider velocity constant at 100 ft. per sec. Find the moment curve (*a*) when chord of the stabilizer is parallel to wing chord, (*b*) when stabilizer is set at $-5°$ to wing chord.

Solution.

0° Stabilizer Setting

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
α	C_L	C.P.	C.P.-27	M_W	α_0	ϵ	α_t	C_{Lt}	t	M_t	M
0	0.360	0.424	0.154	−853	5	2.5	−2.5	−0.180	− 53.5	+ 963	+ 110
4	0.645	0.347	0.077	−766	9	4.5	−0.5	−0.036	− 10.7	+ 192	− 574
8	0.930	0.316	0.046	−660	13	6.5	+1.5	+0.108	+ 32.1	− 577	−1237
12	1.190	0.300	0.030	−547	17	8.5	+3.5	+0.252	+ 75.0	−1350	−1897
16	1.435	0.296	0.026	−573	21	10.5	+5.5	+0.395	+117.2	−2110	−2683

−5° Stabilizer Setting

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
α	C_L	C.P.	C.P.-27	M_W	α_0	ϵ	α_t	C_{Lt}	t	M_t	M
0							−7.5	−0.539	−160.0	+2880	+2027
4							−5.5	−0.395	−117.2	+2110	+1344
8			same as				−3.5	−0.252	− 75.0	+1350	+ 690
12			above				−1.5	−0.108	− 32.1	+ 577	+ 30
16							+0.5	+0.036	+ 14.8	− 260	− 833

Explanation of Table

Column 2 obtained from Fig. 17.
Column 3 obtained from Fig. 17.
Column 4 obtained by subtracting 0.27 from items in column 3.
Column 5 obtained by multiplying (2) × (4) × $\frac{\rho}{2}SV^2c$.
Column 6 obtained by subtracting angle of zero lift ($-5°$) from (1).
Column 7 obtained by halving items in column 6.
Column 8 obtained by inspecting columns 1 and 7.
Column 9 obtained by multiplying items in column 8 by 0.0718.
Column 10 obtained by multiplying (9) × $\frac{\rho}{2}S_tV^2$.
Column 11 obtained by multiplying (10) × 18.
Column 12 obtained by adding (5) and (11).

The results from the above tables are plotted in Fig. 84. While the curve for wing moment has a slightly positive slope, the curve for combined wing and tail moment has a negative slope. With 0°

stabilizer angle the airplane is balanced at approximately 1° angle of attack. With −5° stabilizer angle, the plane is balanced at $12\frac{1}{4}$° angle of attack. In either case, when the angle of attack is increased there is a diving moment tending to bring the nose down, while if the angle of attack is decreased there is a stalling moment tending to bring the nose up.

It will be noticed that a change in the stabilizer angular setting does not appreciably change the slope of the total moment curve but merely shifts it to the left or right.

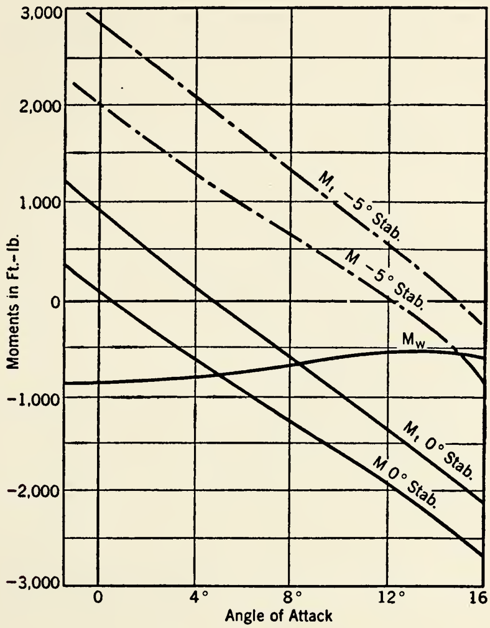


FIG. 84. Moment curves for illustrative example.

Moment Coefficient Curves. It is usual to plot moment coefficients instead of the moments themselves.

The wing moment coefficient, C_{M_0}, as shown in Chapter IV, is such that

$$M = C_M\left(\frac{\rho}{2}SV^2c\right)$$

and

$$C_M = C_L(p - \text{C.P.})$$

c = chord in feet

p = distance in percentage of chord back from L.E. of point on chord about which moment is desired

C.P. = center of pressure location in percentage of chord back from L.E.

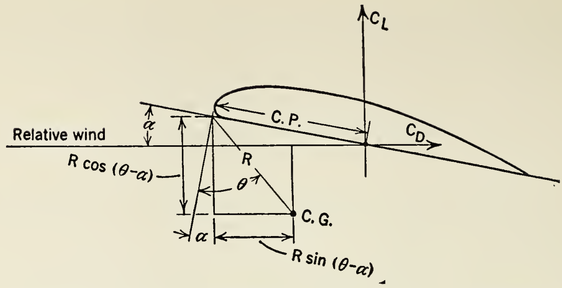


FIG. 85*a*. Moment coefficients about center of gravity.

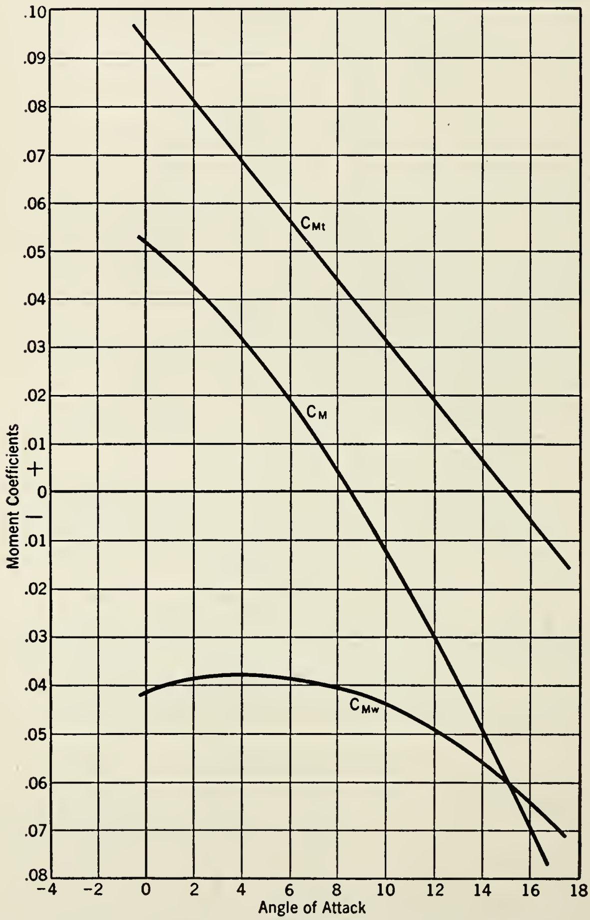


FIG. 85*b*. Moment coefficients versus angle of attack (illustrative example).

If the center of gravity is on the M.A.C., the above expressions are exactly right for moment and moment coefficient, when the distance of the center of gravity backward from the leading edge is substituted for p. When the center of gravity is above or below the wing chord, the wing drag will produce a moment, so that it is necessary to consider the resultant force, instead of merely lift component, and the moment arm should be the perpendicular distance from the center of gravity to the vector of the resultant.

Referring to Fig. 85*a*, if the location of the center of gravity is known, R and θ either are known or can be found. The moment arm of the lift is the perpendicular distance from the center of gravity to the line of direction of lift. From an inspection of Fig. 85*a*, this distance is C.P. $\cos \alpha - R \sin (\theta - \alpha)$, expressed in percentage of chord. The moment arm of the wing drag is the perpendicular distance from the center of gravity to the line of action of the drag. From Fig. 85*a*, this is $R \cos (\theta - \alpha) -$ C.P. $\sin \alpha$. Finding the sum of the moments of the lift and of the drag is exactly the same as finding the moment of the resultant itself.

While earlier in this chapter it was assumed that the velocity of the air past the tail was the same as the velocity of the air past the wing, it has been found that the air stream is slowed up by friction with the fuselage so that the air velocity over the tail surface is less than the airspeed. It is usual to correct for this by assuming that the speed at the tail is 90 per cent of the airspeed. Then the velocity squared at the tail is 81 per cent of the airspeed squared.

$$\begin{aligned} M &= M_{\text{wing}} + M_{\text{tail}} \\ &= C_{M\text{wing}}\left(\frac{\rho}{2} SV^2c\right) + C_{L\text{tail}} \frac{\rho}{2} S_{\text{tail}} V^2_{\text{tail}} d \\ &= C_{M\text{wing}}\left(\frac{\rho}{2} SV^2c\right) + 0.0718 \times \alpha_{\text{tail}} \times F_{\text{A.R. tail}} \times \\ &\qquad \frac{S_{\text{tail}}}{S_{\text{wing}}} \times 0.81 \frac{d}{c}\left(\frac{\rho}{2} SV^2c\right) \end{aligned}$$

$$C_M = C_{M\text{wing}} + 0.0718 \times \alpha_{\text{tail}} \times F_{\text{A.R. tail}} \times \frac{S_{\text{tail}}}{S_{\text{wing}}} \times 0.81 \frac{d}{c}$$

It is quite common to use a tail coefficient, C_f, where

$$C_f = 0.0718 \times \frac{S_{\text{tail}}}{S_{\text{wing}}} \times F_{\text{A.R. tail}} \times 0.81 \times \frac{d}{c}$$

Substituting in above

$$C_M = -C_{M\text{wing}} + C_f \alpha_{\text{tail}}$$

These moments may be plotted for various angles of attack.

Example. Find and plot wing moment coefficient, tail moment, and total moment coefficient for the following high-wing monoplane. The wing is rectangular, Clark Y section, 48-ft. span, 6.5-ft. chord. Total horizontal tail area is 43 sq. ft.; span of tail is 12.5 ft. Stabilizer is set at $-5°$ to main wing. The center of gravity of the airplane is 30 per cent of the chord back of the leading edge of the M.A.C. and 25 per cent of the chord below the M.A.C. Distance from center of gravity to center of pressure of tail is 12 ft.

Solution. Area of main wing = 48 × 6.5 = 312 sq. ft.

$$\text{Aspect ratio of wing} = \frac{48}{6.5} = 7.39$$

From Fig. 39, for aspect ratio of 7.39, $F_{\text{A.R.}} = 1.043$

$$\theta \text{ (see Fig. 85}a) = \tan^{-1}\frac{30}{25}$$

$$= 50.2°$$

$$R \text{ (see Fig. 85}a) = \frac{25}{\cos 50.2°} = 39.0 \text{ per cent M.A.C.}$$

TABLE XVI

(1)	(2)	(3)	(4)	(5)	(6)	(7)
α_{AR_6}	C_L	$\alpha_{AR_6} - \alpha_{AR_{7.39}}$	$\alpha_{AR_{7.39}}$	$C_{D_{AR_6}}$	$C_{D_{AR_6}} - C_{D_{AR_{7.39}}}$	$C_{D_{AR_{7.39}}}$
0	0.36	0.21	− 0.21	0.011	0.001	0.010
4	0.65	0.37	+ 3.63	0.017	0.004	0.013
8	0.94	0.54	+ 7.46	0.033	0.009	0.024
12	1.19	0.68	+11.32	0.060	0.014	0.046
16	1.44	0.82	+15.18	0.139	0.021	0.118

(8)	(9)	(10)	(11)	(12)	(13)	(14)
$(\theta-\alpha)$	$\sin(\theta - \alpha)$	$\cos(\theta - \alpha)$	$R \sin(\theta - \alpha)$	$R \cos(\theta - \alpha)$	C.P.	C.P. $\sin \alpha$
50.41	0.771	0.637	0.301	0.249	0.424	−0.002
46.57	0.726	0.688	0.283	0.268	0.347	+0.022
42.74	0.679	0.734	0.265	0.286	0.316	+0.041
38.88	0.628	0.779	0.245	0.304	0.300	+0.059
35.02	0.574	0.819	0.224	0.319	0.296	+0.078

TABLE XVI (*Continued*)

(15)	(16)	(17)	(18)
C.P. cos α	C_L(C.P. cos α − R sin $(\theta - \alpha)$)	C_D(R cos$(\theta - \alpha)$ − C.P. sin α)	C_{Mw}
0.424	−0.0445	+0.0025	−0.0420
0.346	−0.0410	+0.0032	−0.0378
0.313	−0.0457	+0.0059	−0.0398
0.294	−0.0588	+0.0113	−0.0475
0.286	−0.0891	+0.0285	−0.0606

Explanation of Table

Columns 2 and 5 obtained from Fig. 17.

Column 3 obtained by multiplying items in column 2 by 18.24 (1/6 − 1/7.39).

Column 4 obtained by subtracting items in column 3 from items in column 1.

Column 6 obtained by squaring items in column 2 and multiplying by $1/\pi(1/6 - 1/7.39)$.

Column 7 obtained by subtracting items in column 6 from items in column 5.

$$\text{Tail aspect ratio} = \frac{\overline{12.5}^2}{43} = 3.64$$

From Fig. 39, for A.R. of 3.64, $F_{\text{A.R.}} = 0.874$

$$C_f = 0.0718 \times \frac{43 \times 0.874}{312 \times 1.043} \times 0.81 \times \frac{12}{6.5} = 0.0124$$

TABLE XVII

(1)	(2)	(3)	(4)
$\alpha_{\text{Z.L.(A.R.7.39)}}$	ϵ	α_t	C_{Mt}
4.79	2.395	−7.61	+0.0944
8.63	4.315	−5.68	+0.0704
12.46	6.230	−3.77	+0.0468
16.32	8.160	−1.84	+0.0228
20.18	10.090	+0.09	−0.0011

Explanation of Table

Column 1 obtained by subtracting −5° (angle of zero lift for Clark Y) from items in column 4, Table XVI.

Column 2 obtained by dividing items in column 1 by 2.

Column 3 obtained by adding stabilizer angle (−5°) to items in column 4, Table XVI and from this sum subtracting items in column 2.

Column 4 obtained by multiplying items in column 3 by $C_f(=0.0124)$.

As shown by Fig. 85*b* this gives balance or zero moment at about 8.5° angle of attack. This is a stable condition for locked elevator, i.e., elevator held in same plane with stabilizer; since a greater angle of attack means a diving moment, at a lesser angle of attack, there is a stalling moment.

A change of stabilizer setting does not change the wing moment coefficient, nor does it change slope of tail moment coefficient curve. The curve of total moment coefficient is moved to right or left, so that equilibrium is found at a different angle of attack.

Problems

1. Plot moment coefficient curve for airplane in illustrative example with stabilizer set at $-10°$ to wing chord.

2. Plot moment coefficients for an airplane similar to that described in the illustrative example, except that it is a low-wing monoplane and the center of gravity is 25 per cent above M.A.C., stabilizer set at $-5°$ to wing chord.

Free Elevator. This condition is where the pilot releases the control stick entirely. The elevator being hinged at its forward edge would tend to sag down from its own weight. The air passing under the stabilizer impinges on the under side of the elevator, so that the elevator assumes a position where the moment about its hinge due to its weight just balances the moment due to the air pressure on the under side.

Lateral Stability. Because the airplane is symmetrical about a vertical plane through the longitudinal axis, lateral stability does not present as difficult a problem as does longitudinal stability. A low center of gravity position as in flying boats is a great aid in lateral stability.

When flying straight and level, the air will meet the wing (or wings) on the left side of the airplane at the same angle of attack as on the right side and therefore there will be the same lift on one side of the airplane as on the other side. If for any reason the airplane tips, the wing on the side which is going down will meet air, so that relative to the wing air not only is coming backward at the wing (due to the forward motion of the airplane), but also is coming upward (due to the roll). The relative wind is then not horizontal but backward and upward, so that temporarily the descending wing has a greater angle of attack. The other, rising, wing is meeting air on its upper surface, that is, momentarily the rising wing has a relative wind, which is backward and downward, giving a lesser angle of attack. The descending wing has a greater angle of attack and consequently greater lift; the ascending wing will have less lift, tending to restore the plane to its original attitude. The descending wing will also have more drag, the rising wing less drag, tending to turn the airplane.

This action is effective *only* while the airplane is actually rolling. If an airplane has ceased rolling and is flying level with one wing low, there is the same angle of attack on both the left and the right wings and there is no tendency on the part of the airplane to right itself.

Dihedral. One of the most effective ways of securing lateral stability is with dihedral. Instead of the wings being straight across the span, they slope outward and upward from the center. Dihedral angle is the angle which the wings slope upward from the horizontal. A small dihedral angle of $1\frac{1}{2}°$ or $2°$ is sufficient to give ample lateral stability.

When an airplane tips sideways, it will sideslip. If the wings are tipped up to a vertical position, there will be a great deal of sideslip; but if the plane is only slightly tilted, there will still be some amount of slip as the resultant of lift and weight will have a side component. As soon as any slip takes place, the relative wind, instead of coming directly in front of the wing, will come from a direction which is to one side of dead ahead.

With a straight wing, even when the relative wind comes from the side instead of from dead ahead, the angle of attack will still be the same on the right wing as the left wing. With dihedral, if the right wing drops, the plane will slip to the right. The relative wind is then coming at the airplane from the right of dead ahead. Owing to the dihedral angle, the right wing will have a greater and the left wing a smaller angle of attack. This will give more lift on the low wing and less on the high, tending to restore the airplane to an even keel.

The first Wright airplane happened to have great inherent stability so the upper wings were given negative dihedral, called cathedral, to decrease it. This is never done in modern plane construction. Too much lateral stability makes a cross-wind landing extremely difficult.

An explanation sometimes given of the action of dihedral in aiding lateral stability is that the wing going down has a greater horizontal projected area than the wing going up. As the usual dihedral angle is $1\frac{1}{2}°$ or 2°, the additional projected area of a wing dropping from normal to a horizontal position would be $(1 - \cos 2°) \times$ area or $(1 - 0.9994) \times$ area, which would be a very small part of the area.

Some tapered-wing monoplanes have a straight upper surface, the decrease in thickness coming entirely on the under surface. This helps lateral stability, and is known as effective dihedral.

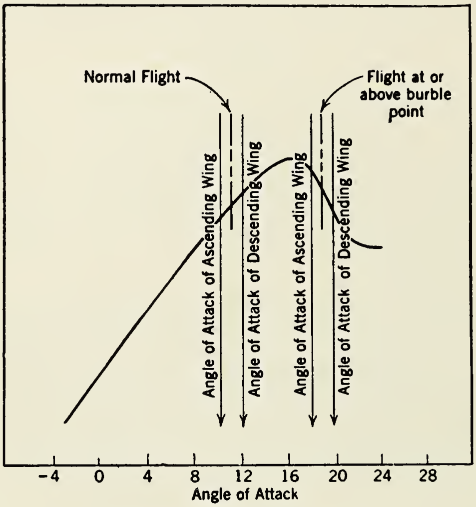


Fig. 86. Autorotation.

Autorotation. As explained in a previous paragraph, when an airplane tips, the descending wing has a greater and the rising wing a lesser angle of attack. At low or medium angles of attack, a wing with greater angle of attack has greater lift. The curve of lift coefficient versus angle of attack is a straight line with a positive slope to near the stalling angle, where it rounds off to a maximum. Beyond the angle of maximum lift, the lift coefficient has negative slope.

When an airplane is flying at or near the angle of maximum lift, if the airplane tips, the descending wing will have a greater angle of attack than the rising; but since beyond the burble point the slope of the lift curve is negative, a greater angle of attack will mean *less* lift. Since the descending wing has the less lift and the ascending wing has the more lift, the rising wing will rise more and the wing which is going down will go down more. This rotation will continue indefinitely unless controls are applied to stop the maneuver. Rotation about the longitudinal axis with wing at or above the angle of maximum lift is called autorotation.

Spins. In a spin, the airplane wing is at a high angle of attack. At any instant, the path of the airplane is vertically downward so that the relative wind is vertically upward. Even though the nose of the airplane is down, the angle of attack is beyond the burble point. With the nose between 20° and 40° below the horizontal, the spin is termed a flat spin; with the nose at a greater angle than 40° below the horizontal, it is a normal spin. In a normal spin the angle of attack is about 35°, in a flat spin the angle of attack may be as high as 70°.

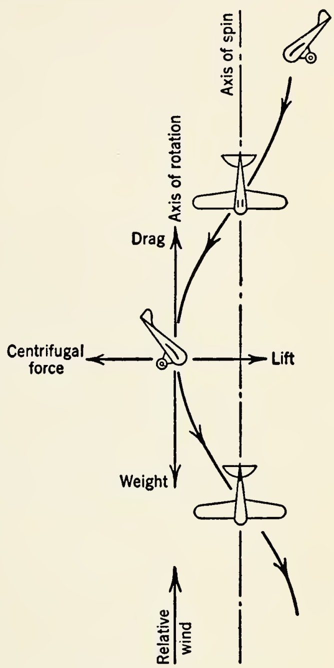


Fig. 87. Forces in a spin.

Combined with the downward motion is autorotation and sideslip. In a steady spin, the actual path is a vertical spiral, the axis of this spiral being termed the axis of spin. Lift, being perpendicular to the relative wind, is horizontal and it balances centrifugal force. Drag, which is vertically upward, is opposing the downward effect of weight. Air striking the under side of the stabilizer tends to throw the nose downward; this is balanced by centrifugal inertia moments which tend to make the airplane assume a more horizontal position.

A spin is usually started from horizontal flight by pulling back the stick till the airplane is in a stall; the rudder is then kicked, causing the airplane to sideslip and autorotate. To come out of a spin, it is first necessary to stop the autorotation. In order to do this, the angle of attack must be reduced to below the stalling angle. This may be accomplished by shoving forward on the stick. This action may be hastened by opening the throttle to send a blast of air against the tail surface. While this will cause the airplane to assume a more vertical position, the autorotation will stop and the plane will be in a simple dive from which recovery is made by merely pulling back on the stick. When the airplane is coming down in a spin, the air is pressing against the under side of the elevator forcing it into an "up" position; with a properly designed plane, bringing the elevator into its mid-position should be sufficient to stop the spin. The Department of Commerce requires that after a six-turn spin, the airplane shall recover in no more than one and a half additional turns after the controls are put in neutral, without the use of the engine.

Directional Stability. Because of the symmetry of the airplane, usually little difficulty is experienced in achieving stability in yaw or directional stability. A deep fuselage will aid in directional stability.

In pursuit planes, which have shallow well-rounded fuselage, use is often made of sweepback. Sweepback or the sloping of the wings backward from the center section aids in the following manner. When an airplane turns to the left, the swept-back right wing will become more nearly at right angles to the direction of flight, its effective span will be greater and its drag will increase, while the drag on the left wing will decrease. A right turn will show an opposite reaction. In each case, the change in relative drag of the two wings will be such as to cause the airplane to return to its original heading.

CHAPTER XV

AUXILIARY LIFT DEVICES

Speed Range. The ratio of maximum velocity to minimum velocity is called the speed range. With retractable landing gear and general clean design, racing planes can have a speed range of 3.3; airplanes not so carefully streamlined or those so heavily loaded that they cannot fly at the angle of attack of minimum drag will have a smaller speed range. Unless design is very poor, or the load is exceedingly heavy, the speed range should be at least 2.5.

The above is based on fixed wings. The landing speed is determined by the maximum lift coefficient and the wing loading. If the maximum lift coefficient can be increased in any way, the wing area can be decreased without changing the minimum speed.

Fixed Slot. G. Lachmann in Germany and F. Handley-Page in England appear to have developed the slotted wing at about the same time. The slot is a narrow opening near the leading edge and parallel to the span. The small section in front of the slot may be considered as a miniature airfoil. When the main wing is at a high angle of attack, the small airfoil in front of the slot is at a small angle of attack. Whereas with a simple wing burbling takes place at 18° to 20° angle of attack, because air is unable to change direction so as to follow the upper surface of the wing, with the slot, the air is given a downward deflection in passing over the small auxiliary wing section, so that it can follow closely the upper surface of the main wing.

By using a slot, the maximum lift angle is increased to 28° or 30°. The slope of the lift curve remains constant, so that the maximum lift coefficient is increased about 50 per cent by the use of a slot.

With a fixed slot, at low angles of attack, there being less pressure on the upper than at the lower end of the slot, air will travel upward through it. This will divert the main air stream and cause burbling so that the drag at low angles of attack is much greater with the fixed slot than with the simple wing.

Automatic Slot. To F. Handley-Page should go the credit for obviating the increased minimum drag of the fixed slot. The auxiliary airfoil is held by a linkage mechanism or a series of studs working in pairs of rollers in the main wing, so that it will move freely from a closed position where the auxiliary is butted against the leading edge of the main wing to an open position where there is a gap of an inch or more between the auxiliary airfoil and the main wing.

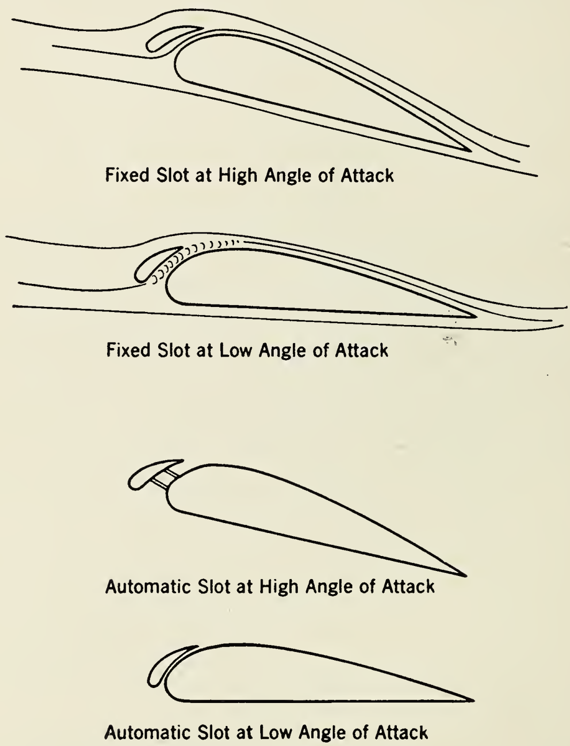


FIG. 88*a*. Wing slots.

By examining Figs. 14, 15, and 16, it will be seen that at low angles of attack there is a pressure on the nose which is utilized to press the auxiliary airfoil tightly against the main wing, while at high angles of attack there is a negative pressure to draw the auxiliary wing to its " open " position. The automatic slot requires no manipulation on the part of the pilot. At low angles of attack, the drag is only very slightly greater than with the ordinary wing.

Flaps. Flaps, in the simplest form, merely mean that the rear part of the main wing is hinged so that it can be swung downward. In appearance, they resemble ailerons, except that ailerons extend only a small portion of the span and are so linked that, when the aileron on one side goes up, that on the other side goes down. Flaps extend across the span, except for the small portion which is aileron, and the flaps on both sides go down together. Flaps are moved by a control mechanism in the pilot's cockpit.

The effect of depressing the flaps is to increase the effective camber of the upper surface of the wing as well as the concavity of the under surface. This has the effect of increasing the lift coefficient, partly because of the action on the under side, where the depressed rear edge acts to hinder the smooth flow of air; pressure will build up at this point, which will cause an increase in lift. It will also cause the center of pressure to move rearward.

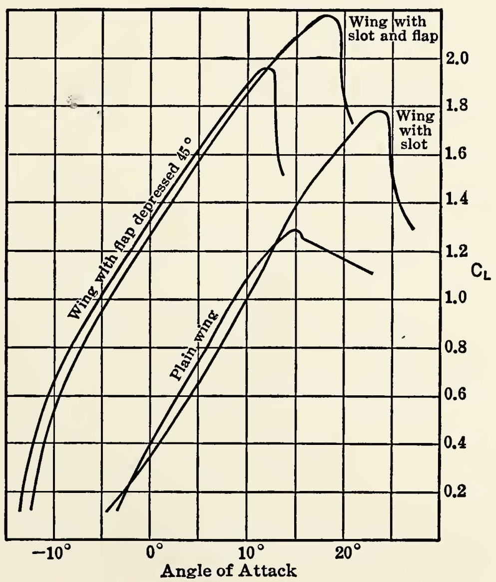


FIG. 88*b*. Effect on lift coefficient of slots and flaps.

This rearward center of pressure movement, when the flaps are swung downward, means that it is usually necessary to interconnect the flaps with the stabilizer; otherwise the airplane will be thrown out of balance longitudinally.

With flaps, the maximum lift coefficient can be increased 50 per cent. At low angles of attack, the flaps are in their normal position so that they do not detract from high speed. Both slots and flaps may be installed on the same wings, and by so doing, maximum lift coefficient may be increased 100 per cent.

Special forms of flaps have been devised and several have proved to be very practical. Among these are the " Zap " flap, in which the lower surface of the rear portion of the wing swings down while the upper surface remains intact. Another type is the Fowler wing; the rear portion of the lower surface slides backward and downward, so that not only the camber but also the wing area are increased.

CHAPTER XVI

UNCONVENTIONAL TYPES OF AIRCRAFT

Autogiro. In the autogiro, lift is derived, not from fixed wings as in airplanes, but from wings rotating in a horizontal plane. In the early models, a small fixed wing gave part of the lift; in the more recent types all the lift is obtained from the rotating vanes. The essential features of the autogiro are shown in Fig. 89.

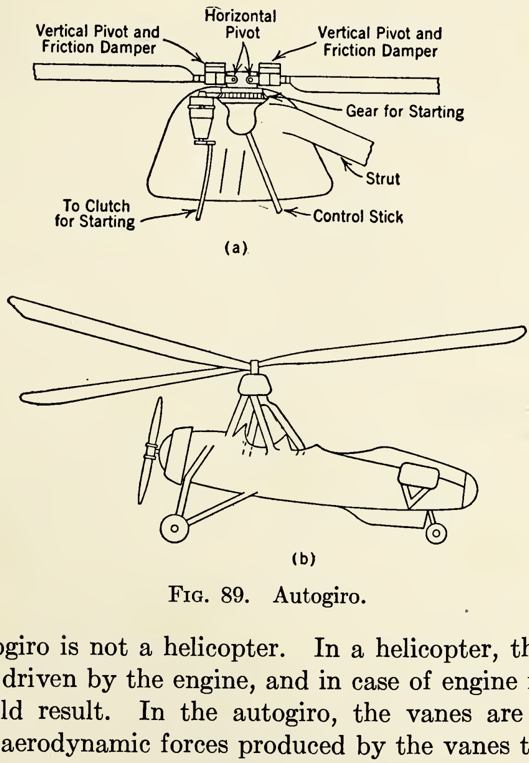


Fig. 89. Autogiro.

An autogiro is not a helicopter. In a helicopter, the rotating vanes are driven by the engine, and in case of engine failure disaster would result. In the autogiro, the vanes are caused to rotate by aerodynamic forces produced by the vanes themselves. To hasten the start of rotation at the beginning of a flight the vanes are connected through a clutch with the engine, but immediately as the vanes rotate at the proper speed the engine is disconnected and the vanes rotate freely.

The rotating vanes are four in the older, three in the newer, types. The latest type, without a fixed wing, permits, by the removal of one bolt on each vane, the swinging together of all three vanes over the fuselage, and in this position the autogiro may be taxied along a road or trundled into an ordinary-sized garage (see Fig. 90).

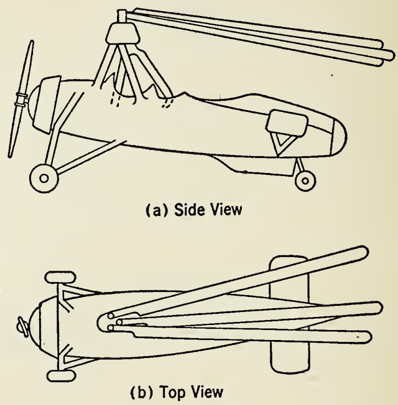


Fig. 90. Autogiro with blades folded.

The airfoil section most favored for the vanes is the Göttingen 429 profile. The normal position of three blades would be 120° apart, but during rotation a blade experiences more drag when moving forward than when moving backward, so that if the blades were fastened rigidly there would be repeated variation in the stresses. For this reason, each blade is hinged about a vertical pin, its horizontal movement being restricted by friction dampers so that only a slight relative motion is permitted.

Each blade is also hinged about a horizontal pin, so as to permit flapping or movement in a plane through the axis of the rotor head. The rotor head supports the inner ends of the three vanes.

In horizontal flight, the advancing blade has a greater relative velocity than the retreating blade; consequently the advancing blade would have greater lift. Rigidity of the blades would tilt the entire gyro, and it is for this reason that the blades are hinged about a horizontal axis. There is no restriction to their upward sweep; care is taken, however, that when rotation is slow the blades cannot sink down so as to strike the propeller. Folding up is impossible, owing to centrifugal force. The angle which the span of any blade makes with the horizontal is due to the resultant of centrifugal force and the lift on that blade. This means that the advancing blade is at a higher angle from the horizontal than the retreating blade. When there is no forward motion and the autogiro is parachuting down, the relative wind is vertically upward and all blades are at the same coning angle.

In vertical descent, there will always be rotation. Even if it could be conceived that the blades were stationary, for the airfoil section used, as well as for most airfoils, zero lift occurs at 93° or 94° angle of attack. At 90° angle of attack there is always a small lift, which acting perpendicular to relative wind, see Fig. 91*a*, will cause rotation. As rotational speed becomes greater, the velocity of the relative wind becomes greater, and since the relative wind is the resultant of the vertical descent velocity and

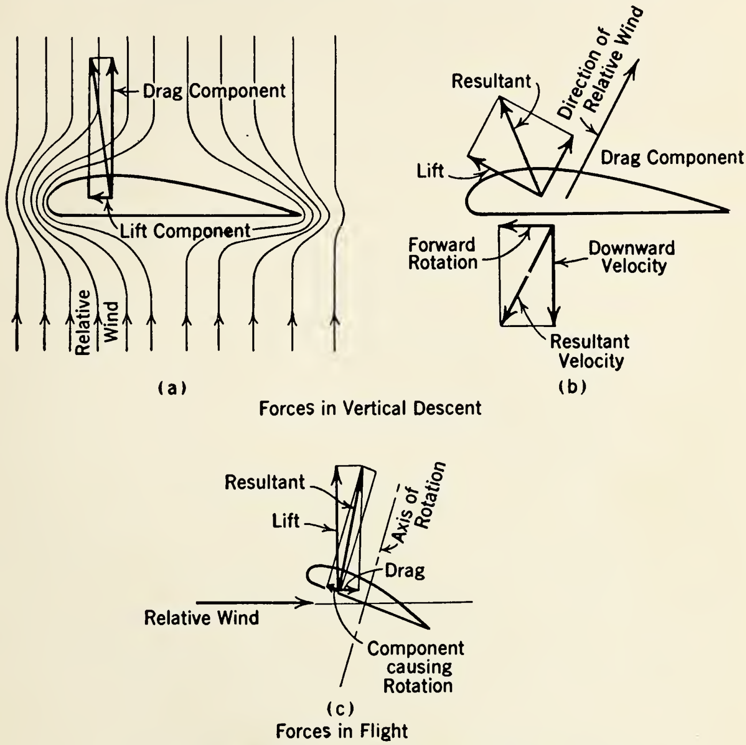


FIG. 91. Forces on autogiro blade.

the rotational velocity, the angle of attack becomes less. The resultant force has a forward component which causes rotation (Fig. 91*b*). As the speed of rotation increases, the direction of the relative wind becomes more and more inclined to the vertical and the angle of attack of the airfoil becomes less. At some one angle of attack the resultant force will be vertical. Under this condition there will be no forward-acting component of force, and without this accelerating force there will be no increase in rotational velocity.

The speed of rotation will be constant at about 100 r.p.m. If, for any reason, the resultant should act backwards from the vertical, it would immediately have a component tending to retard rotation. The decrease in rotational velocity would change the direction of the relative wind to give a larger angle of attack, which would cause the line of action of the resultant to move forward again.

When the autogiro is moving forward in horizontal flight, the main axis of rotation of the rotor head is tilted slightly backward. Rotation of the rotor head is counter-clockwise viewed from above, and this rotation is always in effect before take-off. Then a blade on the right side has greater speed than a blade on the left. If there were no flapping the advancing right blade would have greater lift than the retreating left blade. Being hinged to permit upward movement, the forward-moving blade is rising, giving a smaller angle of attack, while the backward-moving blade is being moved downward by the effect of centrifugal force. These effects tend to make the lifts on each side more nearly equal.

When the axis of rotation is tilted backward to the resultant force produced by the action of the relative wind so that there is a component producing rotation as shown in Fig. 91*c*, the rotor will accelerate. When the resultant force is parallel to the axis of rotation there will be no acceleration, the rotational speed will be constant, and the system will be in equilibrium. If the resultant acts in a direction to give a component decelerating the rotor, the autogiro will start to descend, speeding up the rotor.

The rotor system is supported on a sturdy tripod, the head being mounted on a universal joint, so that it may be tilted in any direction by the control column. This control column hangs down instead of protruding up from the floor as in a conventional airplane. The movement of the pilot's hand is the same as with the ordinary control stick. Pulling back on the stick causes the nose of the autogiro to go up; pushing forward causes it to go down. Moving the stick to the left causes the fuselage to tilt in a manner similar to the way an airplane fuselage would be tilted in banking for a left turn. The rudder on the rear of the fuselage is operated by pedals or rudder bar as in the conventional airplane.

The landing gear resembles that of the airplane except that a much wider tread is given to the wheels. The shock-absorbers have greater travel to withstand harder landings.

Gyroplane. In the autogiro the blade airfoils have a fixed angle with respect to the rotor disk (an imaginary plane perpendicular to the axis of rotation). In the gyroplane this angle can be changed (see Fig. 92).

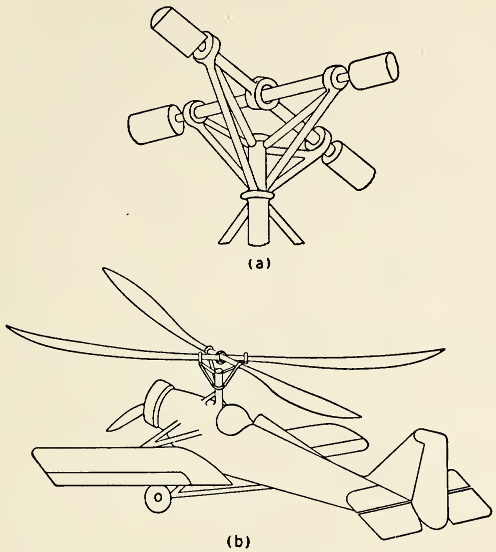


Fig. 92. Gyroplane.

Four blades are used, set 90° apart. The opposite blades are rigidly connected, but each pair is supported in bearings in such a manner that the blades may be " feathered " or have the angle with the rotor disk altered as the blades proceed around the disk. This movement is caused by a central cylindrical cam at the rotor hub.

The gyroplane has a fixed wing as well as the rotating vanes. At high speeds the small fixed wing furnishes sufficient lift, and the rotating blades are turned to the zero lift angle offering a minimum of drag. As the speed is decreased, more of the load is taken by the rotor.

Another feature of the gyroplane is its lateral control. The control stick is connected with the feathering cam by linkage. A movement of the stick to the left will cause the blades on the right side to have an increased angle of attack, and the blades as they proceed to the left side will have a decreased angle of attack, resulting in more lift on the right than on the left. Conversely a stick movement to the left causes the blades on the left side to increase their angle, and on the right side to decrease their angle. The control stick is also connected to conventional ailerons on the fixed wing for control at high speed.

Although the gyroplane is still in the developmental stage, it is aerodynamically sound. It should be as safe as other rotating-wing aircraft and holds possibilities of a greater speed range.

Cyclogiro. The cyclogiro, sometimes termed the "paddle-wheel" airplane, is of the rotating-wing type, having several blades on each side rotating about a horizontal axis perpendicular to the direction of normal flight (see Fig. 93). By means of cams

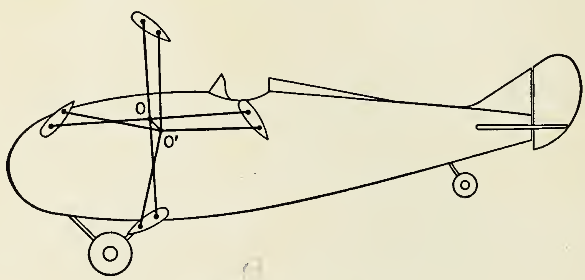


Fig. 93. Cyclogiro.

or linkages, the angle which an element makes with the tangent to its circular path may be changed as the blade rotates. Changing the setting of the cam operating this feathering can be made to produce a resultant in any desired direction.

The combination of wings, i.e., the "paddle-wheels," is driven by the engine, but a clutch mechanism is provided so that in case of engine stoppage the airfoil systems are free to rotate. They autorotate, thus permitting hovering or slow descent. With the engine connected, a reasonable forward speed and climb are obtainable.

The autorotation, with the clutch out, may be understood by examining Fig. 94. The three symmetrical airfoils A, B, and C are pivoted at their centers of pressure at three points 120° apart on a circular framework which rotates about its center O. From eccentric point O' links are connected to each airfoil. The position of O' with respect to O, both in direction and distance, may be controlled by the pilot. In Fig. 94a, it is assumed that the system is stationary. The relative wind will then be vertically upward. The resultant forces on each wing will be as shown. Each of these resultant forces has a tangential component tending to cause the system to rotate in a counter-clockwise manner about center O. After rotation has started, conditions are as represented

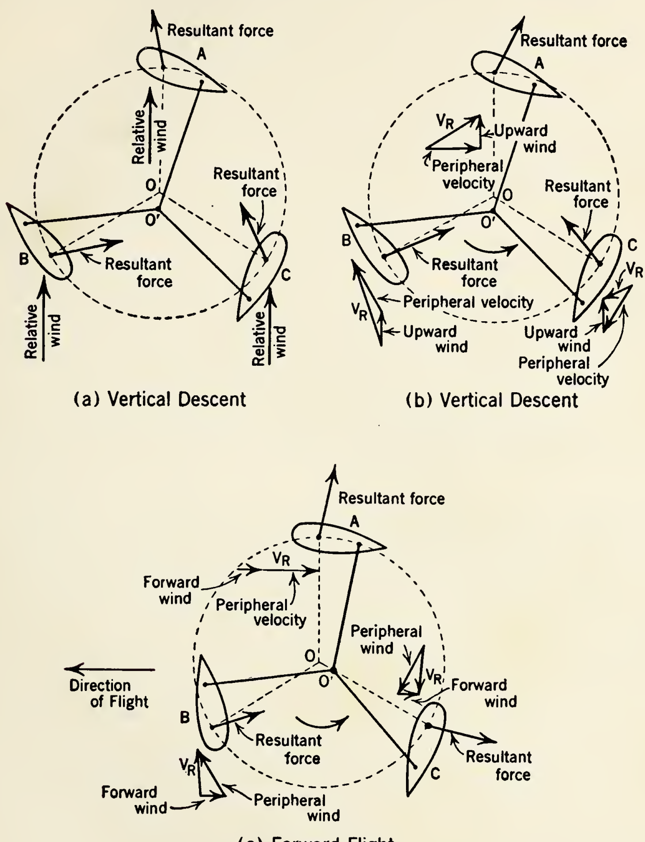


FIG. 94. Operation of cyclogiro.

in Fig. 94*b*. The relative wind (V_R) at each wing is the resultant of that due to the downward motion and to the rotary motion of each individual airfoil. The resultant forces on each wing are shown approximately in the diagram. All these forces have an upward component tending to retard descent. In the diagram, the resultant force on the wing at A has a component tending to retard rotation, but both wings at *B* and *C* have components tending to increase rotation. As long as the components acting to cause counter-clockwise rotation are greater than those tending to cause clockwise rotation, the speed of rotation will increase. At some speed of rotation, equilibrium will be found. Figure 94 shows conditions on the left side of the cyclogiro; on the right side the conditions will be reversed.

To obtain forward motion the position of the eccentric center O' is shifted by the pilot as is shown in Fig. 94*c*. The relative wind at each airfoil is the resultant of the relative wind due to forward motion and that due to the rotation. The resultant force on each airfoil can be resolved into a component tangent to the circular path and radial to the path. The tangential components either aid or retard rotation, the algebraic sum of these components usually retard rotation in forward flight, and it is the function of the engine to furnish torque to overcome this resistance in the same way that the engine furnishes force to overcome drag of the wings in the ordinary airplane. The vectorial sum of the radial components of the resultant forces on the individual airfoils should have an upward component, which is the lift.

Although the cyclogiro is still in experimental stages, it has been proved that the device is capable of slow hovering descent in case of engine failure; that a steep descent, not unlike that of an autogiro, can be made; and that horizontal flight at a fair rate of speed can be maintained.

CHAPTER XVII

MATERIALS AND CONSTRUCTION

Load Factors. In the preceding chapters, it has been shown that, while in flight, external forces are acting on the airplane structure. Some of these forces, lift and weight, are dependent on the size and design; other forces, inertia forces, are dependent on the suddenness with which maneuvers are executed. In one kind of maneuver a certain part of the airplane is stressed highly, and in another kind of maneuver a different part of the airplane may have its greatest stress. In designing parts of an airplane, each part must be planned to withstand the greatest stress that that part may receive in any ordinary maneuver. The term "applied loads" means the actual forces produced on a structure by the accelerations during a maneuver. The load factor is the acceleration expressed in terms of g.

For safety, parts are made of greater ultimate strength than just enough to stand the greatest loading they will undergo. The applied load multiplied by the factor of safety gives the design load.

The load factors in maneuvers are based to a great extent on past experience and are therefore semi-empirical. Many calculated accelerations have been checked by accelerometer readings obtained in airplanes in actual flight. The gust load factors are based on an arbitrarily assumed sharp-edged vertical gust of 30-ft.-per-sec. velocity.

Materials. The materials used in the construction of airplanes cannot be unduly heavy or bulky in giving the required strength, as unnecessary size or weight detracts from the performance. The cost of raw material is small compared with the labor cost, but a material which is extremely high in price would probably not be desirable. The material should, if possible, be adapted to use by workmen of ordinary skill.

The quantity of airplanes of one particular design will to some extent dictate the choice of material. Metal stampings are very satisfactory and cheap on quantity production, but in small lots the expense of dies is not justified.

Wood. Wood was the chief material used in early airplane construction, but it is employed to only a limited extent at the present time. First-quality selected spruce is hard to obtain and is consequently expensive, and other woods are not entirely satisfactory.

Plywood, which is made of several thin sheets of veneer, sometimes serves for flooring and other parts of the fuselage. In one or two airplanes, plywood has been used for wing covering. Plywood is very unsatisfactory in the tropics as the combination of heat and moisture causes the glue to lose its strength.

Probably the main reason that wood has been largely superseded is its lack of uniformity. It must be aged or seasoned before fabrication. It must be protected by varnish from absorbing moisture. There is usually a large waste. Because of its non-uniformity and the ever-present dangers of defects not discernible on the surface, a high safety factor must always be applied in the design of wooden members.

Cloth. The first airplanes had cloth for wing covering, and the first fuselages were cloth-covered. The standard cloth was unbleached mercerized Grade-A cotton, having a minimum tensile strength of 30 lb. per in. After the cloth covering has been sewed in place, it is treated with dope — a colloidal solution of either cellulose acetate or cellulose nitrate. Doping the fabric makes it weatherproof, tightens the fabric, and produces a rigid surface. Without the dope, damp air would make the fabric slack. Because clear dope is transparent, and because sunlight is the chief cause of the deterioration of fabric, it is customary to impregnate the dope with pigment. Even when protected with pigmented dope, cloth loses its life and strength after about a year. Because of this need for frequent renewals, fabric is being used less and less.

Steel. Low-carbon steel is used to some extent in modern airplanes for less important parts of the structure. Its tensile strength is about 55,000 lb. per sq. in. It is used either in the form of tubes or in sheets. It is assembled either by welding or riveting.

Chrome-molybdenum steel, usually termed "Chrome-Moly," is used to a great extent in the fabrication of the modern airplane.

This steel, unheat-treated, has an ultimate tensile strength of 95,000 lb. per sq. in., with a yield point of 60,000 lb. per sq. in. When properly heat-treated it may develop an ultimate tensile strength of 200,000 lb. per sq. in. It is slightly more difficult to machine than the low-carbon steel but it may be welded just as easily.

Aluminum Alloys. Absolutely pure aluminum does not corrode as does impure aluminum, but the addition of other metals greatly increases its strength, so that aluminum alloys are used for aircraft rather than the pure aluminum. The alloys are treated in various ways to minimize the danger of corrosion. Originally the term duralumin was applied to one specific composition alloy; now it is common practice to use the term duralumin loosely to cover all strong aluminum alloys. The most popular alloy has 4 per cent copper, $\frac{1}{2}$ per cent manganese, and $\frac{1}{2}$ per cent magnesium; it is used for tubing and sheets. Changes in the composition are made to give special properties. Alloys suitable for castings usually have silicon added.

Duralumin has a specific gravity of 2.8, while chrome-molybdenum steel has a specific gravity of 7.9, so that, volume for volume, duralumin has 35.4 per cent of the weight of steel. The tensile strength of 17ST duralumin is 58,000 lb. per sq. in., while that of unheat-treated steel is around 100,000 lb. per sq. in. Then a piece of duralumin in tension has 35.4 per cent of the weight and 58.0 per cent of the strength of a piece of steel of the same cross-section. If the duralumin piece had a larger cross-section so the weight were the same as that of the steel piece, the duralumin would be 1.6 times stronger in tension than the steel. In bending such as would be experienced in a beam the extra depth of the duralumin beam for the same weight would give much greater stiffness than that of a steel beam.

Duralumin can be welded or riveted. Sheet duralumin is being used a great deal for wing and fuselage covering, and spot welding appears to be superseding riveting for joining sheets.

Great care must be exercised in heat-treating duralumin, especially if the material is worked cold during fabrication. Aluminum alloy rivets should invariably be driven within a half-hour of being heat-treated. If a longer time has elapsed, they should not be used until after they have been reheat-treated.

To protect aluminum alloys from corrosion, the surface must be protected by a thin coating of oxide. The most popular method is called the " anodic " process. This consists in passing an electric current through the piece of duralumin while it is immersed in a 3 per cent solution of chromic acid. The duralumin is the anode of this electrolytic bath.

Stainless Steel. An alloy of steel containing 18 per cent chromium, 8 per cent nickel, and a little less than 0.18 per cent carbon is known as stainless steel because of its remarkable resistance to corrosion. It has a high tensile strength and is very tough and ductile. By cold rolling, tensile strengths of 400,000 lb. per sq. in. have been obtained, but in this state it is very difficult to fabricate. Ordinarily 200,000 lb. per sq. in. is a fair value of its tensile strength.

The maximum resistance to corrosion is obtained after heat-treatment. The heat of the ordinary welding process causes stainless steel to lose some of its properties. A special method of spot-welding, called " shot-welding," has been developed which does not harm the steel.

In 1932, an amphibian airplane was built in this country entirely of stainless steel. In Russia, stainless steel is used a great deal in airplane construction, and it may be presumed that it will be employed more widely in the United States when its properties and method of handling are better understood.

Construction. The early form of construction of an airplane was to make the fuselage of oblong cross-section, with four lengthwise members called longerons and suitable cross-pieces and bracing. The wings had two wooden spars with ribs or formers spaced at equal intervals to give the desired airfoil shape.

When metal was first introduced as an airplane material, every part was reproduced in metal as an exact replica of the wooden piece. Because of the nature of wood, it absorbs vibrations better than metal, so that troubles due to vibration that had not been experienced with the wooden construction were encountered in the metal construction. More important, however, was the item that wood had little tensile or shear strength, so that the design of the structure had to be extremely simple. Metal can be rolled or worked into innumerable shapes and can be used in very thin sheets still retaining requisite thickness. The entire design of aircraft has therefore been modified to take advantage of all the special properties of metal not possessed by wood.

At higher speeds, parasite resistance becomes of paramount importance, and as fuselages are built of better streamline form, the monocoque form of construction becomes more desirable. The monocoque or stressed-skin construction relies entirely on the skin to carry all the bending moments and shear. The pure monocoque has only vertical bulkheads to reinforce the skin. A type called semi-monocoque has longerons in addition to the bulkheads.

Military Planes. Airplanes for military uses are divided into various types depending on the use to which they are put. Training airplanes should have a fairly low landing speed, and this will mean a moderate maximum speed. The training plane will be a two-seater for the student and the instructor, with dual-control, a small gas supply, and no equipment. The observation plane is a two-seater with fairly high speed. It is essential that there be excellent visibility for the observer. The equipment would include radio, camera, and machine guns for both pilot and observer. The bombing plane is primarily to carry heavy loads of bombs at as high a speed as is consistent with the load carried. It has a crew of four or five. For defense it is equipped with machine guns. Some maneuverability is usually sacrificed in order to obtain load-carrying ability. The pursuit plane is the fast single-seated fighter. It must have high maximum speed, big rate of climb, and high degree of maneuverability. Its engine must be supercharged so that the plane may operate at high altitude. The equipment consists of two fixed synchronized machine guns, a bomb rack for carrying small demolition bombs, radio, and oxygen tank. On some airplanes, fixed machine guns are installed in the wings to be operated from the cockpit.

Figure 95 shows an Army observation airplane. It is a high-wing braced monoplane, designed to give the observer extremely good visibility of the ground.

Figure 96 shows an army attack airplane. It is a low-wing cantilever monoplane. Special attention should be drawn to the cleanness of design, the absence of wire bracing, and the streamlining of the landing gear.

Figure 97 shows an army pursuit airplane. It has an extremely high speed and high degree of maneuverability with full military load.

Figure 98 shows an army two-seater pursuit airplane. Although

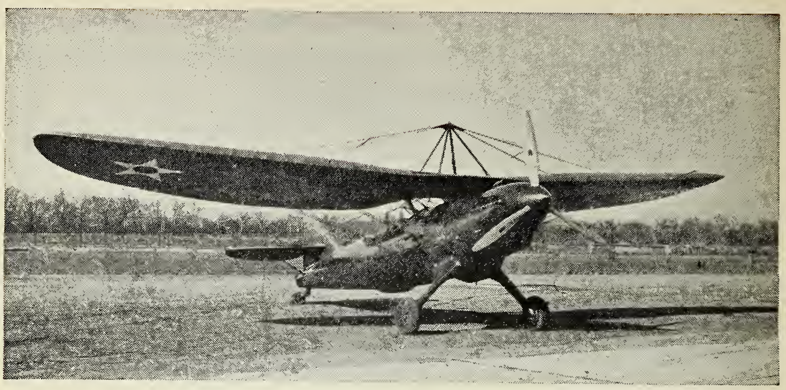


FIG. 95. Douglas Observation Airplane.

FIG. 96. Northrop Attack Airplane.

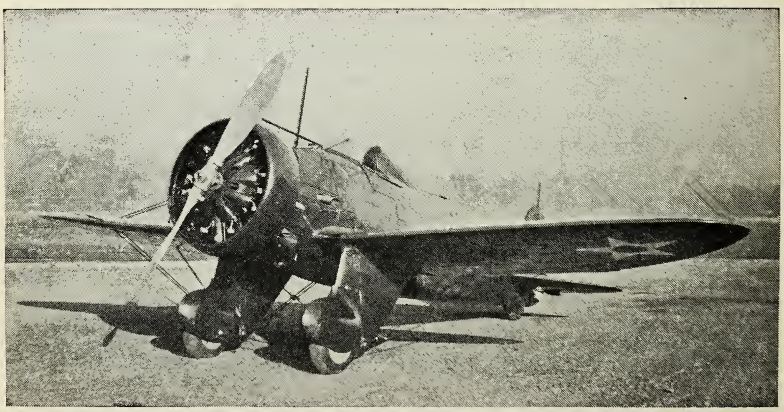


FIG. 97. Boeing Pursuit Airplane.

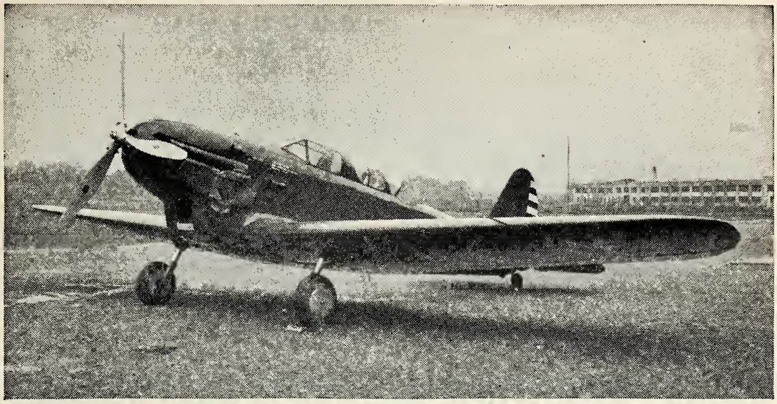


Fig. 98. Consolidated Two-seater Pursuit Airplane.

Fig. 99. Douglas Observation Amphibian.

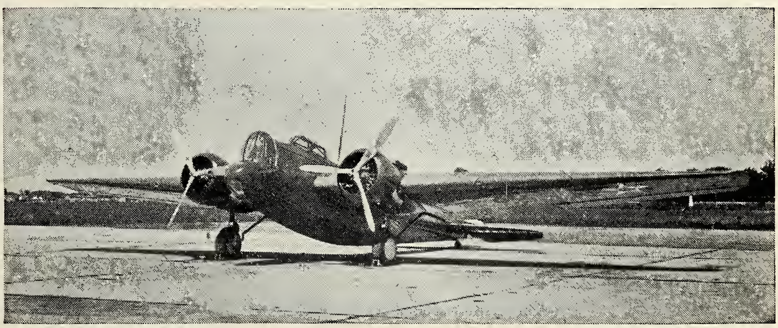


Fig. 100. Martin Bombing Airplane.

the top speed is not as great as that of the single-seater, it is well over 200 miles per hour. The supercharger which enables the engine to give full horsepower at high altitude is shown in this photograph.

Figure 99 shows a twin-motored amphibian observation airplane, adapted for alighting or taking-off from either land or water.

Figure 100 shows a twin-motored bombing airplane. It is a mid-wing cantilever monoplane with retractible landing gear. By reason of the clean design, this plane can carry a heavy bomb load at a high speed.

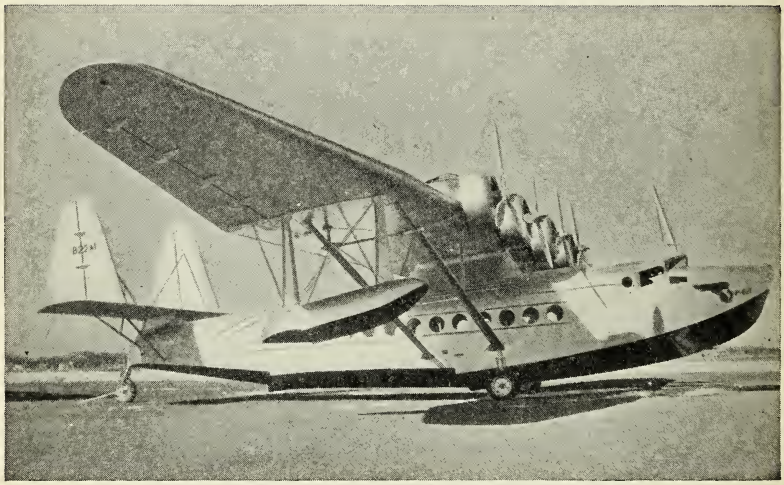


Fig. 101. Sikorsky "Brazilian Clipper."

Non-Military Airplanes. The design of non-military airplanes is influenced by the purpose of the plane whether a sport or a commercial airplane. If it is a commercial plane, it must be decided whether it is to carry a moderate load at high speed or a greater load at a lesser speed. The following paragraphs describe briefly a few commercial airplanes of different types.

Figure 101 shows the Sikorsky S-42, *Brazilian Clipper,* high-wing cabin monoplane seaplane which was used on the pioneering flights from the United States to the Hawaiian, Midway, and Wake Islands. It is equipped with four Pratt and Whitney Hornet engines giving 700 hp. each. The wings are two spar construction with metal skin on both sides of wing forward of the rear spar and with fabric skin rearward of the rear spar. The boat hull is metal-covered over a metal framework, divided for safety into nine watertight compartments. It is designed for thirty-two passengers. The wing span is 114 ft., the wing area is 1,330 sq. ft. Flaps are located between the ailerons and are hydraulically operated. The weight empty is 21,950 lb.; the useful load is 16,050 lb.; the maximum speed at sea-level is 180 miles per hour; the landing speed is 65 miles per hour.

Figure 102 shows the Seversky low-wing twin float, two- or three-seater monoplane amphibian. It is equipped with a 420-hp. Wright Whirlwind engine. The fuselage is all-metal monocoque construction; the wings are multi-box all-metal construction. Flaps are of the split trailing-edge type and extend between the ailerons. The wing span is 26 ft.; the wing area is 209 sq. ft. The weight empty is 2,550 lb., and the useful load, 1,650 lb.; the maximum speed at sea-level is 185 miles per hour, and the landing speed 55 miles per hour.

Figure 103 shows the Lockheed Electra, a twelve-place low-wing cantilever monoplane. It is equipped with two Pratt and Whitney Wasp Junior 400-hp. engines. The fuselage is all-metal monocoque with longitudinal reinforcements. The wings are all-metal stressed skin, with trailing-edge flaps. The wing span is 55 ft.; the wing area is 458 sq. ft. The weight empty is 6,200 lb.; the useful load, 3,550 lb.; the maximum speed is 210 miles per hour, and the landing speed 63 miles per hour. The landing gear is retractable, and the entire airplane is a splendid example of good streamlining.

Figure 104 shows the Fairchild Cargo Carrier, a high-wing braced monoplane. It is equipped with one 750-hp. Wright Cyclone engine. The fuselage is of welded chrome-molybdenum steel tubing, fabric-covered. The wing spars are single solid web type with extruded angles as flanges all of chrome-molybdenum steel; the ribs are built up of aluminum framework; the wing is fabric-covered. Special attention should be drawn to the Zap-type flaps in the illustration in their " down " position. The wing span is 84 ft.; the wing area, 800 sq. ft. The weight empty is 7,320 lb.; the useful load, 5,680 lb.; the maximum speed is 167 miles per hour, and the landing speed 52 miles per hour.

Figure 105 shows the Monocoupe, a high-wing braced cabin

FIG. 102. Seversky Amphibian.

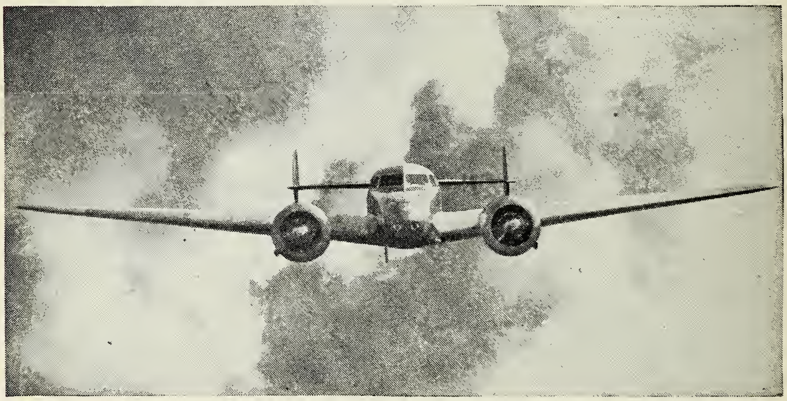


FIG. 103. Lockheed "Electra" Airplane.

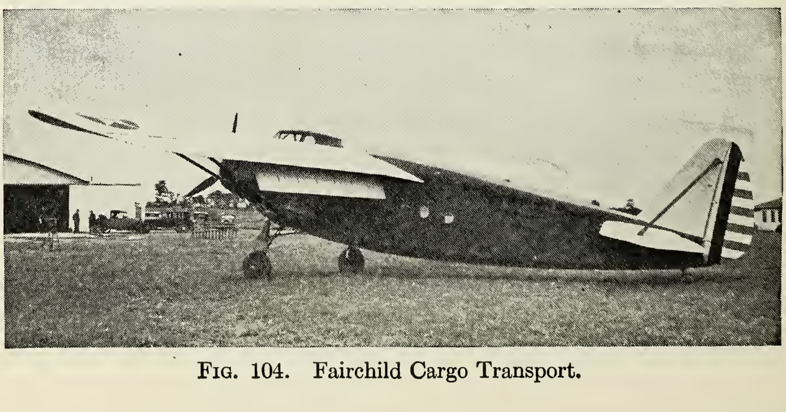


FIG. 104. Fairchild Cargo Transport.

monoplane. This is a typical sport plane for a private owner. It is equipped with a 90-hp. Lambert radial engine. The fuselage is of welded steel tubing, fabric-covered. The wings have two spruce spars, the ribs have basswood webs with spruce cap-strips;

FIG. 105. Monocoupe Sport Airplane.

the wing is fabric-covered except that sheet aluminum is used on the leading edge. The flaps are fabric-covered. The wing span is 32 ft.; the wing area, 132 sq. ft. The weight empty is 935 lb.; the useful load, 650 lb.; the maximum speed is 135 miles per hour, and the landing speed 40 miles per hour.

CHAPTER XVIII

INSTRUMENTS

Introduction. Instruments are practically indispensable for safe flying. Although on short hops around an airport the necessity for instruments is slight, on cross-country flights, safety demands that every possible instrumental aid be given the pilot.

On the Wright brothers' first flights, their only guide was a long streamer of cloth tied to one of the struts. It served as airspeed indicator: at low speeds the rag hung limply down; at higher speeds its position became more nearly horizontal. It also indicated slipping by swinging away from the center of turn and skidding by swinging towards the center of turn.

Many instruments have been added since the days of the early Wright flights, but each has been the result of a distinct need for that instrument. *All* instruments have a definite function, and they are placed on the instrument board because at some time the occasion will arise when the information given by that instrument will be vital.

It is of primary importance that aircraft instruments be accurate at all times. With only the owner flying the airplane, if one instrument reads *consistently* low or high, the pilot can of course make a mental correction; but this is a very poor practice and is extremely dangerous. The Army Air Corps has a fixed policy of "either dead-right or stopped." Interpreted, this means that it is more desirable to have an instrument completely fail to function than to have it function imperfectly. If an instrument is not working at all, the pilot will disregard it; if it apparently is functioning the pilot will rely on its indications and, if they are incorrect, disaster may result.

All aircraft instruments are classed under two headings, engine instruments and avigation instruments. All instruments whose readings do not pertain to the operation of the engine are termed avigation instruments. Since time should not be lost while the pilot's eye is searching about the instrument board, the engine instruments should be grouped together and the avigation instruments should be together.

In considering instruments, care must be taken to differentiate between accuracy and sensitivity. Sensitivity is the ability to detect very small differences in the amount of the quantity being measured. It is not the same as accuracy or correctness. An instrument may be sensitive and at the same time very inaccurate. For example, a tachometer might be extremely sensitive in that it would show a very small change in the number of revolutions per minute, but the actually indicated revolutions per minute might be greatly in error.

Another important feature in instruments is absence of lag. The instrument should respond instantly to any change in the magnitude of the quantity being measured. For example, an altimeter would be of little value if the airplane dropped a thousand feet and the altimeter did not show this decrease for several seconds. A properly designed instrument will have the parts so balanced that the time lag will be reduced to a negligible amount.

Engine Instruments. Engine instruments are either to enable the pilot to operate the engine most efficiently or to warn him of impending trouble. Some engine troubles cannot be alleviated in the air, but the instrument performs the very useful service of indicating wherein the trouble lies, so that, the cause of stoppage being known, the fault can be remedied after landing. Much time can be saved the mechanic in repairing a stopped engine if he knows what first caused the trouble.

Airplane engines that have passed the rigid tests of the Department of Commerce if they have no defective parts and are properly mounted should run for long periods provided that they are oiled properly, proper and sufficient fuel is supplied, etc. The throttle of an airplane engine is a hand throttle. When it is set at a certain throttle-opening to give a certain engine speed, the engine will continue to turn over at that speed unless some part of the engine or its auxiliaries is not functioning properly.

Engines do not stop without cause, nor do they stop instantly. Whatever ultimately causes the engine to quit entirely, first causes it to slow down. The malfunctioning of some part of the operation causes a loss of power which results in a drop in engine speed. If the reason for this malfunctioning is not removed, the loss in power will increase and finally result in stoppage. For example, if a bearing is getting insufficient lubrication, it will get hot and expand. The added friction will slow up the revolutions per minute as well as increase the heating to cause more expansion till finally the bearing and journal " freeze." For another example, clean spark plugs do not suddenly become fouled. There is a gradual accumulation which causes poor firing, lessening the power and decreasing the speed. As the accumulation increases the firing becomes poorer until the spark fails to jump and the cylinder misses completely.

Probably the only reasons for instantaneous failure of an engine are breakage of the fuel-line or breakage of an electric wire, both of which would be due to vibration from improper mounting. Since, on a properly mounted engine, complete failure is always preceded by a more or less brief period of poor functioning, a careful watch should be kept of the engine speed; a slight decrease in revolutions per minute is a warning of impending trouble.

The tachometer is essentially a cautioning instrument; if it shows a decrease in engine speed, the pilot should scan the other engine instruments to locate, if possible, the source of the trouble. If the oil or water is too cool, the pilot can remedy this in the air by closing shutters. Most of the other troubles cannot be rectified in the air, but with the advance warning the pilot can often land before his engine has quit entirely.

Tachometers. The tachometer indicates the revolutions per minute of the engine. Although its most valuable function is to warn of impending engine trouble, it is at all times useful in aiding the pilot to set the engine at the most economical cruising speed, etc. On the line, it helps the mechanic in judging to what extent an engine is " tuned-up." If an airplane engine can be throttled down to 250 r.p.m. and it turns over without missing, it is in very good condition; if it can be throttled down to 200 r.p.m. without missing, it is in excellent condition.

Several types of tachometers are in use, the most popular being the chronometric and the centrifugal types. The chronometric type has more working parts and therefore the production cost should be greater than for the centrifugal type. With wear of the working parts, the centrifugal type loses in accuracy while still functioning; the chronometric type is " either dead-right (within instrumental accuracy) or stopped."

Chronometric Tachometer. This type actually counts the revolutions in 1-sec. periods, but shows on the dial the revolutions per second multiplied by 60 (i.e., the revolutions per minute).

The pointer remains stationary for 1 sec. If the engine speed is constant, the pointer remains fixed; but if the engine speed is varying, at the end of the 1-sec. counting period, the pointer jumps to a new position. This does not mean that the engine speed is erratic or jumpy, but merely that the average speed for 1 sec. is different from that of the previous second.

The mechanism is roughly as follows. A flexible shaft from the engine turns a gear in the tachometer at a fixed ratio to engine speed. This driving gear does two things. First, it winds the mainspring of a clockwork, through a friction clutch, so that the spring will not be wound too tight; as the tachometer operates the clockwork tends to run down but the driving gear keeps it wound up. The other function of the driving gear is to rotate a counting gear. This counting gear, marked G in Fig. 106, is in mesh with

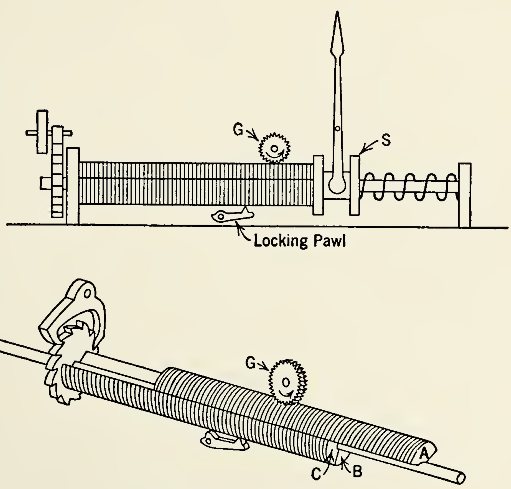


FIG. 106. Chronometric tachometer mechanism.

a cylindrical rack or fine-tooth worm. This worm is split longitudinally into three segments, A, B, and C. The three segments encase a shaft and are so keyed to the shaft that they may move independently along it. On one end (left end in Fig. 106) is a pinion with an escapement operated by the clockwork so that by an intermittent mechanism, at the end of each second, the shaft is rotated through one-third of a revolution. On the other end of the shaft is a loose collar S, which by a spring is held against the end of the segmented worm. The collar S moves the pointer over a scale.

Assuming segment A in mesh with counting gear G, at the beginning of the 1-sec. counting period, the rotation of G starts moving segment A to the right. At the end of the second the linear movement of A has been proportional to the speed of gear G. Synchronously with the end of the second, the clock escapement rotates the shaft and segmented worm through 120°, so that segment A occupies the position previously occupied by segment C, segment C has moved to replace segment B, and segment B has moved to engage with counting gear G. The linear displacement of segment A is maintained by a locking pawl. Collar S, linked with the pointer, is resting against the end of segment A.

During the next second, segment B is being fed to the right by the counting gear. At the end of the second, segment B moves down to the position of A, and segment A slides back along the shaft to its zero position, actuated by a spring not shown.

The above-described type is the "Tel" mechanism; other types use gears instead of the segmented worm.

Even a very cheap clock can be regulated so as not to gain or lose more than half a minute in a day; in 1 sec. the inaccuracy is very small indeed, and the only error should be due to the meshing of the teeth of the counting gear which, on a good instrument, should not exceed plus or minus 10 r.p.m. If a chronometric tachometer is functioning at all, it should be correct within these limits. Sometimes when very cold the oil in the clockwork becomes thick and gummy, stopping the clockwork. Wear of teeth will cause a jam, stopping the operation.

In addition to the merit of either being correct or stopped, the chronometric type is usable when the engine is turning over very slowly. The centrifugal type described in the next paragraph does not give a readable indication much below 500 r.p.m.

Centrifugal Tachometer. The centrifugal type depends on the centrifugal action of a pair of weights disposed about a shaft which is rotated by the engine through a flexible shaft. The weights, see Fig. 107, are connected by links to fixed collar A and movable collar B. As the speed of rotation of the shaft is increased, the weights tend to move outward raising collar B, this motion being resisted by a spring. Centrifugal force varies as the square of the speed, so the upward movement of collar B is not proportional to the changes in revolutions per minute. Usually, since the actuating force is very small at low speeds, no attempt is made to read the instrument at low speed. As the parts wear, the lost motion causes the instrument to be inaccurate, even though apparently functioning. The inertia of the moving parts produces a time lag which may delay the instrument several seconds in indicating a change in rate of revolution.

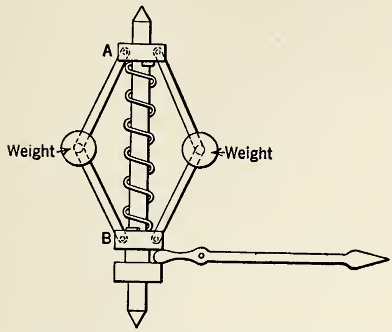


Fig. 107. Centrifugal tachometer mechanism.

Pressure Gages. Pressure gages are used to measure the gasoline pressure near the carburetor in force-feed systems and also to measure the pressure of the oil system. Because loss of pressure means engine failure, the pressure gages should be watched closely, especially before take-off. Overpressure indicates a stoppage in the line; underpressure indicates lack of oil, failure of pump, or broken line.

The gages are usually of the Bourdon type, the expansion member being a seamless drawn bronze tube of elliptic cross-section bent into an arc of more than 180°. One end of this tube is closed, and when internal pressure is applied through the open end, the tube tends to straighten out. The movement of the free end is communicated to a pointer.

Thermometers. Thermometers are used to find the temperature of the cooling liquid, the lubricating oil, or the cylinder walls or heads. The thermometer in the cooling system warns of engine trouble due to overheating, aids in operating the engine at maximum efficiency, warns if the cooling medium is near its boiling or freezing point, and warns if the engine has become too cool in a glide to pick-up readily.

There are two general types of aircraft thermometers: the pressure types and the electric types. All thermometers must be distant-reading; that is, there must be an indication on the instrument board in front of the pilot of the temperature at some point several feet distant. The pressure types are especially suited to finding the temperature of a liquid such as the oil or the cooling medium; the electric type is suited best for finding the temperature of parts of the engine and is used on air-cooled engines.

The pressure-type thermometer has three parts: a bulb, which is immersed in the oil or water; a pressure gage on the instrument board; and a length of tubing which connects the bulb to the gage. There are two kinds of pressure thermometers — the liquid-filled and the vapor-pressure thermometers.

In the liquid-filled type, the bulb, the tubing, and the pressure chamber in the gage are all filled completely with a liquid. When the bulb is heated, the liquid in the bulb expands and causes internal pressure in the system. For an increase of 100° C., the increase in pressure is several hundred pounds per square inch. This pressure is indicated on the pressure gage, the scale, however, reading directly in degrees of temperature. Since only a part of the liquid is heated to the temperature which it is desired to measure, there is liable to be an inaccuracy due to the liquid in the connecting tubing and the gage not being at this temperature. This error is made negligible by having the connecting tubing of very fine bore and by putting a bimetallic compensation on the pressure gage.

In the vapor-pressure type, the bulb contains a volatile liquid, usually ether. The quantity of liquid must be such as not to fill the bulb completely. The remainder of the bulb, the capillary connecting tubing, and the pressure chamber of the gage are filled with the vapor from the liquid. It is essential that the free surface of the liquid be inside the bulb, since in a closed system containing a liquid and its vapor the vapor pressure depends on the temperature of the liquid surface in contact with the vapor.

The liquid-pressure type depends on the expansion of the liquid with heat, so the pressure changes will be lineal with temperature; with the vapor-pressure type, the pressure change will not be linear with temperature but will be greater at higher temperatures, so that the scale will be more open there than at low temperatures. The pressures developed with the liquid-filled type will be much greater at the same temperature than with the vapor-pressure type so that the gage is much more rugged for the liquid-filled type. Owing to these high pressures in the liquid-filled system, the changes in atmospheric pressure due to altitude are insignificant; with the vapor-pressure type there may be an error of several degrees resulting from this. Both types use a fine capillary tube connecting the bulb to the gage, and care must be taken in dismantling an airplane that a careless mechanic does not cut the tubing under the impression that it is a copper wire.

The electric-type thermometer is customarily a thermocouple connected to a millivoltmeter. One wire is iron, the other constantin. The "hot junction" may be a true joint, or the same effect is achieved if the two wires are inserted into holes a short distance apart in the cylinder wall and peined securely in place. In a thermocouple, the difference in potential indicated by the millivoltmeter on the instrument board depends on the difference in temperature between the "hot" and "cold" junctions. This means that some form of temperature compensation must be used or the thermometer will merely indicate the difference between engine temperature and cockpit temperature.

Ice-warning indicators are merely thermometers with special markings for the temperatures between 32° and 29° F., which is the temperature range that is especially dangerous from the standpoint of ice formation.

Avigation Instruments. Avigation instruments are those instruments which aid the pilot in flying the airplane safely and in guiding him to his destination. They tell him his altitude, his speed, his direction, and the attitude of the airplane with respect to the ground or some other reference.

Altimeters. The usual altimeter is a barometer or pressure gage which measures the atmospheric pressure. Other types have been proposed but none have passed beyond the experimental stage. These types are electrical or sonic.

For bombing, it is desirable to know the altitude very accurately at heights of 12,000 to 15,000 ft. in order that the trajectory may be calculated correctly. For photographic mapping, usually done at about 12,000 ft., it is primarily essential that all the photographs be taken at exactly the same altitude in order that each part of the mosaic be the same scale. It is very desirable that this altitude be known correctly to check ground distances.

In flying over mountain ranges in cloudy weather, flight should be made at sufficient altitude to have ample clearance above the mountain tops. The readings of the ordinary airspeed indicator are subject to an altitude correction, but an error of a hundred feet in altitude makes a very small change in the altitude correction to the indicated airspeed.

Therefore, except for bombing and photographic work and the somewhat rare attempts for altitude record, the need for accuracy at medium or high altitudes is not very great. For landing in fog, however, the altimeter should be extremely accurate.

Aneroid Altimeters. The standard type of altimeter is a pressure gage, but since the pressure differences to be observed are very small, the gage must be exceedingly sensitive. For this reason an aneroid is used. An aneroid is a thin metallic box shaped like a poker chip. The flat sides are very thin and flexible. Formerly these sides were corrugated to give flexibility, but better results are obtained with flat sides. When pressure on the outside is removed, the aneroid expands. As the flat sides move outward, this motion is magnified and transmitted to the pointer.

The dial instead of being graduated in pressure units is marked in units of altitude. Since the expansion of the aneroid depends on pressure, a simple linkage would result in an approximately uniform pressure scale. The pressure difference for 1-ft. altitude difference is much greater close to the ground than at altitude, so that if the pointer movement were proportional to pressure difference, the altitude scale would be non-uniform; that is, the graduations would be far apart near the ground and closer together as altitude is increased. To have a uniform altitude scale, a very ingenious arrangement of links and chain and roller is used.

Having a uniform altitude scale makes it possible to have the scale adjustable. With the usual circular dial, a knob is provided outside the case whereby the dial may be rotated irrespective of the position of the pointer.

Formerly there was no fixed barometric scale on the altimeter. Before taking off, the pilot would turn the dial till the zero of his altitude scale was directly back of the pointer. After flying around, in coming down to land he would assume that the altimeter would read zero when his wheels touched the ground. This would be true only if the atmospheric pressure had not changed while he was in the air. If, while he had been flying, the atmospheric pressure at the airport had decreased by, say, 0.2 in., his altimeter would read 200 ft. when he landed. Conversely if the barometric pressure on the ground had risen while he was aloft, his altimeter might indicate zero while he was still several hundred feet in the air.

Modern altimeters have a fixed barometric scale in addition to the usual adjustable altitude scale. This is a desirable adjunct to landing at an airport in fog. While the airplane is still in the air, an operator on the ground, hearing the motor noise, carefully reads the barometer at the airport and radios the reading to the pilot. The pilot thereupon sets the zero of the altitude scale opposite this reading on the fixed barometric scale on his altimeter. He then comes down to land knowing that when the pointer reaches the zero on the altitude scale, his wheels should be touching the ground.

The altitude-pressure relation for altimeter calibration is given in N.A.C.A. Report 246 and is practically identical with the standard atmosphere given in Table I of this text. This relation is based on an arbitrary assumption of the temperature change with altitude. This assumption checks very closely with the average observed variations of temperature with altitude at latitude 40° in the United States. These observations were taken throughout the year. On any one day, the temperature at some altitude may be above or below the average yearly temperature at that altitude.

For an aneroid altimeter to be absolutely correct, the temperature not only must be 15° C. (59° F.) at sea-level but it also must vary in an orthodox manner as the altitude is increased. When these conditions do not exist the altimeter will not read correctly at altitude. A correction to the altimeter reading can be made if the average temperature of the air between the plane and the ground is known. A very close approximation of this mean temperature can be made by reading the temperature every few hundred feet during the climb and averaging these readings. An approximation sufficiently close for most photographic work is made by taking the arithmetic mean between the ground air temperature and the air temperature at the altitude at which the flying is to be done.

Since air inside the aneroid would expand with an increase in temperature and build up a pressure, and vice versa, it is customary to pump out most of the air before sealing the aneroid to decrease the error from this source. The pressure inside the altimeter case, outside the aneroid, should be atmospheric pressure; therefore, there should always be an opening in the case. Ignorant mechanics, seeing a hole in the case, sometimes, with mistaken zeal, plug it up. This should be guarded against.

Frequently the pressure in the cockpit is not exactly the same as the atmospheric pressure immediately around the airplane. This is especially true in cabin planes. For accurate readings a tubing is connected to the altimeter case, the outer open end of the tubing being outside the cabin at some point on the structure where there is only static pressure.

To make the altimeter sensitive, the side of the aneroid is of large area. A very slight difference of pressure will then make an appreciable pointer movement.

It is important that an altimeter have very little lag. Inertia varies with mass, so that, if the aneroid or other moving parts have considerable mass, they will have more inertia or resistance to sudden movement. Sensitive altimeters are built with large aneroids, but sufficient care is taken in balancing the moving parts so that lag is reduced to a minimum.

It should be noted that the word "sensitive" is not synonymous with accurate. A sensitive altimeter is one in which the pointer moves a detectable amount for very small changes in altitude. A "sensitive" altimeter is subject to the same errors as other aneroid altimeters if the atmosphere at the time of flight is not standard, i.e., if the pressure and temperatures do not correspond with the standard pressure-temperature-altitude relation.

Altimeters, Miscellaneous Types. At sea, sonic depth-finders are used, the time interval being measured for a sound to travel to the bottom of the ocean and echo back. Boëhm in Germany and Jenkins in the United States have attempted to adapt this scheme to aircraft but with only moderate success.

A capacity altimeter depends on the principle that the mutual electrical capacity of two adjacent plates increases with the approach of a third plate. Two plates are fastened to the bottom of the fuselage and their capacity increases as the plane nears the earth, which acts as the third plate. This device acts admirably as a landing altimeter because the sensitivity increases greatly as the altitude becomes small. Owing to weight and cost, this type is rarely used.

At night, flying over water, a searchlight with a diverging beam can be aimed directly downward. If alongside the searchlight proper is mounted a telescope equipped with stadia wires, it is possible to measure the diameter of the light circle on the water and compute the altitude. Such a device was used on the *Graf Zeppelin.* By comparing the altitudes obtained by this method with the readings of the aneroid altimeter, the meteorologist ascertained whether the *Graf* was in a low- or high-pressure area to aid him in his weather predictions. Another scheme used on the *Graf* was to drop plaster eggs and time the interval till they shattered on the water.

Airspeed Indicators. The airspeed indicator shows the speed of the airplane with respect to the surrounding air, i.e., the speed of the relative wind. In still air, the airspeed is the ground speed, but if any wind is blowing, the airspeed and ground speed are not the same. The airspeed indicator is a valuable aid to the pilot. It warns him when the plane is near its stalling speed. By variations of its indications, he knows if he is nosing-up or diving. In taking-off, the airspeed indicator tells him when he has attained sufficient speed. He can know when he is at best climbing speed, speed corresponding to L/D_{total} maximum, etc. In dives he is warned when the speed is becoming excessive for structural strains.

Pitot-Static Airspeed Indicators. The standard airspeed indicator makes use of a Pitot tube, which is a tube parallel to the airplane axis, its open end pointing in the direction of flight. The pressure at the open end of a correctly designed Pitot tube will be

$$P_t = P_s + \frac{\rho V^2}{2}$$

P_t is Pitot pressure in pounds per square foot
P_s is static (atmospheric) pressure in pounds per square foot
ρ is mass density of air in slugs per cubic foot
V is airspeed in feet per second

If the airspeed, V, is expressed in miles per hour instead of feet per second this becomes

$$P_t = P_s + \frac{\overline{1.467^2}\,\rho V^2}{2}$$
$$= P_s + 1.075\,\rho V^2$$

If the Pitot and static pressures are in pounds per square inch,

$$P_t = P_s + \frac{1.075}{144}\,\rho V^2$$
$$= P_s + 0.00747\,\rho V^2$$

With air at standard density (ρ = 0.002378 slug per cubic foot), this becomes

$$P_t = P_s + 0.00001776\ V^2$$

In calibrating airspeed indicators against pressure gages, the pressure gage shows the difference between the Pitot pressure and the static pressure, so that

$$P_t - P_s = 0.00001776\ V^2$$

or

$$V = 237.3\sqrt{P_t - P_s}$$

If the calibration of the airspeed indicator is against a U-tube of water, since 1 in. of water is equivalent to a pressure of 0.0362 lb. per sq. in.,

$$h = 0.000491\ V^2 \qquad h \text{ is head of water in inches}$$

or

$$V = 45.2\sqrt{h} \qquad V \text{ is in miles per hour}$$

Because cockpit pressure is not exactly the same as atmospheric pressure, a static tube is mounted immediately adjacent to the Pitot tube, and the other end of the static tube is connected to the inside of the case of the pressure gage. The pressure gage will then be operating on the dynamic pressure, or difference between Pitot and static pressure, at the point where the Pitot and static tubes are mounted. It is important that the tubes be in a proper location where the air flow is not too greatly disturbed. For a biplane, the best location is on a forward interplane strut about midway of the gap. For a monoplane, the preferable location is one-third the semi-span in from a tip and about half a chord length forward of the leading edge in the plane of the under surface of the wing. Mounting the Pitot and static tubes on a rod projecting this far in front of the wing, though it gives more accurate airspeed readings, is undesirable from the liability of their being damaged as the airplane is moved around in the hangar. For this reason it is customary to mount the tubes about a foot below the under surface of the wing, one-third the semi-span in from the tip and vertically below the leading edge.

The tubing connecting the Pitot-static heads to the indicator in the cockpit must be absolutely airtight. All connections should be soldered or a metal union should be used. The former practice of using rubber tubing to make joints is to be condemned as with age the rubber cracks open.

Under certain weather conditions, ice forms on the open forward-projecting end of the Pitot tube. Because it is *pressure,* not *force,* that is being measured, the actual size of the opening makes very little difference. Ice partially blocking the opening of the Pitot tube will not affect the readings of the airspeed indicator as long as there is even a pinhole opening. With the opening totally sealed over by ice, the airspeed indicator fails to function. Some Pitot tubes have means for being electrically heated to prevent ice formation. It should be borne in mind that the same weather conditions that would cause stoppage by ice of the Pitot tube would cause ice to form on the propeller and leading edge of the wing, and a pilot should avoid this.

Pitot-Venturi Airspeed Indicator. At one time, it was almost universal practice to mount a Venturi tube alongside a Pitot tube and connect these by two lines of tubing to a pressure gage airspeed indicator in the cockpit, so that one side or compartment of the pressure gage had the Pitot pressure, the other had the suction from the throat of the Venturi tube. The suction (or negative pressure) at the throat of a properly designed Venturi tube is numerically several times the (positive) pressure of a Pitot tube, the air velocity being the same. The pressure gage operating on the pressure differential between a Pitot tube and a Venturi tube could be much more rugged and sturdy than one operating on the pressure differences between a Pitot tube and a static tube.

The reason for the discontinuance of the manufacture and use of the Pitot-Venturi type of airspeed indicator is that any deposit of ice or mud on the forward lip of the Venturi nozzle destroyed the smooth flow of air through the throat of the Venturi tube. Unless the airflow was devoid of eddies and burbling, the suction would not be proportional to airspeed and the instrument would read incorrectly.

Altitude Correction for Airspeed Indicators. The Pitot-static as well as the obsolete Pitot-Venturi airspeed indicators are termed "dynamic," since the pressures on which the indicators operate vary with the dynamic pressures, i.e., they vary with $(\rho/2)V^2$.

These instruments, when the airplane is traveling at the same airspeed at altitude as at the ground, will be subjected to less pressure difference at altitude since the density (ρ) is less. These airspeed indicators will therefore read low at altitudes.

The mechanism of the indicator is designed to give a uniform airspeed scale, so that, while the pressure varies as the square of the velocity, a linkage is used that gives a pointer movement varying as the square root of the pressure. Then as the density (ρ) decreases with altitude, the pressure decreasing in the same ratio, the indicated airspeed will decrease as the square root of the density.

A rough rule is to add 2 per cent of the indicated airspeed for every thousand feet of altitude to get the true airspeed. More accurate results are obtained by multiplying the indicated airspeed by $\sqrt{\rho_0/\rho}$, using values of ρ/ρ_0 from Table I in this text.

The true airspeed is of little use in flying except, with wind velocity known or by using a wind-star, to obtain groundspeed. For handling the airplane the pilot desires to know not the *true* airspeed but the *dynamic* airspeed or airspeed indicated on a dynamic-type airspeed indicator. Maximum rate of climb, stalling, stresses, etc., are all dependent on dynamic airspeed and not on true airspeed. For example, if an airplane has a certain stalling speed at sea-level, the stalling speed at any altitude will be the sea-level stalling speed multiplied by $\sqrt{\rho_0/\rho}$. The dynamic airspeed indicator at any altitude indicates a speed which is the true airspeed multiplied by $\sqrt{\rho/\rho_0}$, or conversely, the true airspeed is the indicated airspeed multiplied by $\sqrt{\rho_0/\rho}$. If the stalling speed is known at sea-level, whatever the altitude, when the airspeed indicator reads that speed, the airplane is on the point of stalling. In other words, if at some high altitude the airplane is put in a stall and the airspeed indicator read, at any other altitude when the airspeed indicator shows the same reading, the airplane is stalled.

True Airspeed Indicator. Any freely revolving device such as an anemometer or spinner (miniature propeller) will rotate at a rate proportional to the true airspeed provided that there is little or no friction. When friction is not negligible so that the rotor must do some work, its speed is no longer independent of density.

Compasses, Magnetic-Needle Type. A magnetic compass is an instrument which tells direction by means of the earth's magnetism. The earth's magnetic field is represented by magnetic lines of force which, in general, run from the north magnetic pole (located near 70° N., 96° W.) to the south magnetic pole (near 71° S., 148° E.) but with erratic diversions due to local deposits of iron and other causes. These lines slope downward, except at the magnetic equator, so that at the latitude of San Antonio the lines slope 50° from the horizontal; farther north at the latitude of New York the dip is 72°.

A magnetic needle, unaffected by nearby iron or steel, tends to lie along the magnetic lines of force. In the United States, it is customary in making compasses to suspend the needles from a point considerably above the needles so that there will be little tendency for the needles to dip. They will be free to swing into the vertical plane of the magnetic lines of force. At only a few places on the earth are these lines running true north and south. Compasses therefore do not indicate true north, but when they are in the plane of the magnetic lines of force, they are said to be indicating magnetic north. The difference between true north and magnetic north is the magnetic variation (or magnetic declination). Variation is said to be plus or westerly when the compass indicates a north which is west of the true north; variation is easterly or negative when the compass indicates a north which is east of the true north.

Charts or tables tell the variation at various places; a sample chart for the United States is given in Fig. 108. The variation changes from year to year so that the most recent chart should always be used.

The pull of magnetic force is in the direction of the magnetic lines of force; it is only the horizontal component of this force that has a directional effect on the compass needles. Near the earth's magnetic poles the direction of the force is nearly vertical so that the compass needles have little tendency to point in any direction.

Besides variation, compass needles are subject to the additional error of deviation, which is due to the nearby presence of steel or iron. Direct electric currents also affect the compass. The deviating effect of these disturbing elements varies as the mass and inversely as the square of the distance; a small piece of steel close to the compass may have more effect than a large piece of steel at a considerable distance. When deviation is such that the compass indicates a north which is west of *magnetic north,* deviation is called westerly or plus; when the *compass north* is east of the *magnetic north,* deviation is easterly or minus.

The compass needles point in a general northerly direction. As an airplane's heading is changed, the absolute direction with

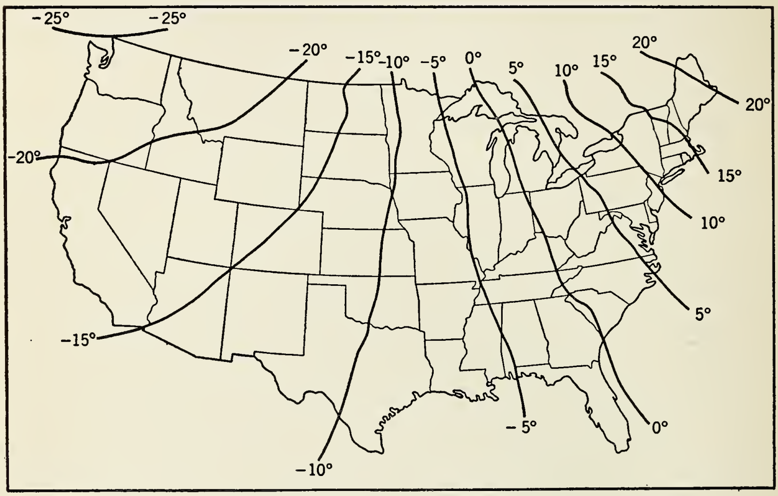


FIG. 108. Magnetic variation for the United States, 1935.

respect to the compass of the engine, fuselage members, and other steel parts is changed. The deviating effect of the engine, etc., is therefore different with each heading of the airplane.

Variation depends on the geographic location of the airplane; deviation depends on the heading.

To ensure that there shall not be large deviations, a compass on an airplane should be compensated, which means placing small magnets close to the compass needles to counteract as much as possible the deviating influence of steel objects on the airplane. It is usually not possible to correct all deviation; the most that is attempted is to compensate on north heading and on east heading so that there shall be zero deviation on these headings. The airplane is then set on other headings (every 30° from north) and the deviation recorded on a card which is fastened on or near the compass. By consulting this deviation card, interpolating if necessary, the pilot can correct his compass reading to obtain his true heading.

Directions are given in degrees clockwise from north, so east is 90°, south is 180°, west is 270°, etc. Headings measured from true north are true headings; measured from magnetic north they are magnetic headings; and measured from the north indicated by the compass they are compass headings.

A course is the direction an airplane should head to reach its destination provided there is no cross-wind; if there is a cross-wind, the proper heading is not the same as the course.

On airway strip maps, the line between airports is marked with the magnetic course (M.C.) at intervals, wherever the variation has changed by as much as 1°. Before these magnetic courses are used by the pilot, they must be changed to compass courses by applying deviation. If a pilot is using maps not so marked but obtains a true course by any means, the true course must be changed to a compass course by applying both variation and deviation.

To find a magnetic course knowing the true course, the variation is added to the true course if the variation is westerly, and subtracted from the true course if variation is easterly. To find the true course knowing the magnetic course, the variation is subtracted from the magnetic course if the variation is westerly, added to the magnetic course if variation is easterly.

To find a compass course knowing the magnetic course, the deviation is added to the magnetic course if the deviation is westerly, and subtracted from the magnetic course if the deviation is easterly. To find a magnetic course knowing the compass course, the deviation is subtracted from the compass course if deviation is westerly, and added to the compass course if deviation is easterly.

Example. It is desired to head due east. Variation (from map) is 14° W. Deviation (from card on instrument board) is 10° E. What should compass read?

Solution. True heading should be 90°; since magnetic N. is 14° west of true N., magnetic heading should be 104°. Since compass N. is 10° east of magnetic N., compass heading should be 94°.

Example. The compass heading is 260°. Variation is 21° E. Deviation is 5° E. What is true heading?

Solution. Compass heading is 260°; since compass N. is 5° E. of magnetic N., magnetic heading is 265°. Since magnetic N. is 21° E. of true N., true heading is 286°.

Problems

1. Compass heading is 80°. Variation 15° W., deviation 10° W. What is true heading?

2. It is desired to head 195° true. Variation 15° W., deviation 10° E. What should compass read?

3. True heading is 290°. Variation 25° E., deviation 5° W. What is compass heading?

4. Magnetic heading is 340°. Deviation is 12° E. What is compass heading?

5. Compass heading is 355°. Variation 12° E., deviation 10° E. What is true heading?

6. Magnetic heading is 5°. Deviation is 20° E. What is compass heading?

7. True course is 160°. Variation is 10° W.; deviation is 15° E.; there is no wind. What should be the compass heading?

8. True course is 15°. Variation is 20° W.; deviation is 8° E.; there is no wind. What should be the compass heading?

9. Magnetic course is 274°. Deviation is 5° W.; there is no wind. What should be compass heading?

10. Magnetic course is 129°. Deviation is 10° E.; there is no wind. What should be compass heading?

Airplane compasses have two parallel bar magnets of tungsten steel fastened to a spider of non-magnetic material. Affixed to the spider is the "card" or scale, a cylindrical strip of celluloid or thin brass, on which the graduations are marked. The compass being mounted so that the pilot is virtually looking backwards at the compass, the point on the card corresponding with the south-seeking end of the magnets is marked N (North). The card is usually marked with 5° graduations.

From the center of the card, a stellite pivot projects downward, resting in a jewel-cup on the top of a post extending up from the bottom of the bowl. A retaining device prevents the pivot from jumping out of the cup (see Fig. 109).

All airplane compasses are of the liquid type. The compass magnets and card are inside a spherical metal bowl which has a glass window on one side, and the entire bowl is full of mineral spirits. The liquid serves two purposes; it buoys up the card so that there is less friction at the pivot, and it damps the swingings of the card.

Oscillations of the card are caused either by the airplane's nose swinging to one side which by friction with the interior of the bowl causes the liquid to swirl, moving the card; by a wing dropping which makes the compass card pivot no longer vertical permitting the magnets to dip; or by vibrations transmitted from the engine. To prevent the last, the entire compass is mounted on felt washers.

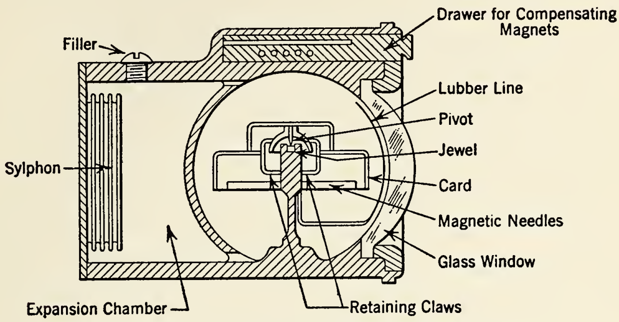


FIG. 109. Compass, magnetic needle type.

Ordinarily compass cards have a regular period for an oscillation and are termed periodic. The period is the time required to swing from an extreme position on one side to an extreme position on the other side and return. Whether the amount of swing is large or small, the period is practically the same. With excessive damping, the periods are not of the same duration; such compasses are termed aperiodic. An aperiodic compass, if disturbed, will require a longer time to return to north than the more common type of compass, but the aperiodic card will not swing so far past north. In a very few swings it will be steady enough to read, whereas an ordinary compass will swing several times before the amplitude has decreased to such a small amount that it may be read. As yet, aperiodic compasses have not been so built as to be adaptable for instrument-board mounting; they serve well as master compasses to be placed back in the cabin, away from magnetic material, to check the pilot's steering compass.

Steering entirely by compass is very tiring to the pilot, and for this reason the compass should be so mounted as to be as nearly as possible in the pilot's ordinary line of vision. The compass should be placed where it will be affected least by local magnetic material such as steel fuselage members, voltage regulators, and guns. Care should be taken when mounting to ensure that the bolts, washers, and nuts are brass or aluminum.

Compasses, Induction Type. The ordinary compass is a direct-reading instrument; the induction-type compass is a distant-reading instrument, that is, the part affected by the earth's magnetism is a considerable distance from the part the pilot reads. The induction-type compass consists of three main parts, a generator, a control-dial, and an indicator. The generator is located in a part of the airplane as free as possible from steel, usually in the extreme rear of the fuselage. The indicator is placed in the cockpit where it can be easily seen by the pilot. The control dial is located conveniently for the avigator, or for the pilot if he is functioning also as avigator.

The induction compass depends on the principle that, if magnetic lines of force are cut by a moving wire, a difference in electric potential is created at the ends of the wire. If the wire moves parallel to the magnetic lines of force, no lines being cut, there will be no difference of potential. If a coil of wire is connected to a galvanometer and the coil is rotated about a vertical axis, at the instant that the coil is in the magnetic north-south plane, the galvanometer will indicate a maximum current, but at the instant that the coil is in the magnetic east-west plane the galvanometer will indicate zero current. Instead of a single coil, one may conceive of a number of coils in different planes all wrapped around a cylindrical core with the ends of each separate coil terminating in commutator segments, and a pair of brushes connecting with the galvanometer. As long as the brushes are in the magnetic east-west plane, they are connected through the commutator with a coil just as it is *not* cutting any lines of force. If the brushes are not in the east-west plane, they will be connecting with a coil when it is cutting the magnetic lines of force and therefore a current will be indicated on the galvanometer. A zero reading on the galvanometer would show that the brushes were in the magnetic east-west plane.

The brushes are mounted on a turntable which may be rotated by the control dial. With the elementary type described above, when it is desired to fly magnetic north, the brushes are set exactly thwartship (perpendicular to the longitudinal axis of the airplane). As long as the airplane is headed due magnetic north, the galvanometer will read zero. If the airplane's heading changes by even a slight amount to one side, the brushes will no longer be in the east-west plane and the galvanometer will be deflected to one side. If it is desired to fly, say, a 30° magnetic heading, the brushes are turned through a 30° angle counter-clockwise viewed from above. With the brushes in this position, right rudder is given till the galvanometer reads zero, which will be when the airplane is headed 30° magnetic. Veering to one side or other from this heading will cause corresponding deflections of the needle.

In the induction compass as actually constructed, instead of a number of separate coils, the armature is drum-wound, greatly adding to the sensitivity of the instrument. The functioning is the same as described above except that the brush positions are shifted through an angle of 90°. The vertical armature shaft projects up through the top of the fuselage, and on the upper end is a paddle-wheel or impeller which is caused to revolve by the slipstream. The axis of the armature coils must be truly vertical at all times; therefore, since the airplane may be flying at different angles of attack, a universal joint is necessary between the upper and lower ends of the armature shaft (see Fig. 110).

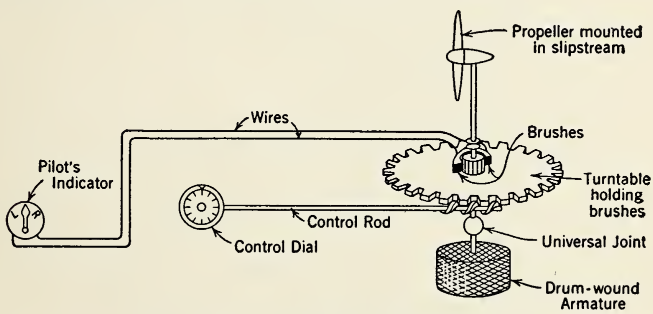


Fig. 110. Compass, induction type.

The induction compass is a magnetic compass, and corrections must be made for variation in order to obtain true directions. The rotor or armature can usually be so located that deviation is negligible, but if deviation is appreciable, compensation can be made by strapping permanent magnets to the vertical shaft-housing of the rotor.

The induction-type compass will read zero either when the airplane is on the desired heading or when it is 180° off from the desired heading. When the airplane is on its correct heading a shift of the heading to the right will make the galvanometer needle deflect to the right, and a shift of heading to the left will make the needle deflect to the left. When the airplane is 180° off from correct heading, a shift of heading to the right will make the needle deflect to the left, and vice versa.

One of the chief advantages of the induction-type compass over the ordinary type is that with the ordinary type on long flights the pilot must constantly bear in mind the heading that he must fly. With the induction type, after the control dial has once been set, the pilot has only to fly so as to keep the needle of the galvanometer always on zero.

Compass, Magneto Type. Because of the weight of the induction compass (14 to 15 lb. for total installation) and because of trouble with failure of the universal joint, the General Electric Company has brought out a modification called the magneto compass, weighing less than 9 lb. In line, one on each side of the armature are two bars or pole pieces of Permalloy, a special alloy of iron which is extraordinarily permeable to magnetic lines of force. If the bars were placed in a north-south plane there would be a very strong magnetic flux across the poles. The poles, however, are so placed by the control dial that, whenever the airplane is on its proper heading, the poles are in the east-west plane. There is then no magnetic flux across the space between the poles where the armature is turning. If the heading is changed even slightly, there will be a magnetic flux cut by the armature coils and a current will be shown on the galvanometer. Slip-rings are used instead of an armature. The armature shaft is rigid through its entire length, no universal joint being necessary. The pole pieces are mounted on a heavy pendulum so that the poles are at all times in a horizontal position.

Directional Gyro. A true gyroscopic compass such as installed on marine vessels is inapplicable to airplane use. Necessarily the rotating mass must be heavy, making the total installation very weighty. On the *Graf Zeppelin,* the only aircraft on which a gyroscopic compass has been tried, the installation was more than 300 lb. in weight. A gyroscopic compass is a north-seeking device in that a weight is suspended by a yoke on the gyroscope shaft so that, if the shaft is ever off from a true north-south line, the weight, in conjunction with the earth's rotation, will cause precession back to a true north indication.

The directional gyro is not a gyroscopic compass in that it has no directive force to make it seek a north heading. It is set by reference to a magnetic or radio compass. After being set, the directional gyro will remain on heading for several minutes, the exact time depending on the bumpiness of the air and the amount of maneuvering of the airplane. The ordinary compass is very difficult to steer by. The card of the magnetic-needle type is constantly swinging from side to side; in extreme cases the card may spin completely around. With the induction type, the pointer of the indicator, except on extremely calm days, is flickering from side to side. The indications of the directional gyro are remarkably steady and consequently easy to stear by.

The operation of the directional gyro may be said to depend on the law of gyroscopic inertia. The energy stored up by the rotation of the gyro wheel tends to keep the axis of rotation in its original position or in a position parallel to its original position.

The directional gyro is in a case which is airtight except for two openings. One opening is connected either to the throat of a Venturi tube located in the slipstream or to an engine-driven pump,

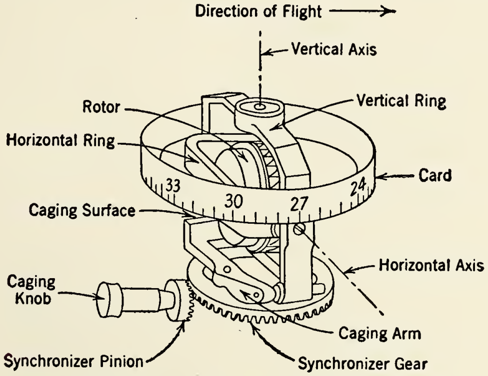


Fig. 111. Directional gyro.

causing a partial vacuum ($3\frac{1}{2}$ in.) inside the case. The other opening is to the outer air but it is so arranged that entering air passes through nozzles to strike against buckets cut in the rim of the gyro rotor, causing the rotor to spin at a speed of 10,000 to 12,000 r.p.m. The rotor wheel weighs 11 oz. Its horizontal shaft is held in bearings in a horizontal gimbal ring. This horizontal ring is pivoted and free to turn about a horizontal axis in the plane of rotation of the gyro rotor.

A view of the directional gyro is shown in Fig. 111. Ordinarily the synchronizer pinion is not in mesh with the synchronizer gear. To effect resetting, the caging knob, which projects outside the front of the case, is pushed inward, accomplishing two things. It engages the bevel pinion with the synchronizer bevel gear so that, by turning the caging knob, the vertical ring is rotated about its vertical axis. Pushing in the caging knob also operates a cam, not shown in the figure, which raises the caging arms. The free ends of the caging arms are united by a cross-bar, the upper surface of which bears against the lower surface of the horizontal gimbal ring, locking it in a horizontal position when the caging arms are raised.

The nozzles, not shown in the figure, are two in number. They are pointed upward, shooting the air up in two parallel, vertical jets. They aid in keeping the rotor shaft horizontal in that, if the shaft is tipped, the nozzle on the high side strikes the side of the buckets, while the nozzle on the low side strikes the face of the buckets in an off-center position, causing a righting moment.

Turn Indicators. The turn indicator was introduced shortly after the World War to overcome the difficulty in flying straight due to the inability of the magnetic compass to detect small turns or turns of short duration. The turn indicator does not indicate any particular direction or heading. If the pointer is on zero it shows that the airplane is not turning about a vertical axis; if the pointer is off zero it shows that the airplane is turning. After the turn has stopped the pointer returns to its zero position.

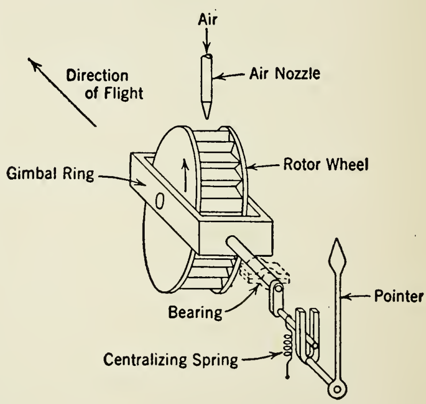


Fig. 112. Turn indicator.

The turn indicator is gyrostatic in principle. The rotating wheel turns about an axis which is perpendicular to the fore-and-aft axis of the airplane and is ordinarily horizontal. The shaft of the rotating wheel is supported at each end in a bearing in the horizontal gimbal ring. This horizontal ring is suspended by pivots at the extremities of the fore-and-aft diameter of the ring;

see Fig. 112. The gyrostat of the turn indicator has therefore only one degree of freedom, the axle may turn about an axis which is parallel to the longitudinal axis of the airplane. This is different from the directional gyro in which the gyroscope has the further freedom of rotation about a vertical axis so that the axle may point in any direction and at any vertical angle. In the turn indicator, the gimbal ring is ordinarily held in a horizontal position by a centrallizing spring. The gyrostat wheel has buckets on its rim. Air is removed from the case by a Venturi tube in the slipstream or by an engine-driven pump, the atmospheric pressure forcing air to enter through a nozzle. The entering air strikes the buckets causing the wheel to spin at approximately 9,000 r.p.m. The direction of rotation is such that the upper rim of the wheel goes in the direction towards the nose of the airplane.

In operation, a horizontal turn of the airplane turns the axle of the gyro wheel. The combination of this turning force and the inertia force of rotation causes the gyro to precess. A right turn of the aircraft causes the right end of the gyro axle to move up or to make the gimbal ring rotate in a counter-clockwise manner as viewed from the pilot's seat. A left turn of the airplane would cause precession to move the right end of the gyro axle downward. The gimbal ring has a projecting lug which moves in a slotted arm connected to the pointer, so that the pointer moves to the same side as does the bottom of the gyro wheel in precessing.

It must be emphasized that the turn indicator shows turns about the apparent vertical or Z axis of the airplane and not about the true vertical in space. It shows both properly banked or skidded turns. In a banked turn, since the tipping of the spin axis of the gyro is opposite to the bank, the instrument is very sensitive. In a sideslip, where the airplane is banked without turning, the turn indicator registers zero. Likewise, in a vertically banked turn where the apparent vertical or Z axis of the airplane is actually horizontal, the turn indicator shows zero. The name " turn indicator " is incorrect; the instrument should be called a yaw indicator since it shows yaw, not actual turn.

Bank Indicators. The banking indicator is a lateral inclinometer. Its purpose is in straight flight to indicate if the airplane is laterally level and in turning flight to indicate if the airplane is properly banked.

In a turn, centrifugal force combines with gravity to form a resultant force which acts on every part of the airplane in the same direction. A pendulum or spirit level indicates an apparent vertical which is the direction of this resultant force. A spirit level mounted on the instrument board, laterally with respect to the airplane, may serve as a bank indicator. In straight flight with wings level, the bubble is in the central highest part of the glass tube; if one wing is low the bubble goes to the high side. In a properly banked turn, the bubble is in the center of the tube. In skidding, where the outer wing is too low, centrifugal force throws the liquid to the outer end of the tube, making the bubble appear at the inner (or too high) end. In slipping, where the inner wing is too low, the liquid goes to the inner or low end, the bubble appears at the high end.

Because with a spirit level a bump will cause the fluid to surge back and forth, and there being no way of damping this bubble movement, the preferred bank indicator is the ball-in-glass-tube type. In this type the glass tube is curved downward at the middle; the steel ball has almost as large a diameter as the inner diameter of the tube and thus has a dashpot action with respect to the liquid in the tube. The fluid is usually a mixture of alcohol and glycerine. By painting the outer back surface of the tube with luminous paint, this type works nicely in night-flying, the ball standing out as a black disk against the luminescent background.

With the ball-in-tube type, the ball goes towards the end of the tube that is too low. If in straight flight the wings are level, or if in turning flight the wings are banked the correct angle, the ball remains in the center of the tube. To correct the airplane, the pilot needs to remember only the simple rule, to move the stick in the same direction that he would move the ball in bringing it to the center of the tube.

Usually the bank indicator and turn indicator are in the same case, the combination instrument being called the bank-and-turn indicator.

Pitch Indicators. The pitch indicator is a fore-and-aft inclinometer to tell whether or not the aircraft is flying level longitudinally. With constant revolutions per minute the airspeed indicator can be used for this purpose, a decreased airspeed meaning climb, an increased airspeed meaning dive, and an unchanged airspeed denoting level flight. The increased airspeed comes, however, after the nose has gone down, and the decreased airspeed comes after the nose has gone up. To eliminate this slight lag which tends to cause overcontrol, and also because in cold weather there is a danger of the Pitot tube becoming clogged with ice, it is desirable to have an instrument that gives directly the angular attitude of the airplane with respect to the horizontal.

The problem of finding longitudinal inclination is extremely difficult because of the effects of accelerations. A spirit level resting on a horizontal table top shows the bubble in the center of the glass. If the spirit level is shoved along the table as the movement is started, owing to inertia the liquid surges backward and the bubble moves forward in the direction the spirit level is being moved. When constant motion is attained, the bubble becomes centered again. When the movement is stopped, the liquid surges onward so that the bubble goes to the rear of the glass. A pendulum hanging in a moving vehicle hangs vertical when the vehicle is not moving or is at a constant speed, swings backward when the vehicle is accelerated and forward under the effect of a deceleration.

If a plumb-bob is suspended at the side of an airplane against a circular scale, it will hang vertically when the speed of the airplane is constant. Suddenly opening the engine throttle to increase the airspeed would cause the plumb-bob to swing backward, and momentarily at least the pilot looking at the reference scale would be led to believe that he had nosed up. If the nose of the airplane drops 10° after the airplane has attained a constant speed consistent with his throttle opening and this angle of dive, the plumb-bob on the side of the cockpit will indicate the 10° downward angle. As the nose goes down, however, the speed increases, and this increase of speed may be at such a rate that the plumb-bob may actually be thrown backward, indicating to the pilot a climb. If the nose of the airplane goes up, the decrease in speed may cause the plumb-bob to swing forward and it will remain forward of its correct position till the lessened speed is constant. A spirit level mounted on the side of an airplane will give wrong indications similar to that of the plumb-bob. Neither a simple pendulum nor a simple spirit level will serve as a longitudinal inclinometer.

A solution that works very effectively is a gyro-controlled pendulum, as shown in Fig. 113. A short-period pendulum is connected by a link to the gimbal ring of a gyrostat. The gyro wheel rotates about an axis ordinarily parallel to the longitudinal axis of the airplane in a direction which is counter-clockwise viewed from the pilot's seat. The axle of the wheel is supported at each end in bearings in a vertical gimbal ring. This ring ordinarily is in a vertical plane parallel to the longitudinal axis of the aircraft, but it may rotate about a vertical axis. A dashpot, not shown in the figure, prevents the mechanism from oscillating back and forth.

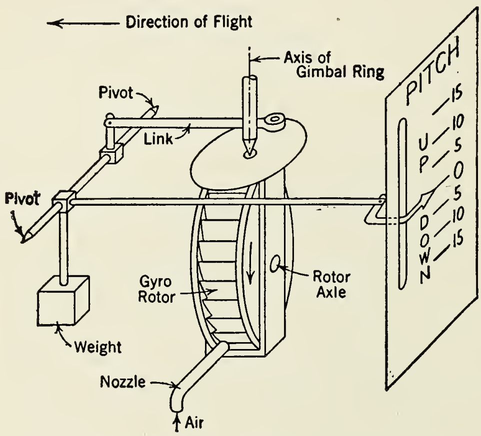


Fig. 113. Pitch indicator.

If the airplane noses up, during the time that the nosing-up is taking place, the clockwise turning of the airplane about its Y axis combined with the inertia force of rotation of the rotor causes the gyro to precess in a counter-clockwise direction viewed from above. This motion causes the link connecting the gimbal with the pendulum to move forward away from the pilot. This tends to move the pendulum backward towards the pilot and show an " up " indication on the dial. During the time that nosing-up is taking place, the speed of the airplane is being slowed down. This deceleration tends to throw the pendulum forward. The precessional force is greater than the decelerating force so that the pointer is moved in the proper direction. When the airplane has assumed a steady upward angle, there are no decelerating or precessional forces and the pendulum determines the setting of the pointer.

In nosing-down, exactly the opposite reactions take place.

The speed acceleration tends to throw the pendulum backward, but the greater precessional force moves the pendulum forward.

When the pitch indicator, bank indicator, and turn indicator are all mounted in one case, the combination is called the flight indicator.

Gyro-Horizon. Although the flight indicator gives the desired information, it is generally conceded that it is easier for the pilot

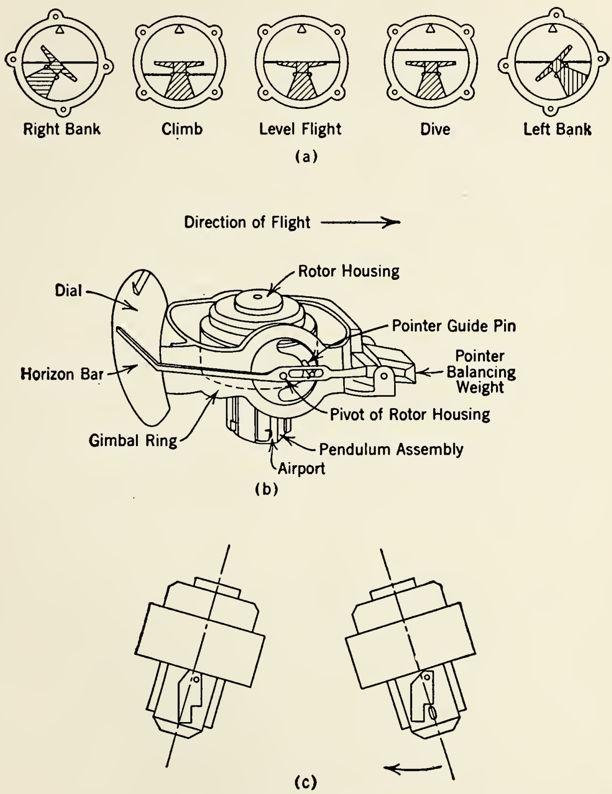


Fig. 114. Gyro-horizon.

to visualize the attitude of the airplane from a miniature replica of the airplane and the horizon than from reading two separate and distinct arbitrary scales. The gyro-horizon gives the angle of pitch and the true angle of bank. On the face of the instrument is a small representation of the airplane, and back of this moves a bar representing the horizon (see Fig. 114).

The rotor in the gyro-horizon rotates about what is ordinarily a vertical axis, in a counter-clockwise direction viewed from above. The rotor is encased in a housing which is swung in a gimbal ring, so that the housing may rotate in a vertical plane parallel with the longitudinal axis of the airplane. The gimbal ring is swung on pivots so that the gimbal may rotate on an axis parallel with the longitudinal axis of the airplane. The front pivot of the gimbal and the left pivot of the rotor housing are so constructed that outside air may enter through the gimbal pivot and pass through a small channel inside the gimbal, then through the gimbal pivot to the interior of the rotor housing. With this construction, no matter what the angular position of the gimbal or the housing, outside air will be able to enter the housing. After entering the housing, the air divides and goes through two passages to nozzle openings on diametrically opposite sides of the rotor. After doing the work of causing the rotor to spin, the air leaves by four ports at the bottom of the rotor housing.

A long arm hinged to the front of the gimbal ring terminates in a " horizon " bar across the face of the instrument. This bar is always parallel to the plane of the gimbal ring. A pin fastened to the rotor housing projects through a slot in the gimbal to actuate the arm of the horizon bar, moving the bar up when the upper part of the rotor housing is moved backward in the case towards the pilot and down when the rotor housing has an opposite relative motion.

At the bottom of the rotor housing are four air-ports or openings, set 90° apart, so that two are on the fore-and-aft axis of the airplane and two on the lateral axis. At each opening is a vane, hung pendulously, so that, when the rotor housing is exactly vertical, each vane covers half of its corresponding opening. An equal quantity of air then issues from each opening. If the gyro departs from its upright position, gravity holding the vanes vertical, one vane completely closes its port as shown in the lower left-hand diagram in figure, while the opposite vane completely opens its port as shown in the lower right-hand diagram. Air streaming out in a jet from an opening when none is issuing from the opposite side causes a reaction tending to swing the bottom of the gyro housing in the opposite direction from the side from which the jet is issuing. This force combined with the inertia force of the gyro rotation causes precession in a direction at right angles to the air reaction which would be in the direction shown by the arrow in the lower right-hand diagram. This corrective movement brings the gyro back to its normal position. Any tendency of the gyro to depart from its true vertical position, caused by acceleration forces or by friction in the bearings, is thus corrected. Although the pilot apparently sees a movement of the horizon bar over the face of the instrument, the horizon bar is actually stationary, and it is the miniature airplane, as well as the real plane, that is moving relatively to the bar.

Rate-of-Climb Indicators. Although pitch indicators and artificial horizons show the attitude of the plane with respect to the horizontal, an airplane can have a constant angle with the horizontal and be either climbing, settling, or flying level, depending on whether the plane has excess speed, insufficient speed, or the correct speed corresponding to the angle of attack. A sensitive altimeter will tell a pilot whether he is going up or down but will not tell him directly the rate at which he is gaining or losing altitude.

In climbing, there is one angle of attack and one airspeed at which the rate of climb is greatest; the airspeed for maximum rate of climb varies with the load on the airplane, the angle of attack bears no simple relation to the angle of the longitudinal axis with the horizontal plane. In gliding, there is one angle of attack and one airspeed for the flattest angle of glide, and this airspeed varies with the load.

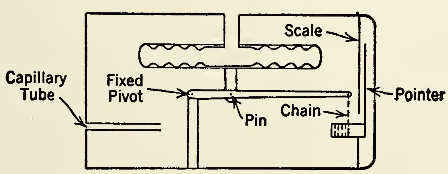


Fig. 115. Rate-of-climb indicator.

For these reasons an instrument giving directly the rate of climb, either positive or negative, is very useful in fog flying and in flight testing. The most satisfactory rate-of-climb indicator is the capillary-leak type, sometimes called the " leaky altimeter " type. An aneroid is mounted in a case with a large opening from the interior of the aneroid box to the outside air. In the case, outside of the aneroid box, is an opening to the outside air through a long, very fine capillary glass tube (see Fig. 115). As long as the aircraft stays at the same altitude, the air in the case outside the aneroid is at the same pressure as that in the interior of the aneroid. When the altitude is changed, the air inside the aneroid because of the large opening instantly assumes the air pressure corresponding to the new altitude, but some time must elapse before sufficient air can escape or enter by the capillary tube for the air in the case outside the aneroid to attain the pressure corresponding to the new altitude.

In climbing, the air inside the aneroid is at less pressure than the air outside, and so the sidewalls contract. This motion is transmitted and magnified by a lever arm and chain to a spring roller to which the pointer is attached. In a glide the air inside the aneroid is at greater pressure than the air outside, so that the side walls expand, giving a reverse motion to the pointer. Flow through a simple orifice depends on density, but flow through a capillary tube is within wide limits practically independent of density and depends only on the viscosity. An instrument, such as described above, can therefore be graduated in rate of climb, and it will retain its calibration at all altitudes.

Drift Indicators. Whenever there is a cross-wind, an aircraft does not travel over the ground in the same direction as the aircraft is headed. The actual path of the aircraft over the ground is called the track. The angle between the aircraft heading and the track is called the drift angle or simply the drift.

To ascertain the drift, it is necessary to see objects on the ground. If the ground objects that are seen directly ahead of the aircraft pass directly below the aircraft and then appear directly astern there is no drift. If the objects as they are passing to the rear appear to be moving over to one side, the airplane is said to drift. To be able to estimate the angle of drift, devices are provided through which the operator sights on objects on the ground. The basic principle of all drift indicators is essentially the same. A bar or wire on the outside of the cockpit or a glass plate with ruled parallel lines set in the floor of the cockpit is so arranged that the bar, wire, or glass plate is rotatable in a horizontal plane. The wire or lines are turned until, to the observer in the airplane, objects on the ground appear to be moving backward along the wire. If the airplane is rolling or pitching, the objects will describe an erratic path as viewed from the airplane.

The observer must mentally average out these deviations from a straight path and so set his wire that the object appears to move equally on either side of the wire.

For over-water flights, smoke bombs by day and flares by night are dropped overboard from the airplane. On the horizontal tail surface are painted lines radiating from a sighting point in the roof of the cabin, where the observer can place his eye. The central line runs along the center line of the fuselage; the other lines are 5° apart. After dropping over the bomb or flare, the observer waits till he sights the smoke or light over the tail.

Gyropilot. The gyropilot is sometimes called the automatic pilot or robot pilot. Thus relieved of the burden of constantly flying the airplane, the human pilot may attend to navigation, radio, and engine control. The human pilot can engage or disengage the automatic device instantly for landing, take-off, or other maneuvers.

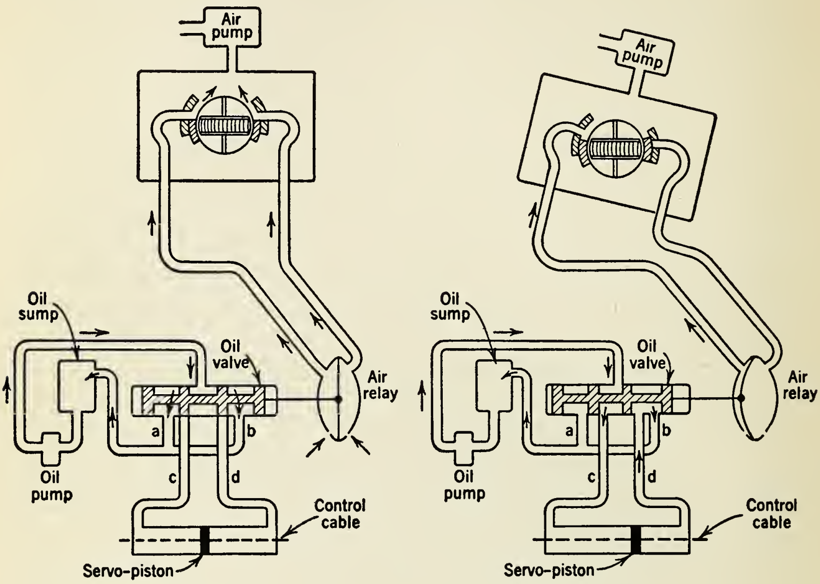
The control mechanism of the gyropilot is a combination of the directional gyro and the gyro-horizon, both of which are described earlier in this chapter. By attaching a pick-up mechanism to these instruments, any deviation from the desired path of the airplane will cause the controls of the airplane to be so operated as to bring the airplane back to its predetermined path.

There are three sets of pick-up devices, one on the directional gyro and two on the gyro-horizon. The device is essentially as follows. Leading from the air-relay to the air-tight box containing the gyroscope are two tubes whose ends are normally open, as shown in Fig. 116*a*. When the position of the airplane is disturbed as in Fig. 116*b*, the gyroscope will tend to remain in its original position and the end of one tube will be closed while the other will be left open.

The air-relay is a small box divided into halves by a thin diaphragm. On each side is a small air inlet and a tube connecting with the gyroscope box. The air inlet openings are smaller than the tubes to the gyro box.

When the gyroscope is in its central position there is equal suction on each side of the diaphragm and it stays in its mid-position. When the airplane tilts or turns, so that there is suction through only one of the two tubes, the diaphragm of the air-relay is moved over to one side as shown in Fig. 116*b*.

Fastened to the center of the air-relay diaphragm is a piston-rod which has on its other end a balanced oil valve. This slide valve is on an oil circulating system. When the valve is in its mid-position, the oil-pump forces oil into the valve and it returns through pipes *a* and *b* to the oil sump.



(*a*) (*b*)

FIG. 116. Mechanism of a gyropilot.

When the valve is to one side as in Fig. 116*b*, the oil from the pump passes through the valve to pipe *c* leading to one side of the servo-piston. From the other side of the servo-piston, oil passes through pipe *d*, the oil valve, and pipe *b* to the sump.

There are three air-relays, three oil valves, and three servo-pistons. The movement of the servo-piston is transmitted to the rudder, elevator, or ailerons.

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CHAPTER XIX

METEOROLOGY

Introduction. The ancients believed that the presence of meteors in our atmosphere was largely responsible for weather changes and therefore gave the name of meteorology to the art or science of weather forecasting. This conception has long since been proved false, but the old name of meteorology is still retained. The United States Navy has adopted the term aerology. It is more appropriate and less tongue-twisting and is being used more and more by aviators.

Meteorology is an important adjunct to aviation. From a standpoint of safety, a study of the meteorological conditions should give warning of flying hazards such as storms or fog. When flying over clouds, consideration of certain meteorological factors should enable the pilot to estimate, at least in a general way, his drift. Pilots of free balloons can judge at what altitude to travel to secure favorable winds. Dirigibles can alter their course in order to secure suitable weather conditions.

The United States Weather Bureau was started many years ago as a division of the Department of Agriculture. Its predictions were primarily for the farmer, to warn him in a general way of impending freezing weather or of rain. In the early days, the observations were taken only at ground level or on the roofs of tall buildings.

With increasing knowledge of the technique of weather forecasting and the growth of flying demanding a more exact and thorough prediction, in recent years there has been improvement in the precision and minuteness of weather prediction. Today, in addition to ground observations made twice daily at some 250 stations in the United States, upper-air measurements of temperature, pressure, and humidity are made daily by the Weather Bureau at seven stations, by the Naval Air Service at seven, and by the Army Air Corps at eight. One important airline has its own staff of meteorologists to supplement the governmental agencies. All these agencies interchange data, and in addition to the Weather

Bureau system of intercommunication, the Department of Commerce maintains a teletype system at the principal airports.

The upper-air measurements are made by means of a meteorograph carried aloft by an airplane or by small unmanned free balloons called sounding balloons. The meteorograph is a small combination instrument containing a clockwork-operated smoked drum on which are recorded temperature, pressure, and humidity. The temperature-indicating unit is a bimetallic strip of dissimilar metals which bends on heating. The pressure-indicating unit is a tiny aneroid. The humidity indicator consists of a number of animal hairs stretched over an elastic framework.

Temperature. The heat radiating from the sun passes first through our atmosphere before reaching the earth. Dry air absorbs practically no heat from the sun, but moist air does absorb it. As half of the air is below 22,000 ft. and the air above that altitude has scarcely any moisture content, practically all the heat absorption by the air occurs at the lower altitudes.

The sun's radiation not absorbed by the atmosphere strikes the earth's surface. Part of the radiation is absorbed and part reflected or re-radiated, the exact proportion depending on whether the surface is rock, field, trees, ice, etc. Dark surfaces, like plowed fields, absorb more heat and are at a slightly greater temperature than reflecting surfaces such as green fields. The air in contact with the ground is heated by direct conduction, and this air heats air with which it is in contact by direct conduction. Also warm air rises, so that warm air may be found at considerable altitude. These rising currents of warm air are called convection currents, and the presence of warm air at altitude from this cause is said to be due to convection.

When a mass of air at a given temperature is compressed adiabatically, that is, without the addition or subtraction of heat, the temperature of the air increases. Conversely, when a mass of air is allowed to expand without heat being added or heat being allowed to escape, the temperature drops. The heat conductivity of atmospheric air being very low, if a mass of dry air moves from one level to a higher level, since the pressure is less at the higher altitude, the air will expand very nearly adiabatically and consequently drop in temperature. This drop will be approximately 1° F. for every 185-ft. increase in altitude. On the other hand, air moving downward 185 ft. should increase 1° in temperature. Actually other factors affect the heating and cooling so that the change in temperature is usually less than 1° for 185-ft. altitude difference.

Both for the reason explained in the last paragraph and because the heating effect of the earth is less with higher altitude, the temperature decreases as one rises above the earth. The rate at which temperature decreases with altitude is called the temperature gradient. The temperature continually decreases until the stratosphere is reached. In the stratosphere, the temperature is presumed to be constant.

At night, the ground cools quickly so that the air adjacent to the ground drops in temperature. When this occurs, the air gets warmer as one ascends a short distance. This is termed reverse temperature gradient. At a comparatively low altitude, this reversal stops and the temperature decreases as further ascent is made.

On weather maps, lines, usually dotted ones, are drawn connecting geographic points that have the same ground temperature. These lines are called isotherms.

Humidity. Warm air can hold more water vapor in suspension than cold air can. The amount of saturation of air with water vapor is called the humidity. Absolute humidity is the actual amount of water vapor present in a cubic inch or cubic centimeter of air. Relative humidity is the ratio of the mass of moisture present in the air to the amount required for complete saturation.

When warm air with high relative humidity is chilled, the absolute humidity may remain the same, but since at lower temperature less moisture is required to saturate the air completely, in lowering the temperature, the relative humidity increases. If the relative humidity increases to more than 100 per cent, precipitation will take place.

Pressure. Atmospheric pressure is due to the weight of the column of air extending from the point of measurement upward to the limit of our atmosphere. Atmospheric pressure is usually given in terms of the height of a mercury column that would be supported by the atmosphere. Even though glass tubes containing mercury are rarely used now except as basic standards, aneroid barometers are marked off in units of height of mercury columns. Standard atmospheric pressure is 29.92 in. or 760 mm.

A unit only used in meteorological work is the millibar (1,000 millibars = 750 mm. of mercury = 29.53 in. of mercury).

On weather maps, lines are drawn connecting places that have the same equivalent sea-level barometric pressure. These lines are called isobars.

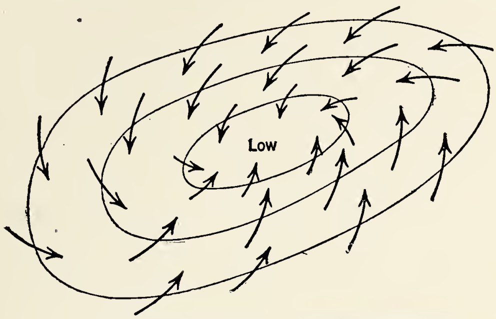
Warm air expands and its density decreases. This would tend to make the barometric pressure less over heated areas and greater over surrounding cooler areas.

Considering the earth as a whole, as the equatorial regions receive nearly three times the amount of heat received by the polar regions, it might be expected that there would be a region of low barometric pressure in the tropics and of high barometric pressure near the poles. This is true in so far that there is low pressure in the tropics, but owing to the earth's rotation, centrifugal force draws the air away from the poles. The resulting effect is to have in general a comparatively high-pressure band around the earth at about 30° latitude and low-pressure areas at the equator and around the poles.

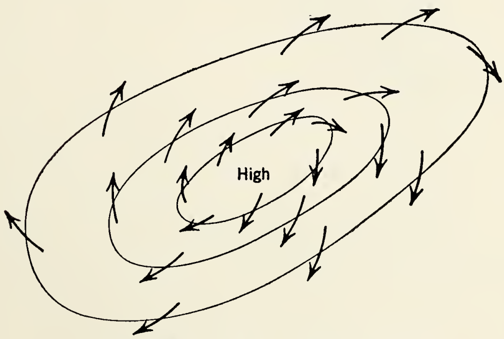
Winds. Air, like any other fluid, travels from a region of high pressure to one of low pressure. A large difference in barometric pressure at two places a short distance apart geographically will cause a strong air movement or wind. A small pressure difference will cause a correspondingly weaker wind. On American weather maps, isobars are drawn for 0.1-in. pressure difference. Where these isobars are close together, there are strong winds; where there are big spaces between the isobars, the winds are very mild.

Because air travels from high-pressure to low-pressure areas, it might be expected that, in the northern hemisphere, at the southern edge of the temperate zone there would be north winds and at the northern edge of the temperate zone there would be south winds. These north and south winds are modified by the earth's rotation as follows. The earth is rotating from west to east. "Stationary" air on the equator actually is moving eastward with a velocity greater than 1,000 miles per hour. Air farther from the equator has less eastward velocity. At any latitude, the ground and "stationary" air has less eastward velocity than the ground and "stationary" air at less latitude. Air tending to move directly northward in the northern hemisphere, owing to its greater eastward velocity, becomes a southwest wind. Air tending to move southward is moving into a region which has a greater eastward velocity, so that the north wind becomes a northeast wind. This is true whether the air movement is on a large or small scale.

When, from local heating or other causes, there is a low-pressure area, usually called simply a "low," in the northern hemisphere, air instead of moving radially inward deviates to the right, so that there is a counter-clockwise rotation of wind about a low, as



(a) Wind Direction in Low, Northern Hemisphere



(b) Wind Direction in High, Northern Hemisphere

FIG. 117. Wind directions (*a*) in low, (*b*) in high.

shown in Fig. 117*a*. About a high-pressure area, or high, the air instead of moving radially outward at right angles to the isobars veers to the right as shown in Fig. 117*b*.

Weather Maps. Areas of high and low barometric pressure are constantly moving across the United States from west to east. A rapid movement and constant succession of highs and lows implies frequent changes in the weather. A slow movement portends a continuation of the present weather.

Although a study of a daily weather map gives a certain amount of information, it is always advisable to study, in conjunction with it, the maps of the two previous days. By doing this, the paths of the disturbances can be visualized and the probable future positions of the disturbances predicted. For special reasons disturbances sometimes appear seemingly spontaneously; but the highs and the lows are usually noted first on the west coast and they follow more or less well-defined paths across the United States.

Lows. In meteorology, the term " cyclone " is synonymous with " low." In some localities, owing to distinct temperature differences from adjacent regions which maintain themselves for long periods, there are lows which are more or less stationary or semi-permanent. Mostly, however, the lows move eastward at a rate of about 25 miles per hour in the summer and about 40 miles per hour in the winter.

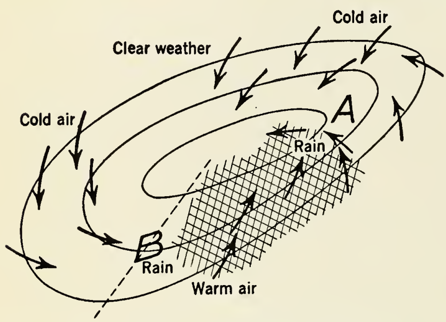


FIG. 117*c*. Weather in a low.

Examining Fig. 117*c*, it will be seen that, if a line is drawn through the center of the low in a northeast to southwest direction as shown by the dotted line, winds on the southeast side of this line come from a southerly direction while those on the other side have a component from the north. Winds from the south bring warm air northward. Winds from the north bring cold air southward.

When the warm air meets the cooler air, approximately in the area marked *A* in Fig. 117*c*, the warm air being of lighter density rises and flows over the cold air. The mass of cold air meeting the warmer air at *B*, Fig. 117*c*, forces its way under the warm air. The leading surface of a mass of air in motion is called a " front "; that at *A* is a warm front, that at *B* a cold front. Fronts are not sharp planes of discontinuity between warm and cold air. Wherever a mass of air at one temperature meets a mass at a different temperature, there is always turbulence and a mixing of warm and cold air. A front may be many miles in length.

In the northeast section of the low, the rising warm front decreases in temperature both because of expansion and because of mixture with the colder air. If the warm air contained moisture, the cooling will result in the water vapor condensing out to form clouds. A continuation of the cooling will cause precipitation in the form of rain or snow. The ascension of warm air is gradual as the air moves northward, so the rainy area may be quite extensive.

In the southwest section of the low, the cold front is much more sharply defined. The change in temperature is quite sudden, and the precipitation is much more intense. The turbulence along the cold front is very intense, and the air is quite " bumpy " to the pilot.

On the northwest side of a low, the air is cold. Cold air holds little moisture in suspension. Any increase in temperature permits the air to absorb more water vapor, so that in this sector are clearing weather and blue skies.

By watching the way in which smoke drifts away from chimneys or flags flutter, a pilot may often be able to ascertain the wind direction on the ground during the day. Knowing the direction from which the wind blows enables the pilot to estimate the direction of the center of the low, and he can head his airplane so as to avoid the extremely bad weather. A very valuable rule to remember is known as Buys-Ballot's law. It is, briefly, to imagine oneself facing into the wind; the barometric pressure on the right hand is lower than the left, and the center of the low will be on the right-hand about 135° from dead ahead.

Highs. A region of high barometric pressure moves eastward at less speed than a low-pressure area, and a large high-pressure area may be stationary for several days. An exception to this is the case where the high is situated between two lows; when this occurs the high has the same velocity as the lows. As a rule, when a high sets in, settled weather may be expected to last for two or three days. Regions of high pressure are sometimes called anticyclones.

In highs, the isobars have the same oval shape as in lows, but the isobars are farther apart. The pressure gradient being less, the winds are less in strength than those found in a low. A high usually covers more territory, never being as concentrated as the lows. The weather in a high is generally clear, but fog sometimes occurs in the autumn. Rains are always of short duration.

Since a high usually follows a low in progression across the United States, when a high comes into a region, the rain area of the preceding low has traveled onward. With the rise in barometer the weather clears and the temperature drops. At the center of the high a perfect calm is usually found with a cloudless sky or at most a few scattered clouds. Temperature is low at night but rises rapidly during the day.

Winds Aloft. While winds at low altitudes cross the isobars at an angle which is somewhat towards the right from going straight across, as altitudes are increased the angle from a perpendicular crossing becomes greater. At about 8,000-ft. altitude, the winds are generally practically parallel to the isobars. Continuing higher the winds change direction so as to become more nearly west winds. If the wind at lower altitude is southeast it will probably veer through south and southwest to west; if the wind is northeast at lower altitude it will probably back through north and northwest to west.

The velocity increases with altitude up to a height of 20,000 ft. Above this, the wind usually decreases in strength. Roughly, the wind at 5,000-ft. altitude has twice as great velocity as at the ground.

Clouds. The suspension of water vapor in the air is determined mainly by the temperature. When the temperature is high, a large amount of water vapor may exist; when the temperature is low only a small amount of water vapor can be held in suspension. Consequently, if a mass of warm air carrying a high percentage of moisture comes in contact with or moves completely into a cooler region, the air is chilled and the water vapor is condensed to liquid water in the form of minute drops. Particles of dust usually act as nuclei about which this condensation collects, but sometimes a drop of water acts as nucleus. These tiny drops form clouds, and if the condensation continues, these drops grow in size until precipitation as rain or snow results.

Clouds may be formed in several ways, such as when:

(*a*) Warm saturated air blows over cold sea or land, as when warm currents pass over mountains especially if the mountains are snow-covered, or in winter, warm south winds. Clouds thus produced are usually of the stratus type; see below.

(*b*) Rising air expands and in expanding it is cooled, condensing out its moisture. This usually produces cumulus clouds.

(*c*) Air is forced to rise by encountering mountain slopes.

(*d*) Air may part with its heat by radiation on cold nights and in cooling cannot contain so much moisture.

(*e*) Air is forced to rise when a warm front meets the cooler air on the northeast sector of a low.

There are many different types of clouds but, all may be classified under or are a combination of four main subdivisions.

Cirrus. These are thin and wispy. They are composed of ice particles and are found at altitudes of 25,000 to 30,000 ft. Cirrus clouds appear like giant, curling feathery plumes and are sometimes called " mares' tails."

Cumulus. These are detached, fleecy clouds. They appear like big lumps of absorbent cotton or shaving-soap suds. They have a flat base. They are composed of water particles and may be found at altitudes from 5,000 up to 25,000 ft.

Stratus. These are low, flat, spread-out clouds, resembling fog but not resting on the ground, and are in distinct layers. This type often merges into nimbus.

Nimbus. These are thick, extensive layer of formless clouds, black on the under side from which rain or snow is falling.

Fog is cloud resting on or close to the ground. Fog is divided into two classes, radiation fog and advection fog.

Radiation fog forms along rivers, creeks, etc., during still cloudless nights of the summer or autumn. During a calm, warm day, water evaporates into the lower atmosphere of such regions, where it remains since there is no wind. During the night the ground cools; the lower, moisture-laden air drops in temperature, and condensation takes place.

Advection fog forms when warm air drifts slowly over a cold surface. This occurs in winter in front of an advancing low. Advection fog is also formed when a cold wind passes under a mass of warm, damp air such as in winter in front of a high.

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CHAPTER XX

AVIGATION

Introduction. Avigation is the guidance of aircraft along a desired path and the ascertainment of the aircraft's geographic position. The term "avigation" has superseded the older expression "air navigation," which is anomalous since the word navigation refers to travel by water.

The chief advantage of air travel over other means of transportation is the speed in reaching the destination. The two reasons for this speed are: first, aircraft are faster than other means of travel; and second, aircraft can travel the shortest possible path. When aircraft follow railroads, highways, or winding rivers, the only gain is due to the superior speed of the aircraft. By cutting across mountain ranges, etc., to follow the most direct route, the distance to be flown is reduced considerably.

Flying by following a definite landmark, such as a railroad, river, or highway, or by sighting recognizable landmarks ahead immediately after a known landmark has passed astern is a form of avigation; but the term avigation is usually applied to the conducting of an aircraft from place to place and finding position when recognizable landmarks are not continuously in sight. Dead reckoning (contracted from deduced reckoning) is the term applied to the method of finding position by means of the direction and distance flown from the last recognized landmark. It is sometimes called compass flying. When long distances are flown, dead reckoning is liable to considerable error, owing to the effect of the wind not being known accurately, and the position should be checked if possible by astronomic means or by the aid of radio.

Maps and Charts. The earth is an oblate spheroid, the polar diameter of which is 7,900 miles and the equatorial diameter 7,926 miles. For all avigation work the earth is assumed to be a sphere. The earth rotates daily about an axis, the ends of which are the north and south poles. The equator is the circle formed by a plane cutting the earth perpendicularly to the axis and equidistant from the poles. The equator is everywhere 90° distant from each pole. Circles passing through both poles are meridians. Their direction is everywhere north and south. Circles formed by planes cutting through the earth perpendicular to the axis are called parallels of latitude.

The geographic position of any place on the earth may be denoted by its latitude and longitude. The latitude of a place is the length of the arc of the meridian through the place, between the equator and the place. It is measured in degrees and minutes, and is designated north or south according as the place is north or south of the equator. The longitude of a place is the angle between the plane of the meridian through the place and the plane of the meridian through Greenwich, England. The angle is measured in degrees and minutes, and longitude is designated west or east according as the place is west or east of Greenwich.

A circle formed by any plane passing through the center of the earth is called a *great circle.* The shortest path between two places on the earth is along the great circle passing through the two points.

A map or chart represents in miniature, upon a flat surface and according to a definite system of projection, a portion of the earth. Although the two names are used indiscriminately, a chart pictures a portion of the earth the greater part of which is water, whereas a map depicts a portion of the earth's surface which is mostly land.

No spherical surface can be depicted correctly on a flat surface. There can be no accurate maps.

There are four uses for maps: (*a*) to find distances, (*b*) to find directions, (*c*) to recognize natural features, and (*d*) to compare areas. No map is ideal for all these purposes; a map may furnish correct information for one of these uses and approximately correct information for one or two of the others, but it cannot be correct for all four.

The scheme or arrangement on which the parallels of latitude and meridians are laid out is called the system of projection. An infinite number of systems of projection may be thought of. At one time or another several hundred have been used, and at present over three dozen are actually in use in the maps made by various nations and by various agencies and for various purposes. The three most common projections which are widely used are the mercator, the polyconic, and the great circle projections. For areas under 200 square miles the earth may be considered flat; for larger areas the curvature of the earth becomes important. It is important to know the system of projection of the map being used and the limitations and inaccuracies of that system.

Polyconic Projection. The polyconic projection is used in the United States Coast and Geodetic Survey maps, United States Post Office maps, and the airway strip maps issued by the Department of Commerce and Army Air Corps. This projection can usually be identified by being the only one of the three common projections having a scale of miles printed on it. This scale of miles is correct in the central portion of the map but is likely to be slightly inaccurate near the sides of a map of such size as to cover the area of a continent.

It is an ideal projection for areas the size of a single state. It is best suited to maps which are long in the north-south dimension and narrow in the east-west dimension, thus making it a satisfactory projection for maps of the Atlantic or Pacific coast. For maps of the entire United States there is considerable distortion near the seacoasts.

The parallels of latitude are circles of different radii. If the map is large scale, short portions of the circular parallels may be considered as being straight lines. When longer distances are being dealt with, the curvature enters into the problem. On the earth, the parallels of latitude are due east-west lines. On the polyconic maps, since the parallels are represented by circles, east-west lines have a different inclination in different parts of the map; therefore, if a line is drawn between two places on the map a considerable distance apart, the direction of this line cannot be ascertained. The polyconic projection depicts more accurately than other projections the actual surface of the ground so that landmarks, rivers, and shorelines can be recognized readily.

Mercator Projection. The mercator projection is used in the maps issued by the United States Navy Hydrographic Office and is the standard projection for ocean-sailing charts. This projection can be identified by being the only one of the three common projection having the meridians as straight vertical equidistant lines and the parallels of latitude as straight horizontal lines, the degrees of latitude increasing in length farther from the equator. It is the only one of these three projections on which the entire inhabited portion of the earth can be represented on one map.

Since its limit of construction is about 70° latitude, polar regions cannot be shown on the ordinary mercator chart.

On the mercator projection, the meridians and parallels intersect at right angles; north is the same direction in all parts of the map. It is therefore very easy to find the direction of a line drawn on a mercator chart by measuring the angle at which the line crosses a meridian. A straight line drawn on a mercator chart is called a rhumb line.

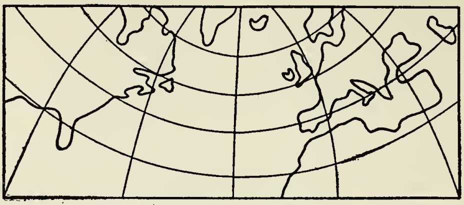
On the sides of mercator charts is a scale of latitude. This scale is not constant. To measure short distances on a mercator chart, the length of a degree of latitude on the side scale, at the average latitude between the two places, is used as measuring unit. This gives the distance in degrees and fractions of a degree of latitude. A degree of latitude is 60 nautical miles in length, a nautical mile being 6,080 ft. long.

A rhumb line is not the shortest path between two points on the earth's surface; it is merely a line which has a constant direction. The shortest path is a great circle, which is shown as a curve on a mercator chart.

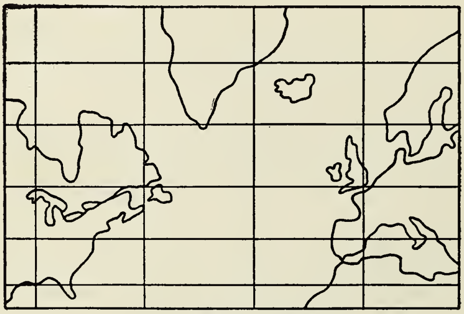
Great-Circle Charts. For the use of seamen, the Hydrographic Office has published five great-circle charts, namely, of the North Atlantic, South Atlantic, North Pacific, South Pacific, and Indian Oceans. These charts have the property that a straight line between two points on the chart represents the great circle through these points on the earth, and shows all the localities through which this most direct route passes. The meridians are straight lines, either parallel as shown in Fig. 118, or converging towards the pole. The parallels of latitude are represented as conic sections.

It is very complicated to measure directions on this projection. The usual procedure, after drawing a straight line on this chart as a great-circle path between two points, is to spot a number of points on this line. By reading the latitude and longitude of each point, they may be transferred to a mercator chart. On the mercator chart, a smooth curve is then drawn through these points. On the great circle, direction is constantly changing. In practice, instead of actually following the great circle, a series of straight tracks are flown approximating the great circle, the heading of the aircraft being changed whenever the direction of the great circle changes a degree.

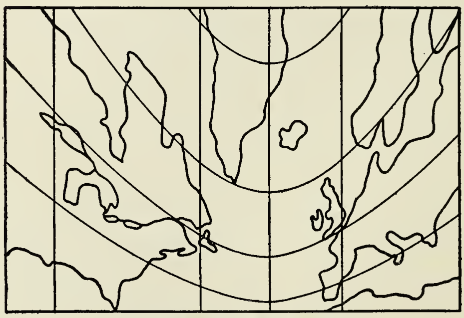
The rules of the Fédération Aéronautique International state that, in making a record for distance flown, the distance from starting to landing point shall be measured along the great-circle route. Directions given by radio direction-finders, such as the



(a) Polyconic Projection



(b) Mercator Projection



(c) A Great Circle Projection

FIG. 118. Common map projections.

radio beacon or the radio compass, are along the great-circle course. For short hops of only a few hundred miles the mileage saved is inappreciable, but for transcontinental or transoceanic flights the distance along the great-circle path is many miles shorter than along the rhumb-line path.

Distances cannot be scaled off directly on a great-circle chart, and though an auxiliary graph is provided, it is quite complicated so that the great-circle distance is usually computed.

Great-Circle Distance. The distance along the arc of the great circle between two points A and B may be found by the following formula:

$$\cos D = \sin L_A \sin L_B + \cos L_A \cos L_B \cos LO_{AB}$$

D is distance
L_A is latitude of point A
L_B is latitude of point B
LO_{AB} is longitude difference between points A and B

Example. Find the great circle distance from Curtiss Field, Long Island (40° 45′ N., 73° 37′ W.), to Le Bourget Field, Paris (48° 50′ N., 2° 20′ E.).

Solution.

$$\begin{aligned} L_A &= 40° \, 45' \text{ N.} \\ L_B &= 48° \, 50' \\ LO_{AB} &= 73° \, 37' + 2° \, 20' = 75° \, 57' \end{aligned}$$

(Note: Since longitudes are of opposite names, the difference is found by adding.)

$$\begin{aligned} \sin 40° \, 45' \sin 48° \, 50' &= 0.6528 \times 0.7528 \\ &= 0.4914 \\ \cos 40° \, 45' \cos 48° \, 50' \cos 75° \, 57' &= 0.7576 \times 0.6582 \times 0.2428 \\ &= 0.1211 \\ \cos D &= 0.4914 + 0.1211 \\ &= 0.6125 \\ D &= 52° \, 14' \end{aligned}$$

On a great circle, a degree is 60 nautical miles and a minute is 1 nautical mile in length.

$$\begin{aligned} \text{Distance } (D) &= 52 \times 60 + 14 \\ &= 3{,}134 \text{ nautical miles} \\ &= 3{,}134 \times \frac{6{,}080}{5{,}280} \text{ or } 3{,}609 \text{ land miles} \end{aligned}$$

Problems

1. Find the distance from Rome (41° 54′ N., 12° 29′ E.) to Natal, Brazil (5° 37′ S., 35° 13′ W.). (Del Prete's flight.)

2. Find the distance from San Francisco (37° 47′ N., 122° 28′ W.) to Honolulu (21° 22′ N., 157° 48′ W.). (Hegenberger's flight.)

3. Find the distance from Cranwell, England (53° 05′ N., 0° 25′ W.), to Walvis Bay, South Africa (22° 43′ S., 14° 20′ W.). (Flight of Gayford and Nicholetts.)

4. Find the distance from Burbank, California (34° 12′ N., 118° 18′ W.), to Floyd Bennett Field, N. Y. (40° 40′ N., 73° 50′ W.). (Flight of Haizlip and others.)

5. Find the distance from Harbor Grace, Newfoundland (47° 43′ N., 53° 08′ W.), to Clifden, Ireland (53° 30′ N., 10° 04′ W.). (Flight of Alcock and Brown.)

Velocity Triangle. The velocity of an aircraft with respect to the ground is the resultant of two velocities: that of the aircraft with respect to the air, and that of the air with respect to the ground, the latter being the wind velocity. Velocity has two characteristics, speed and direction.

The velocity of the aircraft with respect to the air has a speed which is the reading of a properly corrected airspeed indicator and a direction which is the true heading of the aircraft determined from a properly compensated compass reading corrected for variation. The velocity of the aircraft with respect to the ground has a speed which is the ground speed and a direction which is the track or actual path of the aircraft over the ground. Drift is the angle between the heading and the track.

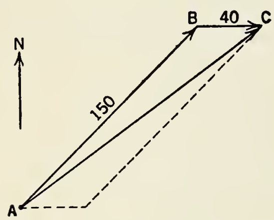


FIG. 119. Velocity triangle, finding ground velocity.

To find the resultant of the air velocity and the wind velocity, use is made of the parallelogram of velocities, the two component velocities being drawn to the same scale and in the proper respective directions as two sides of a parallelogram, and the resulting velocity being its diagonal. By omitting two sides of the parallelogram, the remaining velocity triangle is sufficient to obtain a solution. Care must be taken that the arrows indicating direction of velocity are correctly marked to give continuity. The use of the velocity triangle is illustrated by the following example in conjunction with Fig. 119.

Example. An airplane is heading due northeast with a corrected airspeed of 150 miles per hour. There is a 40-mile-per-hour west wind. What are the ground speed, track, and drift?

Solution (see Fig. 119). Draw line AB, 150 units in length; let direction of AB be northeast. From point B, draw line BC, 40 units in length, direction of BC being east (since a west wind blows towards east). Connect points A and C.

Then the line AC represents ground velocity.

The length of AC being 181 units, the ground speed is 181 miles per hour.

The line AC makes an angle of 54° from north, so that track is 54°.

Angle BAC is 9°, which is drift of 9° right.

Problems

1. An airplane is headed east with an airspeed of 125 miles per hour. There is a 30-mile-per-hour northeast wind. What are ground speed, track, and drift?

2. An airplane is headed 30° with an airspeed of 150 miles per hour. There is a south wind of 25 miles per hour. What are ground speed, track, and drift?

3. An airplane is headed 340° with an airspeed of 140 miles per hour. There is a northeast wind of 20 miles per hour. What are ground speed, track, and drift?

4. An airplane is headed 265° with an airspeed of 100 miles per hour. There is a 35-mile-per-hour wind from 320°. What are ground speed, track, and drift?

5. An airplane is headed 220° with an airspeed of 120 miles per hour. There is a south wind of 30 miles per hour. What are ground speed, track, and drift?

The desired track being known, the aircraft must be headed into the wind from this track in order that the wind in combination with the aircraft's own motion shall carry the aircraft along the desired track. The angle by which the heading is altered from the desired track into the wind is called the angle of crab.

In taking-off on a cross-country flight, if the aircraft is first headed on the desired track and the drift measured, the heading should not be crabbed by the drift angle since when the heading is changed the wind will be at a different angle and consequently there will be a different drift angle. The heading can, of course, be altered in this way to an approximately correct heading, but the drift on the new heading should be measured and the heading re-altered for the difference between this drift and the drift on the former heading. This is called the cut-and-try method.

When the wind speed and direction are known, the proper heading can be found by a velocity triangle. In this form of problem, the airspeed is known but not its direction; the track is known but not the ground speed. Various devices have been made for solving this problem mechanically. The graphic solution is illustrated by the following example.

Example. It is desired to fly northeast. Airspeed is 150 miles per hour. There is a 40-mile-per-hour west wind. What should be the proper heading, and what will be the ground speed?

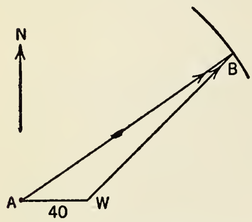


Fig. 120. Velocity triangle, finding heading.

Solution (see Fig. 120). Draw line AW, 40 units in length, in a direction representing due east. Through point A draw line AX of indefinite length, in a direction representing northeast. With W as center and 150 units as radius, strike an arc, intersecting line AX at point B.

Then triangle ABW is the velocity triangle. The length of AB is the ground speed, 176 miles per hour. The direction of line WB, 34°, is the heading.

Problems

1. It is desired to fly due north. Airspeed is 150 miles per hour. The wind is from 80°; wind speed is 40 miles per hour. What should be proper heading, and what will be ground speed?

2. It is desired to fly northeast. Airspeed is 135 miles per hour. The wind is from west; wind speed is 30 miles per hour. What should be heading, and what will be ground speed?

3. It is desired to fly 5°. Airspeed is 100 miles per hour. Wind is from 300°; wind speed is 40 miles per hour. What should be heading, and what will be ground speed?

4. It is desired to fly 160°. Airspeed is 125 miles per hour. Wind is from 70°; wind speed is 35 miles per hour. What should be heading, and what will be ground speed?

5. It is desired to fly 230°. Airspeed is 110 miles per hour. Wind is from 170°; wind speed is 35 miles per hour. What should be heading, and what will be ground speed?

Wind-Star. Although, at many airports, information is available as to winds at altitude, there is no certainty that winds will not have changed after the observations were taken. The wind conditions may be entirely different a few miles away from the airport.

Drift indicators installed on the aircraft can be used to measure drift angle with a fair degree of accuracy. Finding ground speed by use of two cross-hairs on the drift indicator presupposes knowledge of the actual height of the aircraft above the ground. Altimeters give barometric height, i.e., altitude above sea-level, not height above the ground. When the location of the aircraft is known, the height above sea-level of the ground below the aircraft can be found from contour maps; but when the geographic location of the aircraft is recognizable on a map, the pilot resorts to landmark flying, not dead reckoning.

Ground-speed indicators have been devised on the principle of synchronizing the speed of an endless celluloid belt on the aircraft with the apparent rearward speed of the ground. These devices have not yet passed out of their developmental status.

A single drift measurement does not give ground speed, nor does it give sufficient information to obtain wind speed and direction. Measurements of drift on two headings will give ground speed and wind velocity by utilizing the principle of the wind-star.

For example — see Fig. 121*a* — if an airplane is headed north with an airspeed of 150 miles per hour and a 15° right drift is measured, this may be due to a 40-mile-per-hour west wind or to a 45-mile-per-hour northwest wind or to a 78-mile-per-hour southwest wind. There may be an infinite number of winds which will produce a 15° right drift on a north heading.

If the heading is changed to east and the drift is found to be, say, 5° left, the wind is then determined, because there can be only one wind that will give a 15° right drift on a north heading and a 5° left drift on an east heading. By drawing line AB — see Fig. 121*b* — 150 units long (airspeed is 150 miles per hour) in a direction representing north and drawing line AX of indefinite length at an angle of 15° to the right of AB, the track on the first heading is represented. From point B, a line BA' is drawn in a direction opposite to the second heading (i.e., westward); the length BA' is 150 units. Then $A'B$ represents the airspeed and heading after heading has been changed. Through A' a line $A'X'$ is drawn of indefinite length making an angle of 5° left of $A'B$. Call the intersection of AX and $A'X'$ the point W, and connect points B and W. Then the vector BW represents the wind speed of 48 miles per hour and wind direction of 249°, this being the only wind velocity that will give a drift of 15° right on a north heading and a drift of 5° left on an east heading. The length of AW is the ground speed 173 miles per hour on first heading, and the length of $A'W$ is the ground speed 195.5 miles per hour on second heading.

After the drifts on two headings are found so that the wind is known, the drift can be found on any heading by drawing a line representing the heading into point B, and from the other or starting end of the vector drawing a line through point W.

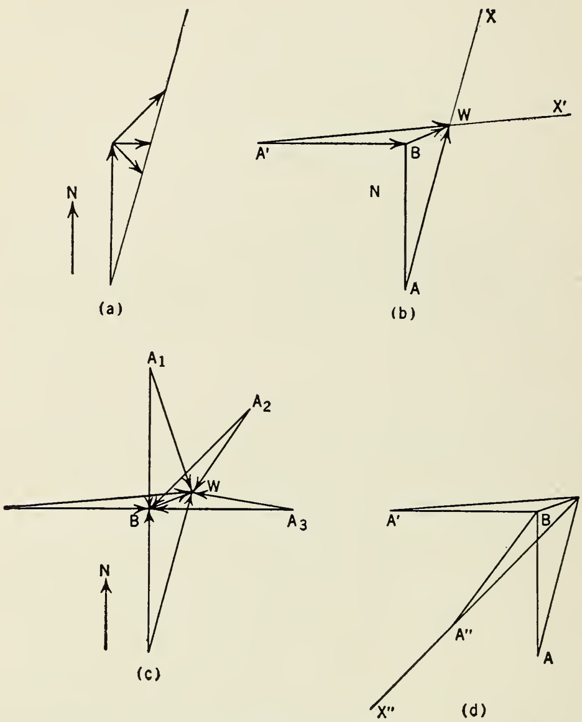


Fig. 121. Wind-star.

Figure 121c shows the finding of drift on south, southwest, and west headings, A_1B, A_2B, and A_3B representing the respective headings and A_1W, A_2W, and A_3W representing the respective tracks and ground speeds. When headings and tracks are drawn for several different headings as in Fig. 121c, the drawing bears a crude resemblance to the conventional star-form, which is the reason for giving this process the name of the wind-star.

After the wind velocity has been found by measuring the drift on two headings, it is possible to find the proper heading for the aircraft to follow a desired track by the following procedure. From the arrow end of the wind-vector draw a line of indefinite length in a direction opposite to that of the desired track. From the non-arrow end of the wind-vector as center and with the airspeed as radius strike an arc intersecting the line just drawn. The radius from this point of intersection gives the proper heading. For example, the wind having been found on a north heading and an east heading as Fig. 121*b*, it is desired to find the heading which will give a true northeast track. Through point W draw a line of indefinite length WX'' in a southwest direction; see Fig. 121*d*. With B as center and 150 units as radius, intersect line WX'' at point A''. Then direction of $A''B$ is the proper heading 37.5°, and length of $A''W$ is the ground speed, 192 miles per hour.

In using this method, it is customary to head 30° to the right of the desired track and 30° to the left, since by so doing a good intersection is obtained. The wind-star method does not involve knowledge of altitude.

Problems

1. On a 30° heading there is a 10° left drift; on a 330° heading there is a 15° left drift. If airspeed is 125 miles per hour: (*a*) what is wind speed and (*b*) direction; (*c*) what heading should be flown to obtain a due north track; (*d*) what will be ground speed?

2. On a 15° heading there is a 10° left drift; on a 75° heading there is a 20° left drift. Airspeed is 160 miles per hour. (*a*) What is wind speed and (*b*) direction? (*c*) What heading should be flown to obtain a northeast track? (*d*) What will be ground speed?

3. On a 60° heading there is a 5° right drift; on a 120° heading there is a 10° left drift. Airspeed is 140 mile per hour. (*a*) What is wind speed and (*b*) direction? (*c*) What should heading be to fly east. (*d*) What will be ground speed?

Astronomic Avigation. For centuries sailboats and more recently steamships have navigated by means of observations on the sun, moon, or stars. The angular height of a celestial body above the horizontal plane being measured by a sextant, an oblique spherical triangle is solved and a line partially determining the ship's position is obtained. Two such lines exactly fix the position.

On marine vessels, the horizon, where sky and water meet, is used to determine the horizontal plane. Unless an aircraft is at an altitude less than 1,000 ft., the observer is unable to see the horizon because of haze. For this reason, and also to enable observations to be taken at night, some form of artificial horizon is used; generally a spirit level is utilized. In bumpy air the bubble dances about badly, but by taking the average of a number of observations the error can be reduced to a minimum.

Computation of the position from the sextant observations was formerly a long, tedious matter. Short methods have been evolved which permit the calculations to be made in a minute or less.

Astronomic avigation is used on long overwater flights, such as trans-Atlantic or trans-Pacific hops.

Radio Avigation. Radio is an important adjunct to flying. A simple receiving set enables the pilot to receive storm-warning and other weather information as well as to use the radio beacon. A two-way set enables the pilot to report his progress and otherwise keep completely in touch with the ground.

To make use of radio a battery is required, so that an airplane whose engine operates from a magneto must add a battery. This is necessary for reception; to transmit, a source of high-voltage current is needed in addition. Apparatus have been developed which are very compact and light weight.

The ordinary aircraft engine has a spark gap in each cylinder for each ignition. This sparking sets up radio oscillations which would interfere with the reception of radio messages if the engine were not shielded. This means that every electric wire must be covered with a coating of braided metal, the plugs themselves shielded with a metal cup, and every other part of the electrical equipment surrounded with a metal covering.

The metal structure of the airplane is made to serve as a ground, which means that every metallic part of the airplane must be connected electrically to every other metallic part. This is called bonding. Proper bonding prevents trouble caused by intermovement of parts and eliminates possible sparking.

Radio is used in avigation in several ways. If two or more stations on the ground are equipped with radio compasses, the pilot can request them to get his bearings, then sending out a series of dots from the plane enables the ground operators to get the direction of the airplane each from his own station. With the bearings from two or more ground stations, either the pilot or one of the ground operators can draw corresponding lines on a map and the intersection gives the geographic location of the airplane. If the ground operator does the plotting he promptly radios the position to the airplane.

Radio Beacon. Another form of radio avigation is the utilization of the radio beacon. The Department of Commerce has

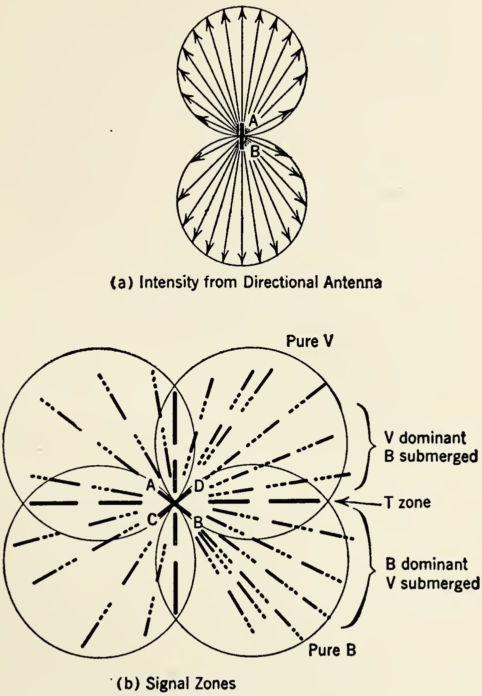


Fig. 122. Radio beacon.

installed radio beacons at many airports, and it has been very successful in guiding airplanes from one airport to another. No special equipment is needed on the airplane; a receiving set is required, but a transmitter is not necessary. No special skill is required by the pilot.

The radio beacon transmitter is shown diagrammatically in Fig. 122. In broadcasting, the transmitter is designed to send signals in all directions with equal strength, but it is relatively simple to erect an antenna which will send stronger signals in one direction than another. The directional antenna usually has "figure 8" characteristics as shown in Fig. 122*a*. This diagram means that, if *AB* is the antenna, the strongest signals will be heard in a vertical plane through *AB*. At any point not in this plane the signal heard on the plane will not be as strong as it would be if the airplane were the same distance away from *AB* but in the plane *AB*. The intensity of the signal received on the airplane, which is a measure of the audibility, is proportional to the length of the arrow on the diagram in the direction of the airplane.

When two directional antennas, *AB* and *CD*, are crossed as in Fig. 122*b*, but not connected electrically in any way, signals may be sent out simultaneously from each. Two code letters are used, which are easily recognizable but are so selected that, if they are both started at the same time, the dash of one code letter is sounded at the same instant as the dot or dots of the other letter. The letters A (· —) and N (— ·) answer this requirement, as do D (— · ·) and U (· · —), or B (— · · ·) and V (· · · —). If the radiations from each antenna are received with equal strength, the receiver will respond to both equally, and the signal heard by the pilot will be a T or long dash (——). If the reception of one signal is of greater intensity than the other, the powerful signal will be heard while the weaker signal will be either faint or inaudible.

For example, if antenna *AB* is emitting a characteristic signal B and antenna *CD* is sending out V, in the two planes, which bisect the two angles made by *AB* and *CD*, both signals would be received with equal strength, and the T signal would be heard. If the airplane's position is nearer the plane through *AB* than the plane through *CD*, though the V signal might be heard faintly, the B signal would predominate.

Because it is somewhat difficult to distinguish the difference in signals when the pilot is in or close to the T zone, a visual-type radio beacon is frequently used. One antenna transmits with a frequency of 66 cycles, the other with a frequency of 86$\frac{2}{3}$ cycles. Located on the instrument board is a glass-faced box containing two springy metal strips or "reeds." One responds to a frequency of 66 cycles; the other to 86$\frac{2}{3}$ cycles. When receiving the modulation frequencies, the reed vibrates rapidly in vertical plane. The greater the intensity received, the greater is the amplitude of vibration. The pilot looks at the ends of the reeds, which are painted white; when vibrating rapidly they appear to be two white vertical lines. The greater the amplitude of vibration, the longer is the line. When the airplane is in the T zone, the circuits for reeds are receiving the same intensity, and the two white lines are the same length. When the airplane is off to one side of the T zone, the circuit for one reed is receiving greater intensity so that one white line is longer and the other is shorter. The visual beacon is less tiring for the pilot, but since the receiving hook-up is slightly more complicated there is more danger of failure.

The radio beacon guides a pilot in a proper direction. With a cross-wind, the pilot will probably zigzag a bit till he finds experimentally a heading which does not direct him from the B zone across the T zone into the V zone, or vice versa. Although there is a zone of silence directly over the beacon, a pilot flying in fog cannot rely on silence meaning he has reached his destination; his receiving set might have developed trouble. The beacon gives no indication of the distance to the transmitting station. In mountainous districts, the radio waves suffer peculiar distortions and echoes which affect the accuracy of the beacon. The beacon requires a special transmitting installation.

Radio Compass. By constructing the receiving antenna in the form of a coil or loop, and by suitably designing the tuned circuits in rotating this loop about a vertical axis till a signal fades out completely, the direction of the transmitter of that signal can be ascertained. Such an arrangement is called a radio compass.

A radio compass can be used on any broadcasting transmission; the sending set need have no directional effect. The radio compass is useful in fog flying, since the pilot can take bearings on two or more broadcasting stations and plot these bearings on his map; the intersection gives the plane's position. The radio compass can also be used as a " homing " device. The pilot can find the direction of a broadcasting station located near his destination and then head his airplane in that direction.

Radio Landings. The following method of making a landing in fog, etc., was devised by Captain A. F. Hegenberger, A.C., and is termed the Army method.

Two transmitters are positioned near the landing field. These transmitters are portable so that the line connecting them can be put in the direction of the prevailing wind. Transmitter *A* is located outside the landing area but not more than 1,000 ft. outside the boundary of the field. Transmitter *B* is located farther from the field at a distance of a mile to a mile and a half from *A*. If an airplane flies over *B* headed for *A*, continuing his flight, without changing his heading, should bring the airplane directly over the landing area. The transmission from *A* and *B* are on different frequencies.

The transmitters at *A* and *B* are not especially powerful. It is sufficient if signals can be picked up at a distance of 25 miles. The antenna for this transmission is non-directional; that is, the signals are broadcast with equal intensity in all directions.

At each station, *A* and *B*, is also a marker beacon. This beacon uses a low-power ultra-high-frequency transmitter, the wave length of the oscillator being 4 meters. A very short horizontal doublet antenna is used, located one-quarter wavelength above the ground. This form of antenna is very directional. The antenna is placed so that it is in line with the approach to the field, i.e., along the line *AB*. Then the radiation is a fan-shaped wave in a plane through *A* or through *B* perpendicular to the line *AB*.

The procedure for utilizing the above-described equipment is as follows. The pilot, by making use of radio beacon, radio compass, or dead reckoning, arrives within receiving distance of station *A*. He then tunes in on *B* with his radio compass, and flies directly towards it, setting his directional gyro on his heading. He is then flying directly away from the field. After passing over *B*, the pilot makes a gradual flat turn of 180°, using his directional gyro to tell him when he has exactly reversed his heading. Altitude should be gradually lost so that the altitude is approximately 700 ft. above the ground when the airplane passes the outer station *B*. The exact time of passing *B* is told by the signal denoting passing the standing wave of the marker beacon. The receiver of the radio compass is then tuned on *A* and the airplane headed in that direction. In this, the pilot is aided by his directional gyro which he had previously set on the reverse heading. In proceeding from *B* to *A* the throttle is set back approximately at 1,000 r.p.m. and the plane is put in a steady glide at an airspeed about 30 per cent above stalling speed. If this procedure has been followed the plane should arrive at station *A* at an altitude of 200 ft., and continuing the steady glide the plane should pass over all obstacles and reach the landing area properly.

As soon as the wheels touch the ground, the pilot cuts the throttle, permitting the plane to come to rest.

The pilot knows when he is passing over station A either by the " zone of silence " or from the signal from the marker beacon located at A. If his altitude at that time is less than 200 ft., the pilot merely opens the engine, putting the airplane in level flight for a brief interval of time, and then permits the plane to settle into its glide again. If the plane is higher than 200 ft. in passing over A, the pilot may either momentarily glide more steeply before resuming his normal glide or he may return to B and start his glide again at a slightly steeper angle. The altitudes mentioned in this paragraph are read from a sensitive altimeter set at correct ground barometer reading from information radioed up to the pilot by the ground operator.

This method of landing without seeing the ground has been tested by many hundred landings either in hooded cockpits or in actual fog. No accidents have ever been reported in using this method.

CHAPTER XXI

AEROSTATICS

Introduction. Aerodynamics treats of air in motion; aerostatics deals with air at rest. In aviation, aerostatics deals with the problems of lighter-than-air craft.

All lighter-than-air craft are balloons, but present-day usage is to employ the term balloon only for craft which has no motive power and the term dirigible balloon or simply dirigible for a balloon supplied with motive power. The term airship is synonymous with dirigible; an airplane should not be called an airship.

Balloons are classified in two types, captive and free balloons. Captive balloons are moored to the ground and are used for observing artillery fire, etc. Captive balloons, if spherical in shape, have a strong tendency to rotate, so that they are usually elongated in shape with protuberances so designed that the balloon lies in the direction of the wind. This shape is called a kite-balloon. Free balloons are usually the so-called spherical shape. The upper part is a true hemisphere; the lower half is hemispherical except that at the extreme lower part the skin cones down to a long narrow tube called the appendix. In the earlier forms a rope netting was arranged over the upper half of the bag, the ends coming down just below the appendix where they were fastened to a metal concentration-ring from which the basket for passengers was hung. Because of the weight of the rope and the criss-cross of the netting forming many tiny pockets to retain rainwater, the netting arrangement was discarded for the suspension band or " bellyband," a strip of fabric around the equator of the bag, ropes from the concentration-ring being attached to this band. The most modern method is to fasten each rope from the concentration-ring to a piece of fabric, called a finger-patch, cemented to the bag at the point where the curve of the suspension rope is just tangent to the sphere of the envelope. A valve, normally held shut by springs, is opened by pulling the valve-rope which hangs down inside the bag, coming through the appendix, the lower end being at the basket.

Dirigibles are classified according to their method of construction as rigid, semi-rigid, and non-rigid. The rigid ones are fre-

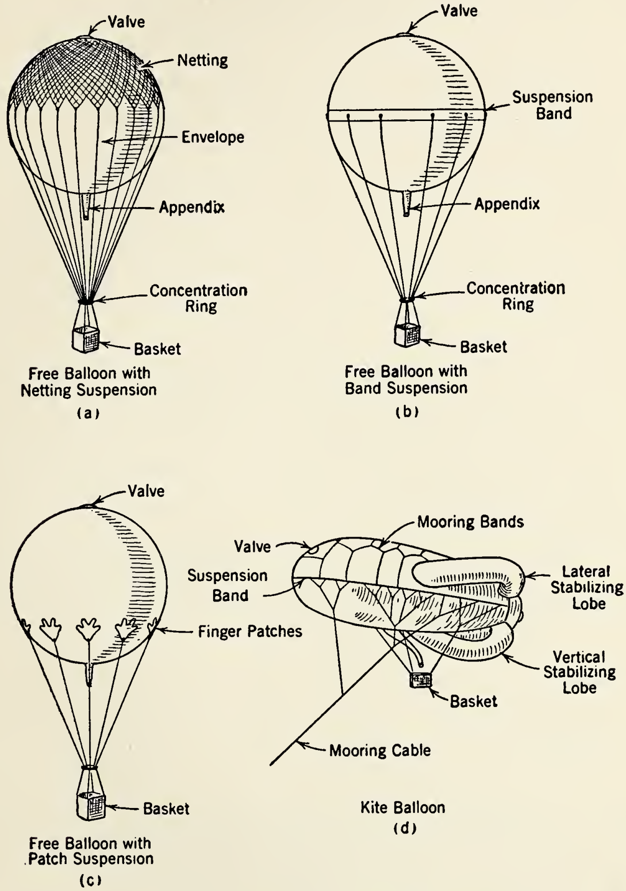


FIG. 123. Free and captive balloons.

quently called Zeppelins after Ferdinand Count Zeppelin who was the first successful designer of this type. The rigid type has a cloth-covered metal framework which gives the airship its shape, the gas being contained in a number of individual cells. The semi-rigid type has a metal keel the entire length of the ship, taking care of the transverse loads, the gas being in a single large envelope. The non-rigid is a single rubberized-cloth gas-tight bag containing the gas. Since, in non-rigid and semi-rigid types, the pressure of the gas inside the bag is relied upon to give the bag its shape, both these types are termed pressure airships.

Gases. Hot-air, coal-gas, hydrogen, and helium are the principal gases used for lighter-than-air work. Heated air is used sometimes for parachute jumps at carnivals; as soon as the air cools lift is lost. Coal-gas or ordinary illuminating gas is used sometimes for free balloons, the hydrogen contained in these gases being chiefly responsible for the lifting power. Hydrogen has been used for more than a century and is best from the point of lift but has the disadvantage that, if it mixes with air, it is highly inflammable and explosive. Helium though furnishing less lift than hydrogen is perfectly inert so that in its use there is not the fire hazard always present with hydrogen.

Previous to 1915, helium cost approximately $2,000 per cubic foot. During the war the cost to the United States government was reduced to $400 per 1,000 cu. ft. At present, owing to improvements in the method of extraction from natural gas, the cost is about $10 per 1,000 cu. ft. The cost of hydrogen is about $5 per 1,000 cu. ft.

Under standard conditions of 59° F. temperature and 29.92 in. of mercury pressure, the weight per cubic foot of the gases important in aerostatics is as follows

Air	0.07651 lb.
Hydrogen	0.00532 lb.
Helium	0.01056 lb.
Average illuminating gas	0.0323 lb.

Laws of Aerostatics. Six physical laws find application in aerostatic work. Full explanations are given in any standard textbook on physics. Briefly these laws are as follows.

Archimedes' law: the buoyant or upward force exerted upon a body immersed in a fluid is equal to the weight of the fluid displaced.

Boyle's law: at a constant temperature, the volume of a gas varies inversely as the pressure.

Charles' law: at a constant pressure, the volume of a gas varies directly as the absolute temperature.

Dalton's law: the pressure of a mixture of several gases in a given space is equal to the sum of the pressures which each gas would exert by itself if confined in that space.

Joule's law: gases in expanding do no interior work.

Pascal's law: the fluid pressure due to external pressure on the walls of the containing vessel is the same at all points throughout the fluid.

Lift. A body completely immersed in a fluid displaces its own volume of fluid. If the weight of the fluid displaced equals the weight of the body, the body is in equilibrium. If the weight of the fluid displaced is greater than the weight of the body, the body rises. If the weight of the fluid displaced is less than the weight of the body, the body falls.

The operation of a balloon and a submarine is somewhat similar in that both are entirely submerged in a fluid. The submarine is entirely sealed when under water, and the skin must be of sufficient strength to withstand stresses due to difference in pressure between inside and outside the hull. Balloons either are open to the air or have other arrangements so that there will be little or no pressure difference.

Lift is obtained directly from Archimedes' principle. Unit lift is the difference between the weight of a cubic foot of air and the weight of a cubic foot of the gas. The gross lift or buoyancy of a balloon or airship is expressed by the following equation:

$$L = V(D_a - D_g)$$

L = gross lift in pounds
V = volume of gas in cubic feet
D_a = weight of a cubic foot of air
D_g = weight of a cubic foot of gas

The net lift or useful load is the difference between the gross lift and the dead weight of the bag, ropes, basket, etc.

From the laws of Boyle and Charles, volume varies inversely as pressure and directly as absolute temperature. If P_0 is the standard pressure, T_0 the standard temperature, and V_0 the standard volume, then V_1, the volume when the pressure is P_1 and the absolute temperature is T_1, may be found by the following,

$$\frac{V_1}{V_0} = \frac{T_1}{T_0} \times \frac{P_0}{P_1}$$

The foregoing is true only if the gas is in a perfectly elastic container.

Example. A cloth bag contains 1,000 cubic feet of air, the temperature being 59° F. and the pressure 29.92 in. What is the volume if the pressure is increased to 40.0 in. and the temperature decreased to 0° F.?

Solution.

$$\begin{aligned} V_1 &= V_0 \frac{T_1}{T_0} \times \frac{P_0}{P_1} \\ &= 1{,}000 \frac{460}{519} \times \frac{29.92}{40.0} \\ &= 663 \text{ cu. ft.} \end{aligned}$$

In practice the balloon bag is made of rubberized cloth. If there is not sufficient volume of gas to fill the bag, the cloth sides will fold in and the bag will be flabby. When the volume of the gas is just equal to the cubical content the bag will be fully inflated. Any further increase in volume of the gas means that gas will escape through the appendix. If the appendix is closed, a decrease in the air pressure outside or an increase of temperature of the gas inside will cause the bag to burst. For this reason, though the appendix is usually tied shut while the balloon is being handled on the ground, the tie-off is broken as soon as the balloon leaves the ground.

Density varies inversely as volume, so that if D_0 is the density under normal pressure P_0 and normal absolute temperature T_0, the density D_1 under pressure P_1 and temperature T_1 is found by the following relation:

$$\frac{D_1}{D_0} = \frac{T_0}{T_1} \times \frac{P_1}{P_0}$$

Under normal conditions, the temperature of the gas inside the envelope is the same as the temperature of the adjacent air. The open appendix ensures that the pressure of the gas and air are the same.

Considering the pressure of the gas the same as the pressure of the surrounding air and the temperature of the gas the same as that of the air, the expression for lift under other than standard conditions becomes

$$L = V\left(D_{a0}\frac{T_0}{T_1}\frac{P_1}{P_0} - D_{g0}\frac{T_0}{T_1}\frac{P_1}{P_0}\right)$$
$$= V(D_{a0} - D_{g0})\frac{T_0}{T_1}\frac{P_1}{P_0}$$

L = gross lift in pounds
V = volume of bag in cubic feet
D_{a0} = weight of a cubic foot of air under standard conditions
D_{g0} = weight of a cubic foot of gas under standard conditions
P_0 = standard pressure
P_1 = actual pressure
T_0 = standard absolute temperature
T_1 = actual absolute temperature

Example. A 10,000-cu.-ft. free balloon is filled with pure hydrogen. Air and gas are at a temperature of 32° F. and a pressure of 28 in. What is the lift?

Solution.

$$\text{Lift} = V(D_{a0} - D_{g0})\frac{T_0}{T_1} \times \frac{P_1}{P_0}$$
$$= 10{,}000\ (0.07651 - 0.00532)\frac{519}{492} \times \frac{28}{29.92}$$
$$= 703 \text{ lb.}$$

Problems

1. A 30,000-cu.-ft. free balloon is full of pure hydrogen; air and gas are at a temperature of 80° F. and a pressure of 26.3 in. What is the lift?

2. A 10,000-cu.-ft. free balloon is full of pure helium; air and gas are at a temperature of 75° F. and a pressure of 27.5 in. What is the lift?

3. A 20,000-cu.-ft. free balloon is full of pure hydrogen; air and gas are at a temperature of −10° F. and a pressure of 21.3 in. What is the lift?

4. A 20,000-cu.-ft. free balloon is full of pure helium; air and gas are at a temperature of −10° F. and a pressure of 21.3 in. What is the lift?

5. A 30,000-cu.-ft. free balloon is full of pure hydrogen; air and gas are at a temperature of 20° F. and a pressure of 23.7 in. What is the lift?

Ascension of a Free Balloon. When a free balloon, full of gas on the ground, has lift greater than its weight, it rises, and because the atmospheric pressure decreases with altitude, gas will expand and tend to occupy greater volume. Since the fabric does not stretch, the volume is fixed and gas is forced out of the appendix. As the altitude increases, the density of the air decreases; but the density of the lifting gas decreases at the same rate, so that the difference between the two weight densities, which is the unit lift, also decreases at the same rate. The gross lift which is the constant volume multiplied by the unit lift decreases. Ascension will continue until the gross lift just equals the weight.

If the bag is only partially inflated at the ground, on rising, the atmospheric pressure being less, the gas will expand, rounding out the bag more fully. There will be no loss of gas till the bag is fully rounded out. The altitude where the bag is completely full and where any further increase in altitude will cause gas to escape is called the pressure height.

A free balloon which is fully inflated at the ground will start to lose lift immediately on leaving the ground and there will be a continuous loss of lift as the balloon rises. A free balloon which is partially inflated at the ground will have a constant lift until the pressure height is reached; above that altitude, lift will decrease with altitude.

While inflating a balloon, sufficient sandbags are placed in the basket or hung on the netting to ensure that the balloon will not leave the ground. When ready to start, the balloon is weighed-off. This process is the removing of ballast until the total weight of the balloon just equals the total lift. In this condition, a man standing on the ground can with practically no exertion move the balloon up in the air a foot or so, where it will stay in equilibrium. Adding a fraction of an ounce of sand will make the balloon sink slowly to the ground. Tossing overboard a little ballast will cause the balloon to rise slowly; heaving over a lot of sand will cause the bag to rise swiftly.

If the bag is fully inflated at the ground, after weighing-off, weight equals lift. If ballast is then jettisoned, the weight is lessened. The balloon will rise. As it rises, gas will escape, so that the lift will decrease. At the start, the accelerating force will be equal to the difference between the weight and the lift; i.e., the force in pounds will equal the pounds of sand put over. In ascending, since the difference between the weight of the balloon and the lift will be less owing to the decreasing lift, the upward accelerating force diminishes. At the height where the lift has decreased till it equals the weight, there will be no unbalanced upward accelerating force and the balloon will be in equilibrium. To ascend further, more ballast must be put overboard. The ascent to a high altitude can be made by easy stages, by first dropping a little ballast, then a little more, and a little more till the desired height is reached.

A bag, which is only partially inflated at the ground is weighed-off in the same manner as a fully inflated bag. When a little ballast is dropped, however, the balloon will immediately rise to its pressure height, the upward accelerating force being constant to that altitude. On reaching the pressure height gas will begin to escape, and with decreasing lift, the difference between lift and weight will get smaller, till equilibrium is reached.

A partially inflated balloon will be at pressure height when its volume has increased to equal the capacity of the balloon. As the volume of a gas varies inversely with density, the reciprocal of the ratio of the new volume to the old is the ratio of new density to the old. Using Table I, interpolating if necessary, the altitude may be found corresponding to this density.

If it is desired to ascend to some predetermined moderately high altitude, there is no special merit in fully inflating the bag for gas will start to escape immediately as the balloon rises. A partially inflated balloon on reaching its pressure height is exactly in the same condition as if it had been fully inflated on the ground. It should be noticed that there is a difference in maneuvering. On the ground, the fully inflated bag has more lift so that more ballast can be carried. In ascending with the full bag, ballast is gradually dropped so that, when the altitude is reached corresponding to the pressure height of a partially inflated bag, the amount of ballast remaining in each case would be the same. The bag which was fully inflated on the ground can be brought up to any desired altitude slowly. A bag only partially inflated on the ground will have a constant upward accelerating force; therefore there will be a constantly increasing upward velocity. Arriving at the pressure height, the balloon will have considerable upward momentum, the product of its mass times its velocity, and this momentum will tend to carry the balloon on upward beyond its equilibrium point. At the equilibrium point, the lift equals the weight. If momentum carries the balloon above its pressure height, the gas having already expanded to fill the bag, gas will be forced out and lift will decrease. Lift being less than the weight, there is a downward accelerating force which acts first to decelerate the upward velocity, and, when this has been reduced to zero, the downward force will cause a downward velocity with ever-increasing speed. A skilful pilot will time his actions so that, just when the balloon has reached its highest point and is about to start on its downward plunge, just enough ballast is thrown over so that the weight remaining equals the then-existing lift.

Example. A 10,000-cu.-ft. free balloon is inflated with 8,000 cu. ft. of pure hydrogen at the ground, temperature 59° F., pressure 29.92 in. What is lift at 5,000-ft. altitude (temperature 45° F., pressure 24.8 in.)? At 10,000-ft. altitude (temperature 15° F., pressure 21.1 in.)? What is the pressure height?

Solution.

At ground

$$\begin{aligned} L &= 8{,}000\ (0.07651 - 0.00532) \\ &= 570 \text{ lb.} \end{aligned}$$

At 5,000-ft. altitude: To determine if bag is full

$$\begin{aligned} V_1 &= V_0 \frac{P_0}{P_1} \times \frac{T_1}{T_0} \\ &= 8{,}000\ \frac{29.92}{24.8} \times \frac{505}{519} \\ &= 9{,}391 \text{ cu. ft. (volume at 5,000-ft. alt.)} \\ L &= 9{,}391\ (0.07651 - 0.00532) \frac{519}{505} \times \frac{24.8}{29.92} \\ &= 570 \text{ lb.} \end{aligned}$$

At 10,000-ft. altitude: To determine if bag is full

$$\begin{aligned} V_1 &= V_0 \frac{P_0}{P_1} \times \frac{T_1}{T_0} \\ &= 8{,}000 \times \frac{475}{519} \times \frac{29.92}{21.1} \\ &= 10{,}951 \text{ cu. ft.} = \text{volume of original gas at 10,000-ft.} \\ &\qquad \text{altitude; i.e., 951 cu. ft. has escaped.} \\ L &= 10{,}000\ (0.07651 - 0.00532) \frac{519}{475} \times \frac{21.1}{29.92} \\ &= 549 \text{ lb.} \end{aligned}$$

To find pressure height:

$$\begin{aligned} \frac{P_1}{P_0} &= \frac{8{,}000}{10{,}000} \\ &= 0.8 \end{aligned}$$

From Table I by interpolation P_1/P_0 is 0.8 at 7,075-ft. altitude.

Problems

(See Table I for pressures and temperatures at altitude.)

1. A 10,000-cu.-ft. free balloon is inflated at the ground with 7,000 cu. ft. of pure hydrogen under standard atmospheric conditions. (*a*) What is the lift at the ground? (*b*) What is the lift at 5,000-ft. altitude?

2. A 25,000-cu.-ft. free balloon is inflated at the ground with 15,000 cu. ft. of pure helium under standard atmospheric conditions. (*a*) What is the lift at the ground? (*b*) What is the lift at 5,000-ft. altitude? (*c*) What is the lift at 10,000-ft. altitude?

3. The total weight of a 20,000-cu.-ft. balloon, including bag, basket, crew, instruments, and 300 lb. ballast, but less gas, is 1,450 lb. The bag is filled with pure hydrogen on a day when the temperature is 65° F. and the barometer is 29.6 in. (*a*) How much ballast must be dropped off in weighing-off, i.e., having weight just equal lift at the ground? (*b*) How much more ballast must be dropped in order for the bag to be in equilibrium at 5,000-ft. altitude?

4. The balloon described in problem 3 is inflated with only 18,000 cu. ft. of hydrogen under the same atmospheric conditions as in 3. (*a*) How much ballast must be dropped in weighing-off? (*b*) What is the pressure height? (*c*) How much more ballast must be dropped in order to be in equilibrium at 5,000-ft. altitude?

Descent of a Free Balloon. In descending, the atmospheric pressure increases. Any gas that has escaped from the appendix or that has been valved is gone. When the balloon is in equilibrium, if gas is valved, the lift becomes less than the weight, so that balloon starts downward under an accelerating force which is the excess of the weight over the lift. The effect of the increase in atmospheric pressure while descending will be to decrease the volume of the gas in the bag. The same weight of gas as at the beginning of the descent will displace less and less volume of air, but the density of the gas and air will increase as the volume decreases. The lift will remain the same all the way down, and there will therefore be a constant accelerating force downward which will cause ever-increasing downward velocity.

The only way to check descent will be to introduce a decelerating force, that is, an upward force. This can be accomplished only by making the weight less than lift, which is done by dropping ballast. The upward excess of lift will decrease the downward speed to zero and then cause an upward acceleration.

If it is desired to descend to a definite altitude and remain there in equilibrium, the pilot, after valving a slight amount of gas to start descent, must drop ballast as he nears the desired altitude. At the instant that his downward progress is checked completely and before he starts to rise again, he must valve just enough gas to gain equilibrium.

The faster the balloon is descending, the greater is the amount of ballast needed to check the speed. A good pilot valves only a little gas at a time, so that only a little ballast must be sacrificed in maneuvering.

In landing, the pilot comes down towards the ground, and at the proper height, which he has learned by experience, the pilot tosses over ballast so that his downward speed is zero when he is just a few feet off the ground. At that instant, he is pulling the rip-cord, which entirely opens one seam of the bag, completely emptying it of gas.

Example. A 10,000-cu.-ft. bag is full of hydrogen at 12,000-ft. altitude (temperature 16.2° F., pressure 19.03 in.) and is in equilibrium. (*a*) What is the total weight of the bag including ballast? (*b*) If 100 cu. ft. of gas is valved, what is the loss of lift at this altitude? (*c*) At 7,000-ft. altitude, what is the volume of the gas in the bag? (*d*) What is lift at 7,000-ft. altitude? (*e*) What ballast must be dropped at 7,000-ft. altitude to secure equilibrium?

Solution.

(*a*) $$L_{12,000} = 10{,}000\,(0.07651 - 0.00532)\,\frac{19.03}{29.92} \times \frac{519}{476} = 494 \text{ lb.}$$

(*b*) $$L = 100\,(0.07651 - 0.00532)\,\frac{19.03}{29.92} \times \frac{519}{476} = 5 \text{ lb.}$$

(*c*) $$V_{7,000} = 9{,}900 \times \frac{19.03}{23.09} \times \frac{494}{476} = 8{,}468 \text{ cu. ft.}$$

(*d*) $$L_{7,000} = 8468\,(0.07651 - 0.00532)\,\frac{23.09}{29.92} \times \frac{519}{494} = 489 \text{ lb.}$$

(*e*) Ballast to be dropped $W - L_{7,000} = 5$

Problems

1. A 30,000-cu.-ft. balloon is only partially inflated with 20,000 cu. ft. of pure hydrogen under standard atmospheric conditions at the ground. (*a*) What is the lift at the ground? (*b*) What is the lift at 15,000-ft. altitude? (*c*) If balloon is in equilibrium at 15,000-ft. altitude, what is excess weight after 150 cu. ft. of gas are valved? (*d*) What is volume of bag after descending to 10,000-ft. altitude? (*e*) What is lift at 10,000-ft. altitude?

2. A 20,000-cu.-ft. balloon is filled with pure helium at the ground under standard atmospheric conditions. (*a*) What is the lift at the ground? (*b*) What weight of ballast must be tossed over to rise to 5,000-ft. altitude; (*c*) to 10,000-ft. altitude? (*d*) If 200 cu. ft. of helium are valved at 10,000-ft. altitude, what is the remaining lift? (*e*) When balloon has descended to 5,000-ft. altitude, 200 cu. ft. additional of helium are valved; what is the remaining lift?

Superheat. The sun's radiation consists not only of the long heat waves but also of the shorter light and electric waves. It is the heat radiation that warms the earth and to a smaller extent the atmosphere.

Light radiation may change into heat radiation in passing through surfaces. This effect may be noticed in a tent which has the flaps down; the confined air is at a higher temperature than the outside air because the light in passing through the tent fabric is changed to heat. In greenhouses there is no ventilation, and the air inside is hotter than outside, as the result of light changing to heat in passing through the glass. Naturally the foregoing is true only on sunny days. In free balloons, the sun shining on the bag heats up the gas inside the bag to a higher temperature than the surrounding air. This is called superheat.

Free balloons are suspended in air. If the air is moving, the balloon travels with it. Air does not travel past the balloon; the only heat lost is by direct conduction to the surrounding air.

Kite balloons are moored to the ground for observational purposes. In perfectly still air, the same conditions apply as in a free balloon. If any breeze is blowing, the air passing the balloon takes up heat, so that any superheat is quickly lost, the temperature of the gas being reduced to that of the surrounding air. Dirigibles in motion are forcing their way through air, and this air acts to cool off any superheat.

Superheat give additional lift to a balloon. It is a false, treacherous lift, since if the sun goes behind a cloud or the sun sets, the additional lift is quickly lost.

Black surfaces absorb heat and light; shiny surfaces reflect heat and light. Balloons whose envelopes are of dark material have been found to have in the center of the bag a temperature 70° hotter than the outer air. With lighter-colored surface the superheat will be less.

The increase in temperature due to superheat will cause the gas to expand. Below the pressure height, this will increase the fullness of the bag; for a fully inflated bag, superheat will force out gas; in either case, lift is increased.

If a balloon or airship is brought out of a hangar, the gas is presumably at the same pressure and temperature as the atmosphere. If the balloon is weighed-off immediately, the unit lift is due to the difference in weight density of the air and gas at the same temperature. Should the balloon stay on the ground with the sun shining brightly, the gas will receive superheat and will expand. The additional lift involves adding ballast to prevent the balloon or airship from rising. The weight of ballast that must be added exactly represents the additional lift due to superheat. When rising, the balloon moves upward through the air, and this air passing the envelope tends to cool it slightly.

The additional lift can be calculated if the number of degrees of superheat is known. If the balloon is initially inflated, the increased temperature will cause expansion, reducing the density, the volume being the same. If the balloon is only partially inflated, the expansion of the gas will cause a greater displacement of the bag until the bag is fully inflated. Further expansion will cause gas to escape. For the partially inflated bag, gas expanding but not escaping will not change the weight of the gas contained in the envelope; more air will be displaced, however, so that the lift will be increased.

It will be noted that, when a fully inflated bag is superheated, the increase in lift is exactly equal to the weight of the gas forced out of the bag by the expansion.

The finding of the increased lift from superheat is illustrated by the following two examples.

Example. An 8,000-cu-ft. balloon is fully inflated with pure hydrogen under standard conditions. What is the additional lift due to superheat of 40°?

Solution.

$$
\begin{aligned}
\text{Lift without superheat} &= 8{,}000\ (0.07651 - 0.00532) \\
&= 569.5 \text{ lb.} \\
\text{Weight of displaced air} &= 8{,}000 \times 0.07651 \\
&= 612.1 \text{ lb.} \\
\text{Weight of superheated gas} &= 8{,}000 \times 0.00532 \times \frac{519}{559} \\
&= 39.5 \text{ lb.} \\
\text{Lift with superheat} &= 612.1 - 39.5 \\
&= 572.6 \text{ lb.} \\
\text{Gain} &= 572.6 - 569.5 \\
&= 3.1 \text{ lb.}
\end{aligned}
$$

Check.

$$
\begin{aligned}
\text{Volume of superheated gas} &= 8{,}000 \times \frac{559}{519} \\
&= 8{,}616 \text{ cu. ft.} \\
\text{Volume of escaping gas} &= 8{,}616 - 8{,}000 \\
&= 616 \text{ cu. ft.} \\
\text{Weight of escaping gas} &= 616 \times 0.00532 \times \frac{519}{559} \\
&= 3.1 \text{ lb.}
\end{aligned}
$$

Example. A 10,000-cu.-ft. balloon is inflated with 8,000 cu. ft. of pure hydrogen under standard conditions. What is the additional lift due to superheat of 40°?

Solution.

$$
\begin{aligned}
\text{Lift without superheat} &= 8{,}000\ (0.07651 - 0.00532) \\
&= 569.5 \text{ lb.} \\
\text{Volume of superheated gas} &= 8{,}000 \times \frac{559}{519} \\
&= 8{,}616 \text{ cu. ft.} \\
\text{Weight of displaced air} &= 8{,}616 \times 0.07651 \\
&= 659.2 \text{ lb.} \\
\text{Weight of superheated gas} &= 8{,}000 \times 0.00532 \\
&= 42.6 \text{ lb.} \\
\text{Lift with superheat} &= 659.2 - 42.6 \\
&= 616.6 \text{ lb.} \\
\text{Gain} &= 47.1 \text{ lb.}
\end{aligned}
$$

Problems

1. A 10,000-cu-ft. hydrogen-filled balloon is brought out of the hangar. Both air and hydrogen are at 29.9 in. pressure and 45° F. temperature. (*a*) What is the lift? (*b*) Owing to sun's rays, the hydrogen experiences 30° superheat; what is then the lift?

2. On a cold day a 25,000-cu.-ft. balloon is filled with pure hydrogen. Air and gas are at 30.1 in. pressure and 18° F. temperature. (*a*) What is the lift? (*b*) If the hydrogen is superheated 45° F., what is the lift?

3. A 10,000-cu.-ft. balloon contains 7,000 cu. ft. of pure hydrogen. Air and gas are at 29.9 in. pressure and 70° F. temperature. (*a*) What is lift? (*b*) What is lift if the hydrogen is superheated 35° F.?

4. A 10,000-cu.-ft. balloon contains 9,000 cu. ft. of pure helium. Air and gas are at 29.9 in. pressure and 32° F. temperature. (*a*) What is lift? (*b*) What is the lift if the helium is superheated 50° F.?

5. A 10,000-cu.-ft. balloon is full of hydrogen at 75° F. temperature. Atmospheric pressure is 29.9 in. and atmospheric temperature is 40° F. (*a*) What is lift? (*b*) What is the lift if the sun goes behind clouds and the gas loses all its superheat?

Purity. Any gas which remains inside a balloon for any length of time contains impurities owing to air seeping in through the appendix and mixing with the gas and to other causes. Although the impurities may be dry air, water vapor, carbon dioxide, or other substances, only a slight error is involved if all impurities are considered as being dry air.

The purity of a gas is defined as the ratio of the volume of pure gas in the mixture to the total volume of impure gas. Considering all impurities as being dry air, they merely support themselves and furnish no lift. The impurities merely subtract from the total volume of lifting gas. A volume of gas of x per cent purity is x per cent of the volume of pure gas, giving lift, and $(100 - x)$ per cent of the volume of dry air giving no lift.

Example. A 10,000-cu.-ft. balloon is inflated with hydrogen of 95 per cent purity under standard conditions. What is the lift?

Solution.

Gas is 95 per cent pure hydrogen
5 per cent dry air

$$\begin{aligned} \text{Lift} &= 0.95 \times 10{,}000 \times (0.07651 - 0.00532) \\ &= 676.3 \text{ lb.} \end{aligned}$$

Check.

$$\begin{aligned} \text{Weight of 9,500 cu. ft. of pure hydrogen} &= 9{,}500 \times 0.00532 \\ &= 50.5 \text{ lb.} \\ \text{Weight of 500 cu. ft. of dry air} &= 500 \times 0.07651 \\ &= 38.3 \text{ lb.} \\ \text{Weight of 10,000 cu. ft. of impure gas} &= 88.8 \text{ lb.} \\ \text{Weight of 10,000 cu. ft. of displaced air} &= 10{,}000 \times 0.07651 = \mathbf{765.1\ lb.} \\ \text{Difference in weight of displaced air and gas} &= \mathbf{676.3\ lb.} \end{aligned}$$

Non-Rigid Airship. A non-rigid airship is propelled through the air by one or more engines. To reduce the drag resistance of this motion, the gas bag is made of streamline shape. The rubberized cloth is tailored to the correct shape, and the pressure inside the bag is relied upon to fill out the bag properly. Use is made of two or more ballonets to preserve the shape of the bag.

Without ballonets, a non-rigid airship, which might have been fully inflated on the ground, on ascending would need to have gas valved to prevent bursting, on descending would be flabby. In free balloons there is no special objection to flabbiness; but in dirigibles, flabbiness will destroy the streamline contour of the envelope. Partial inflation means that the nose will be cupped or dished in, increasing the drag enormously.

A ballonet is a bag or compartment in the main gas bag that is formed by a diaphragm made of the same kind of gas-tight rubberized cloth as the outer skin. The ballonet compartment is to hold air, and it has a valve opening to the atmosphere. The main gas bag is sealed to the outer air except for a gas valve used only in emergency.

During inflation the ballonet is filled with air while the main bag is being filled with gas. On rising, the expansion of the gas causes the flexible wall of the ballonet chamber to collapse, expelling air. When the ballonet wall has entirely flattened out against the skin of the envelope so that all the air has been expelled, the non-rigid airship is at its pressure height. On descending, air is introduced into the ballonet, thus maintaining the rigidity of shape of the main envelope. To force air into the ballonet a scoop hangs down from the ballonet with its open end in the slipstream of the propeller. Sometimes a small auxiliary blower is used to pump air into the ballonet.

If the non-rigid airship ever ascends above its pressure height, gas will have to be valved, and on descending, even with full ballonets, the main envelope will still be flabby. For this reason great care needs to be exercised that pressure airships do not rise to too great altitudes.

The aerostatics of pressure airships does not vary much from that of free balloons below the pressure height, bearing in mind that the air in the ballonets contributes no lift.

Superheat affects the operation of an non-rigid airship in the following manner. Bringing the airship out of the hangar into the bright sunshine, the pilot will notice immediately by his pressure gage or manometer that the pressure inside the gas chamber is increasing. The valve releasing air from the ballonet must be opened at once to prevent the gas bag from bursting. Each pound of air valved increases the lift by 1 lb.

When the airship starts to move forward, the superheat is reduced by the air circulating past the outside of the envelope. The gas contracts, air is forced into the ballonet, lift is reduced, and therefore ballast must be discarded.

The ballonet capacity is small compared with the gas capacity of an airship. Because air is heavier than the gas, the ballonets are always located on the lower side of the hull. In this location the sun's rays do not shine on the ballonets but on the upper side of the hull; any heating of the air would be by direct conduction from the warm gas through the separating wall to the air. Any heating of the air would add to the lift in the same way that lift was obtained in the old-style hot-air balloons; ordinarily this additional lift is so small that it is neglected.

Example. A non-rigid airship of 200,000 cu.-ft.-capacity is brought out of the hangar. Atmospheric temperature is 50° F. and pressure is 30.2 in. Ballonets have 40,000-cu.-ft. capacity. How much added lift will the dirigible pick up if gas is superheated 30° F. before take-off?

Solution.

$$\begin{aligned}
\text{Volume of unsuperheated gas} &= 200{,}000 - 40{,}000 \\
&= 160{,}000 \text{ cu. ft.} \\
\text{Original lift} &= 160{,}000\,(0.07651 - 0.00532)\frac{519}{510}\times\frac{30.2}{29.9} \\
&= 11{,}700 \text{ lb.} \\
\text{Volume of superheated gas} &= 160{,}000 \times \frac{540}{510} \\
&= 169{,}410 \text{ cu. ft.} \\
\text{Volume of displaced air} &= 169{,}410 - 160{,}000 \\
&= 9{,}410 \text{ cu. ft.} \\
\text{Weight of displaced air} &= 9{,}410 \times 0.07651 \times \frac{519}{510} \times \frac{30.2}{29.9} \\
&= 738 \text{ lb. gain in lift}
\end{aligned}$$

Check.

$$\begin{aligned}
\text{Original lift} &= 160{,}000\,(0.07651 - 0.00532)\frac{519}{510}\times\frac{30.2}{29.9} \\
&= 11{,}700 \text{ lb.}
\end{aligned}$$

$$
\begin{aligned}
\text{Weight of air displaced after superheat} &= 169{,}410 \times 0.07651 \times \frac{519}{510} \times \frac{30.2}{29.9} \\
&= 13{,}314 \text{ lb.} \\
\text{Weight of gas after superheat} &= 160{,}000 \times 0.00532 \times \frac{519}{510} \times \frac{30.2}{29.9} \\
&= 874 \text{ lb.} \\
\text{Lift after superheat} &= 13{,}314 - 874 \\
&= 12{,}440 \text{ lb.} \\
\text{Gain in lift} &= 12{,}440 - 11{,}700 \\
&= 740 \text{ lb.}
\end{aligned}
$$

Problems

1. A non-rigid airship of 300,000-cu.-ft. capacity has its 50,000-cu.-ft. ballonets full of air; the remaining space is filled with hydrogen at standard conditions. What is the pressure height?

2. A non-rigid airship of 225,000-cu.-ft. capacity has its 30,000-cu.-ft. ballonets full of air. The airship is inflated with pure helium. Air and gas are at 60° F. temperature and 30.1 in. pressure. (*a*) What is the lift if there is no superheat? (*b*) What is the lift with 20° F. superheat?

3. A non-rigid airship of 200,000-cu.-ft. capacity is filled with hydrogen of 95 per cent purity; ballonets are 25,000-cu.-ft. capacity; superheat is 35° F. (*a*) What is the total lift? (*b*) How much ballast must be tossed over when all superheat is lost?

Rigid Airships. A rigid airship has a metal framework which consists of a series of huge rings of varying diameter connected by longitudinal girders extending from the nose to the tail. During the war, for ease and cheapness in construction, the rings, except for those at the nose and tail, were of the same diameter. This gave the " pencil-shape " to the war-time Zeppelins. The modern Zepps have a nicely streamlined contour even though this requires separate design of each ring and more intricate tailoring of the envelope.

There are three keels running lengthwise of the modern rigid ship. The framework of these keels is triangular in cross-section, permitting their use as passageways from stem to stern. In addition to these keels and other longitudinal members, the structure is braced by a multiplicity of wires and cross-pieces.

The gas is contained in ten to twenty individual cells instead of one large cell as in the pressure-type airship. This gives protection similar to the bulkheads of marine vessels: if one or two cells become punctured, the remaining cells will have sufficient buoyancy to keep the ship in the air. A network of wires around each cell prevents undue expansion.

Over the entire framework is stretched the outer cover, which is a doped fabric designed to reflect heat and give a smooth flying surface. The air space between the outer cover and the walls of the gas cells acts as ventilation space.

The cells are only partially inflated on the ground so that expansion of the gas may take place when the airship rises. Flabbiness of the gas cells is permissible in the rigid type, since the streamline shape is preserved by the framework and outer cover.

The dimensions of three famous rigid airships are given below.

	Los Angeles	*Graf Zeppelin*	*Akron*
Nominal gas volume (cu. ft.)	2,470,000	3,700,000	6,500,000
Over-all length (ft.)	658	776	785
Maximum diameter (ft.)	91	100	132
Slenderness ratio	7.2	7.7	6.0
Gross lift (lb.)	153,000	258,000	403,000
Useful lift (lb.)	60,000		182,000
Maximum velocity (miles per hour)	73	80	84

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APPENDIX A

NOMENCLATURE

(From N.A.C.A.–T.R. 474.)

accelerometer — An instrument that measures the accelerations of an aircraft in a defined direction.

acrobatics — Evolutions voluntarily performed with an aircraft other than those required for normal flight.

adjustable propeller — *See* propeller, adjustable.

aerodynamic center, wing section — A point located on or near the chord of the mean line approximately one-quarter of the chord length aft of the leading edge and about which the moment coefficient is practically constant.

aerodynamics — The branch of dynamics that treats of the motion of air and other gaseous fluids and of the forces acting on solids in motion relative to such fluids.

aerostatics — The science that treats of the equilibrium of gaseous fluids and of bodies immersed in them.

aileron — A hinged or movable portion of an airplane wing, the primary function of which is to impress a rolling motion on the airplane. It is usually part of the trailing edge of a wing.

Frise aileron — An aileron having the nose portion projecting ahead of the hinge axis, the lower surface being in line with the lower surface of the wing. When the trailing edge of the aileron is raised, the nose portion protrudes below the lower surface of the wing, increasing the drag.

airfoil — Any surface, such as an airplane wing, aileron, or rudder, designed to obtain reaction from the air through which it moves.

airfoil profile. — The outline of an airfoil section.

airfoil section — A cross-section of an airfoil parallel to the plane of symmetry or to a specified reference plane.

airplane — A mechanically driven fixed-wing aircraft, heavier than air, which is supported by the dynamic reaction of the air against its wings.

canard airplane — A type of airplane having the horizontal stabilizing and control surfaces in front of the main supporting surfaces.

pusher airplane — An airplane with the propeller or propellers aft of the main supporting surfaces.

tailless airplane — An airplane in which the devices used to obtain stability and control are incorporated in the wing.

tractor airplane — An airplane with the propeller or propellers forward of the main supporting surfaces.

airship — An aerostat provided with a propelling system and with means of controlling the direction of motion. A dirigible balloon; lighter than the airship.

non-rigid airship — An airship whose form is maintained by the internal pressure in the gas bags and ballonets.

rigid airship — An airship whose form is maintained by a rigid structure.

semi-rigid airship — An airship whose shape is maintained by means of a rigid or jointed keel in conjunction with internal pressure in the gas containers and ballonets.

air speed — The speed of an aircraft relative to the air.

airspeed head — An instrument which, in combination with a gage, is used to measure the speed of an aircraft relative to the air. It usually consists of a Pitot-static tube or a Pitot-Venturi tube.

airway — An air route along which aids to air navigation, such as landing fields, beacon lights, radio direction-finding facilities, intermediate fields, etc., are maintained.

airworthiness — The quality of an aircraft denoting its fitness and safety for operation in the air under normal flying conditions.

altimeter — An instrument that measures the elevation of an aircraft above a given datum plane.

altitude:

absolute altitude — The height of an aircraft above the earth.

density altitude — The altitude corresponding to a given density in a standard atmosphere.

pressure altitude — (1) The altitude corresponding to a given pressure in a standard atmosphere. (2) The altitude at which the gas bags of an airship become full.

amphibian — An airplane designed to rise from and alight on either land or water.

angle:

aileron angle — The angular displacement of an aileron from its neutral position. It is positive when the trailing edge of the aileron is below the neutral position.

blade angle — The acute angle between the chord of a section of a propeller and a plane perpendicular to the axis of rotation.

dihedral angle — The acute angle between a line perpendicular to the plane of symmetry and the projection of the wing axis on a plane perpendicular to the longitudinal axis of the airplane. If the wing axis is not approximately a straight line, the angle is measured from the projection of a line joining the intersection of the wing axis with the plane of symmetry and the aerodynamic center of the half-wing on either side of the plane of symmetry.

downwash angle — The angle through which an air stream is deflected by any lifting surface. It is measured in a plane parallel to the plane of symmetry.

drift angle — The horizontal angle between the longitudinal axis of an aircraft and its path relative to the ground.

elevator angle — The angular displacement of the elevator from its neutral position. It is positive when the trailing edge of the elevator is below the neutral position.

gliding angle — The angle between the flight path during a glide and a horizontal axis fixed relative to the air.

zero-lift angle — The angle of attack of an airfoil when its lift is zero.

angle of attack — The acute angle between a reference line in a body and the line of the relative wind direction projected on a plane containing the reference line and parallel to the plane of symmetry.

absolute angle of attack — The angle of attack of an airfoil, measured from the attitude of zero lift.

critical angle of attack — The angle of attack at which the flow about an airfoil changes abruptly as shown by corresponding abrupt changes in the lift and drag.

effective angle of attack — *See* angle of attack for infinite aspect ratio.

induced angle of attack — The difference between the actual angle of attack and the angle of attack for infinite aspect ratio of an airfoil for the same lift coefficient.

angle of attack for infinite aspect ratio — The angle of attack at which an airfoil produces a given lift coefficient in a two-dimensional flow. Also called "effective angle of attack."

area, equivalent flat-plate — The area of a square flat plate, normal to the direction of motion, which offers the same amount of resistance to motion as the body or combination of bodies under consideration.

area, measurement of (performance calculations):

horizontal tail area — The horizontal tail area is measured in the same manner as the wing area, that is, with no deduction for the area blanketed by the fuselage, such blanketed area being bounded within the fuselage by lateral straight lines that connect the intersections of the leading and trailing edges of the stabilizer with the sides of the fuselage, the fairings and fillets being ignored.

vertical tail area — The area of the actual outline of the rudder and the fin projected in the vertical plane, the fairings and fillets being ignored.

wing area — Wing area is measured from the projection of the actual outline on the plane of the chords, without deduction for area blanketed by fuselage or nacelles. That part of the area, so determined, which lies within the fuselage or nacelles is bounded by two lateral lines that connect the intersections of the leading and trailing edges with the fuselage or nacelle, ignoring fairings and fillets. For the purpose of calculating area, a wing is considered to extend without interruption through the fuselage and nacelles. Unless otherwise stated, wing area always refers to total area including ailerons.

area, propeller-disk — The total area swept by a propeller; i.e., the area of a circle having the same diameter as the propeller.

aspect ratio — The ratio of the span to the mean chord of an airfoil; i.e., the ratio of the square of the span to the total area of an airfoil.

effective aspect ratio — The aspect ratio of an airfoil of elliptical plan form that, for the same lift coefficient, has the same induced-drag coefficient as the airfoil, or the combination of airfoils, in question.

atmosphere:

standard atmosphere — An arbitrary atmosphere used in comparing the performance of aircraft. The standard atmosphere in use in the United States at present represents very nearly the average conditions found at latitude 40° and is completely defined in N.A.C.A. Report 218.

autogiro — A type of rotor plane whose support in the air is chiefly derived from airfoils rotated about an approximately vertical axis by aerodynamic forces, and in which the lift on opposite sides of the plane of symmetry is equalized by the vertical oscillation of the blades.

automatic pilot — An automatic control mechanism for keeping an aircraft in level flight and on a set course. Sometimes called "gyro pilot," "mechanical pilot," or "robot pilot."

axes of an aircraft — Three fixed lines of reference, usually centroidal and mutually perpendicular. The horizontal axis in the plane of symmetry is called the longitudinal axis; the axis perpendicular to this in the plane of symmetry is called the normal axis; and the third axis perpendicular to the other two is called the lateral axis. In mathematical discussions, the first of these axes, drawn from rear to front, is called the X axis; the second, drawn downward, the Z axis; and the third, running from left to right, the Y axis.

axis, wing — The locus of the aerodynamic centers of all the wing sections.

balance — A condition of steady flight in which the resultant force and moment on the airplane are zero.

bank — The position of an airplane when its lateral axis is inclined to the horizontal. A right bank is the position with the lateral axis inclined downward to the right.

bank — To incline an airplane laterally; i.e., to rotate it about its longitudinal axis.

biplane — An airplane with two main supporting surfaces placed one above the other.

blade element — A portion of a propeller blade contained between the surfaces of two cylinders coaxial with the propeller cutting the propeller blades.

blade face — The surface of a propeller blade that corresponds to the lower surface of an airfoil. Sometimes called "thrust face" or "driving face."

blade section — A cross-section of a propeller blade made at any point by a plane parallel to the axis of rotation of the propeller and tangent at the centroid of the section to an arc drawn with the axis of rotation as its center.

boundary layer — A layer of fluid, close to the surface of a body placed in a moving stream, in which the impact pressure is reduced as a result of the viscosity of the fluid.

camber — The rise of the curve of an airfoil section, usually expressed as the ratio of the departure of the curve from a straight line joining the extremities of the curve to the length of this straight line. "Upper camber" refers to the upper surface; "lower camber" to the lower surface; and "mean camber" to the mean line of the section.

ceiling:

absolute ceiling — The maximum height above sea level at which a given airplane would be able to maintain horizontal flight under standard air conditions.

service ceiling — The height above sea level, under standard air conditions, at which a given airplane is unable to climb faster than a small specified rate (100 ft. per min. in the United States and England). This specified rate may differ in different countries.

cellule (or cell) — In an airplane, the entire structure of the wings and wing trussing of the whole airplane on one side of the fuselage, or between fuselages or nacelles if there are more than one.

center of pressure of an airfoil — The point in the chord of an airfoil, prolonged if necessary, which is at the intersection of the chord and the line of action of the resultant air force.

center-of-pressure coefficient — The ratio of the distance of the center of pressure from the leading edge to the chord length.

center section — The central panel of a wing; in the case of a continuous wing or any wing having no central panel, the limits of the center section are arbitrarily defined by the location of points of attachment to the cabane struts or fuselage.

chord — An arbitrary datum line from which the ordinates and angles of an airfoil are measured. It is usually the straight line tangent to the lower surface at two points, the straight line joining the ends of the mean line, or the straight line between the leading and trailing edges.

chord, mean aerodynamic — The chord of an imaginary airfoil which would have force vectors throughout the flight range identical with those of the actual wing or wings.

chord length — The length of the projection of the airfoil profile on its chord.

cockpit — An open space in an airplane for the accommodation of pilots or passengers. When completely enclosed, such a space is usually called a cabin.

control column — A lever having a rotatable wheel mounted at its upper end for operating the longitudinal and lateral control surfaces of an airplane. This type of control is called "wheel control."

controllability — The quality of an aircraft that determines the ease of operating its controls and/or the effectiveness of displacement of the controls in producing change in its attitude in flight.

decalage — The difference between the angular settings of the wings of a biplane or multiplane. The decalage is measured by the acute angle between the chords in a plane parallel to the plane of symmetry. The decalage is considered positive if the upper wing is set at the larger angle.

dive — A steep descent, with or without power, in which the airspeed is greater than the maximum speed in horizontal flight.

downwash — The air deflected perpendicular to the direction of motion of an airfoil.

drag — The component of the total air force on a body parallel to the relative wind.

induced drag — That part of the drag induced by the lift.

parasite drag — That portion of the drag of an aircraft exclusive of the induced drag of the wings.

profile drag — The difference between the total wing drag and the induced drag.

profile drag, effective — The difference between the total wing drag and the induced drag of a wing with the same geometric aspect ratio but elliptically loaded.

drag force or component (stress analysis) — A force or component, in the drag direction, i.e., parallel to the relative wing.

drag strut — A fore-and-aft compression member of the internal bracing system of an aircraft.

dynamic pressure — The product $\frac{1}{2}\rho V^2$, where ρ is the density of the air and V is the relative speed of the air.

engine:

compression-ignition engine — A type of engine in which the fuel is sprayed into the cylinder and ignited by the heat of compression of the air charge.

right-hand engine — An engine whose propeller shaft, to an observer facing the propeller from the engine end of the shaft, rotates in a clockwise direction.

supercharged engine — An engine with a compressor for increasing the air or mixture charge taken into the cylinder beyond that inducted normally at the existing atmospheric pressure.

engine, dry weight of — The weight of an engine exclusive of fuel, oil, and liquid coolant.

engine weight per horsepower — The dry weight of an engine divided by the rated horsepower.

equivalent monoplane — A monoplane wing equivalent as to its lift and drag properties to any combination of two or more wings.

fairing — An auxiliary member or structure whose primary function is to reduce the drag of the part to which it is fitted.

fin — A fixed or adjustable airfoil, attached to an aircraft approximately parallel to the plane of symmetry, to afford directional stability; for example, tail fin, skid fin, etc.

fineness ratio — The ratio of the length to the maximum diameter of a streamline body, as an airship hull.

fishtail — A colloquial term describing the motion made when the tail of an airplane is swung from side to side to reduce speed in approaching the ground for a landing.

flap — A hinged or pivoted airfoil forming the rear portion of an airfoil, used to vary the effective camber.

split flap — A hinged plate forming the rear upper or lower portion of an airfoil. The lower portion may be deflected downward to give increased lift and drag; the upper portion may be raised over a portion of the wing for the purpose of lateral control.

flight path — The flight path of the center of gravity of an aircraft with reference to a frame fixed relative to the air or with reference to the earth.

flow:

laminar flow — A particular type of streamline flow. The term is usually applied to the flow of a viscous liquid near solid boundaries, when the flow is not turbulent.

streamline flow — A fluid flow in which the streamlines, except those very near a body and in a narrow wake, do not change with time.

turbulent flow — Any part of a fluid flow in which the velocity at a given point varies more or less rapidly in magnitude and direction with time.

flutter — An oscillation of definite period but unstable character set up in any part of an aircraft by a momentary disturbance, and maintained by a combination of the aerodynamic, inertial, and elastic characteristics of the member itself.

fuselage — The body, of approximately streamline form, to which the wings and tail unit of an airplane are attached.

monocoque fuselage — A fuselage construction which relies on the strength of the skin or shell to carry either the shear or the load due to bending moments. Monocoques may be divided into three classes (reinforced shell, semi-monocoque, and monocoque), and different portions of the same fuselage may belong to any one of these classes. The reinforced shell has the skin reinforced by a complete framework of structural members. The semi-monocoque has the skin reinforced by longerons and vertical bulkheads, but has no diagonal web members. The monocoque has as its only reinforcement vertical bulkheads formed of structural members.

gap — The distance separating two adjacent wings of a multiplane.

glide — To descend at a normal angle of attack with little or no thrust.

ground loop — An uncontrollable violent turn of an airplane while taxying, or during the landing or take-off run.

horn — A short lever attached to a control surface of an aircraft, to which the operating wire or rod is connected.

horsepower of an engine, rated — The average horsepower developed by a given type of engine at the rated speed when operating at full throttle or at a specified altitude or manifold pressure.

Immelman turn, normal — A maneuver made by completing the first half of a normal loop, then, from the inverted position at the top of the loop, half-rolling the airplane to the level position, thus obtaining a 180° change in direction simultaneously with a gain in altitude.

impact pressure — The pressure acting at the forward stagnation point of a body, such as a Pitot tube, placed in an air current. Impact pressure may be measured from an arbitrary datum pressure.

inclinometer — An instrument that measures the attitude of an aircraft with respect to the horizontal.

induction system, rotary — A carburetor induction system used on radial engines, in which a rotary fan assists in distributing the fuel charge to the cylinders.

instability, spiral — A type of instability, inherent in certain airplanes, which becomes evident when the airplane assumes too great a bank and side-slips; the bank continues to increase and the radius of the turn to decrease.

instrument flying — The art of controlling an aircraft solely by the use of instruments; sometimes called "blind flying."

interceptor — A lateral-control device consisting of a small plate placed just back of a wing slot to spoil the effect of the slot at high angles of attack.

interference — The aerodynamic influence of two or more bodies on one another.

landing gear — The understructure which supports the weight of an aircraft when in contact with the land or water and which usually contains a mechanism for reducing the shock of landing. Also called " undercarriage."

retractable landing gear — A type of landing gear which may be withdrawn into the body or wings of an airplane while it is in flight, in order to reduce the parasite drag.

leading edge — The foremost edge of an airfoil or propeller blade.

level-off — To make the flight path of an airplane horizontal after a climb, glide, or dive.

lift: dynamic — The component of the total aerodynamic force on a body perpendicular to the relative wind.

lift/drag ratio — The ratio of the lift to the drag of any body.

load:

full load — Weight empty plus useful load; also called gross weight.

pay load — That part of the useful load from which revenue is derived, viz., passengers and freight.

useful load — The crew and passengers, oil and fuel, ballast other than emergency, ordnance, and portable equipment.

loading:

power loading — The gross weight of an airplane divided by the rated horsepower of the engine computed for air of standard density, unless otherwise stated.

span loading — The ratio of the weight of an airplane to its equivalent monoplane span.

unsymmetrical loading (stress analysis) — A design loading condition for the wings and connecting members, representing the conditions as in a roll.

wing loading — The gross weight of an airplane divided by the wing area.

longeron — A principal longitudinal member of the framing of an airplane fuselage or nacelle, usually continuous across a number of points of support.

loop — A maneuver executed in such a manner that the airplane follows a closed curve approximately in a vertical plane.

maneuverability — That quality in an aircraft which determines the rate at which its altitude and direction of flight can be changed.

mean line (of an airfoil profile) — An intermediate line between the upper and lower contours of the profile.

mixture control, altitude — A device on the carburetor for regulating the weight proportions of air and fuel supplied to the engine at different altitudes.

monoplane — An airplane with but one main supporting surface, sometimes divided into two parts by the fuselage.

high-wing monoplane — A monoplane in which the wing is located at, or near, the top of the fuselage.

low-wing monoplane — A monoplane in which the wing is located at, or near, the bottom of the fuselage.

midwing monoplane — A monoplane in which the wing is located approximately midway between the top and bottom of the fuselage.

parasol monoplane — A monoplane in which the wing is above the fuselage.

multiplane — An airplane with two or more main supporting surfaces placed one above another.

nacelle — An enclosed shelter for personnel or for a power plant. A nacelle is usually shorter than a fuselage, and does not carry the tail unit.

nose-down — To depress the nose of an airplane in flight.

nose-over — A colloquial expression referring to the accidental turning over of an airplane on its nose when landing.

oleo gear — A type of oil-damping device that depends on the flow of oil through an orifice for its shock-absorbing effect in a landing gear.

oscillation:

phugoid oscillation — A long-period oscillation characteristic of the disturbed longitudinal motion of an aircraft.

stable oscillation — An oscillation whose amplitude does not increase.

unstable oscillation — An oscillation whose amplitude increases continuously until an attitude is reached from which there is no tendency to return toward the original attitude, the motion becoming a steady divergence.

over-all length — The distance from the extreme front to the extreme rear of an aircraft, including the propeller and tail unit.

overhang — (1) One-half the difference in span of any two main supporting surfaces of an airplane. The overhang is positive when the upper of the two main supporting surfaces has the larger span. (2) The distance from the outer strut attachment to the tip of a wing.

overshoot — To fly beyond a designated mark or area, such as a landing field, while attempting to land on the mark or within the area.

panel (airplane) — A portion of an airplane wing constructed separately from the rest of the wing to which it is attached.

pitch — An angular displacement about an axis parallel to the lateral axis of an aircraft.

pitch of a propeller:

effective pitch — The distance an aircraft advances along its flight path for one revolution of the propeller.

geometrical pitch — The distance an element of a propeller would advance in one revolution if it were moving along a helix having an angle equal to its blade angle.

zero-thrust pitch — The distance a propeller would have to advance in one revolution to give no thrust. Also called "experimental mean pitch."

pitch ratio (propeller) — The ratio of the pitch to the diameter.

pitot-static tube — A parallel or coaxial combination of a Pitot and a static tube. The difference between the impact pressure and the static pressure is a function of the velocity of flow past the tube.

Pitot tube — A cylindrical tube with an open end pointed upstream, used in measuring impact pressure.

Pitot-Venturi tube — A combination of a Pitot and a Venturi tube.

plane (or hydroplane) — To move through the water at such a speed that the support derived is due to hydrodynamic and aerodynamic rather than to hydrostatic forces.

plan form, developed — The plan of an airfoil as drawn with the chord lines at each section rotated about the airfoil axis into a plane parallel to the plane of projection and with the airfoil axis rotated and developed and projected into the plane of projection.

plan form, projected — The contour as viewed from above.

profile thickness — The maximum distance between the upper and lower contours of an airfoil, measured perpendicularly to the mean line of the profile.

propeller — Any device for propelling a craft through a fluid, such as water or air; especially a device having blades which, when mounted on a power-driven shaft, produce a thrust by their action on the fluid.

adjustable propeller — A propeller whose blades are so attached to the hub that the pitch may be changed while the propeller is at rest.

automatic propeller — A propeller whose blades are attached to a mechanism that automatically sets them at their optimum pitch for various flight conditions.

controllable propeller — A propeller whose blades are so mounted that the pitch may be changed while the propeller is rotating.

geared propeller — A propeller driven through gearing, generally at some speed other than the engine speed.

pusher propeller — A propeller mounted on the rear end of the engine or propeller shaft.

tractor propeller — A propeller mounted on the forward end of the engine or propeller shaft.

propeller efficiency — The ratio of the thrust power to the input power of a propeller.

propeller rake — The mean angle which the line joining the centroids of the sections of a propeller blade makes with a plane perpendicular to the axis.

propeller root — That part of the propeller blade near the hub.

propeller thrust — The component of the total air force on the propeller which is parallel to the direction of advance.

propeller thrust, effective — The net driving force developed by a propeller when mounted on an aircraft, i.e., the actual thrust exerted by the propeller, as mounted on an airplane, minus any increase in the resistance of the airplane due to the action of the propeller.

propeller thrust, static — The thrust developed by a propeller when rotating without translation.

propeller tipping — A protective covering of the blade of a propeller near the tip.

propulsive efficiency — The ratio of the product of the effective thrust and flight speed to the actual power input into the propeller as mounted on the airplane.

pull-out — The maneuver of transition from a dive to horizontal flight.

pull-up — A maneuver, in the vertical plane, in which the airplane is forced into a short climb, usually from approximately level flight (*cf.* zoom).

sudden pull-up (or sudden pull-out) (stress analysis) — A loading condition for the tail surfaces resulting from a sudden application of up-elevator (*cf.* dive).

purity (of gas) — The ratio of the partial pressure of the aerostatic gas in the container to the total pressure of all the contained gases.

range, maximum — The maximum distance a given aircraft can cover under given conditions, by flying at the economical speed and altitude at all stages of the flight.

range at maximum speed — The maximum distance a given aircraft can fly at full speed at the altitude for maximum speed under given conditions.

rate-of-climb indicator — An instrument that indicates the rate of ascent or descent of an aircraft.

Reynolds number — A non-dimensional coefficient used as a measure of the dynamic scale of a flow. Its usual form is the fraction Vl/v in which V is the velocity of the fluid, l is a linear dimension of a body in the fluid, and v is the kinematic viscosity of the fluid (*cf.* scale effect).

righting or restoring moment — A moment that tends to restore an aircraft to its previous attitude after any small rotational displacement.

ring cowling — A ring-shaped cowling placed around a radial air-cooled engine to reduce its drag and improve cooling.

roll — A maneuver in which a complete revolution about the longitudinal axis is made, the horizontal direction of flight being approximately maintained.

aileron roll — A roll in which the motion is largely maintained by forces arising from the displacement of the aileron.

outside roll — A roll executed while flying in the negative angle-of-attack range.

snap roll — A roll executed by a quick movement of the controls, in which the motion is largely maintained by autorotational couples on the wings.

rudder — A hinged or movable auxiliary airfoil on an aircraft, the function of which is to impress a yawing moment on the aircraft.

rudder bar — The foot bar by means of which the control cables leading to the rudder are operated.

scale effect — The change in any force coefficient, such as the drag coefficient, due to a change in the value of Reynolds number.

seaplane — An airplane designed to rise from and alight on the water.

sesquiplane — A form of biplane in which the area of one wing is less than half the area of the other.

sideslipping — Motion of an aircraft relative to the air, in which the lateral axis is inclined and the airplane has a velocity component along the lateral axis. When it occurs in connection with a turn, it is the opposite of skidding.

sinking speed — The vertical downward component of velocity that an aircraft would have while descending in still air under given conditions of equilibrium.

slip — The difference between the geometrical pitch and the effective pitch of a propeller. Slip may be expressed as a percentage of the mean geometrical pitch, or as a linear dimension.

slip function — The ratio of the speed of advance through the undisturbed air to the product of the propeller diameter and the number of revolutions per unit time: V/nD.

slipstream — The current of air driven astern by a propeller.

slot — The nozzle-shaped passage through a wing whose primary object is to improve the flow conditions at high angles of attack. It is usually near the leading edge and formed by a main and an auxiliary airfoil.

span — The maximum distance, measured parallel to the lateral axis, from tip to tip of an airfoil, of an airplane wing inclusive of ailerons, or of a stabilizer inclusive of elevator.

effective span — The true span of a wing less corrections for tip loss.

speed:

ground speed — The horizontal component of the velocity of an aircraft relative to the ground.

landing speed — The minimum speed of an airplane at the instant of contact with the landing area in a normal landing.

minimum flying speed — The lowest steady speed that can be maintained, with any throttle setting whatsoever, by an airplane in level flight at an altitude above the ground greater than the span of the wings.

rated engine speed — The rotative speed of an engine at which its reliability has been determined for continuous performance.

stalling speed — The speed of an airplane in steady flight at its maximum coefficient of lift.

take-off speed — The airspeed at which an airplane becomes entirely airborne.

spin — A maneuver in which an airplane descends along a helical path of large pitch and small radius while flying at a mean angle of attack greater than the angle of attack at maximum lift (*cf.* spiral).

flat spin — A spin in which the longitudinal axis is less than 45° from the horizontal.

inverted spin — A maneuver having the characteristics of a normal spin except that the airplane is in an inverted attitude.

normal spin — A spin which is continued by reason of the voluntary position of the control surfaces, recovery from which can be effected within two turns by neutralizing or reversing all the controls. Sometimes called " controlled spin."

uncontrolled spin — A spin in which the controls are of little or no use in effecting a recovery.

spinner — A fairing of approximately conical or paraboloidal shape, which is fitted coaxially with the propeller hub and revolves with the propeller.

spiral — A maneuver in which an airplane descends in a helix of small pitch and large radius, the angle of attack being within the normal range of flight angles.

split S — A maneuver consisting of a half snap roll followed by a pull-out to normal flight, thus obtaining a 180° change in direction accompanied by a loss of altitude.

spoiler — A small plate arranged to project above the upper surface of a wing to disturb the smooth air flow, with consequent loss of lift and increase of drag.

stability — That property of a body which causes it, when its equilibrium is disturbed, to develop forces or moments tending to restore the original condition.

automatic stability — Stability dependent upon movable control surfaces automatically operated by mechanical means.

directional stability — Stability with reference to disturbances about the normal axis of an aircraft, i.e., disturbances which tend to cause yawing.

dynamic stability — That property of an aircraft which causes it, when its state of steady flight is disturbed, to damp the oscillations set up by the restoring forces and moments and gradually return to its original state.

inherent stability — Stability of an aircraft due solely to the disposition and arrangement of its fixed parts; i.e., that property which causes it, when disturbed, to return to its normal attitude of flight without the use of the controls or the interposition of any mechanical device.

lateral stability — Stability with reference to disturbances about the longitudinal axis.

longitudinal stability — Stability with reference to disturbances in the plane of symmetry; i.e., disturbances involving pitching and variation of the longitudinal and normal velocities.

static stability — That property of an aircraft which causes it, when its state of steady flight is disturbed, to develop forces and moments tending to restore its original condition.

stabilizer (airplane) — Any airfoil whose primary function is to increase the stability of an aircraft. It usually refers to the fixed horizontal surface.

stagger — A term referring to the longitudinal position of the axes of two wings of an airplane. Stagger of any section is measured by the acute angle between a line joining the wing axes and a line perpendicular to the upper wing chord, both lines lying in a plane parallel to the plane of symmetry. The stagger is positive when the upper wing is in advance of the lower.

stall — The condition of an airfoil or airplane in which it is operating at an angle of attack greater than the angle of attack of maximum lift.

static pressure — The force per unit area exerted by a fluid on a surface at rest relative to the fluid.

streamline — The path of a particle of a fluid, supposedly continuous, commonly taken relative to a solid body past which the fluid is moving; generally used only of such flows as are not eddying.

streamline form — The form of a body so shaped that the flow about it tends to be a streamline flow.

strut — A compression member of a truss frame.

supercharge — To supply an engine with more air or mixture than would be inducted normally at the prevailing atmospheric pressure. The term supercharged is generally used to refer to conditions at altitudes where the pressure in the intake manifold is partly or completely restored to that existing under normal operation at sea-level.

supercharger — A pump for supplying the engine with a greater weight of air or mixture than would normally be inducted at the prevailing atmospheric pressure.

centrifugal-type supercharger — A high-speed rotary blower equipped with one or more multiblade impellers which, through centrifugal action, compress the air or mixture in the induction system.

positive-driven-type supercharger — A supercharger driven at a fixed ratio from the engine shaft by gears or other positive means.

Roots-type supercharger — A positive-displacement rotary blower consisting of two double-lobed impellers turning in opposite directions on parallel shafts within a housing, the impellers rolling together except for a small clearance, meanwhile alternately trapping incoming air or mixture in the ends of the housing and sweeping it through to the outlet.

sweepback — The acute angle between a line perpendicular to the plane of symmetry and the plan projection of a reference line in the wing.

tachometer — An instrument that measures in revolutions per minute the rate at which the crankshaft of an engine turns.

tail, airplane — The rear part of an airplane, usually consisting of a group of stabilizing planes, or fins, to which are attached certain controlling surfaces such as elevators and rudders; also called "empennage."

tail boom — A spar or outrigger connecting the tail surfaces and the main supporting surfaces.

tailheavy — The condition of an airplane in which the tail tends to sink when the longitudinal control is released in any given attitude of normal flight.

tail skid — A skid for supporting the tail of an airplane on the ground.

take-off — The act of beginning flight in which an airplane is accelerated from a state of rest to that of normal flight. In a more restricted sense, the final breaking of contact with the land or water.

taper in plan only — A gradual change (usually a decrease) in the chord length along the wing span from the root to the tip, with the wing sections remaining geometrically similar.

taper in thickness ratio only — A gradual change in the thickness ratio along the wing span with the chord remaining constant.

taxi — To operate an airplane under its own power, either on land or on water, except as necessarily involved in take-off or landing.

thickness ratio — The ratio of the maximum thickness of an airfoil section to its chord.

trailing edge — The rearmost edge of an airfoil or of a propeller blade.

turn indicator — An instrument for indicating the existence and approximate magnitude of angular velocity about the normal axis of an aircraft.

velocity, terminal — The hypothetical maximum speed that an airplane could attain along a specified straight flight path under given conditions of weight and propeller operation, if diving an unlimited distance in air of specified uniform density. If the term is not qualified, a vertical path angle, normal gross weight, zero thrust, and standard sea-level air density are assumed.

Venturi tube (or Venturi) — A short tube of varying cross-section. The flow through the Venturi causes a pressure drop in the smallest section, the amount of the drop being a function of the velocity of flow.

visibility — The greatest distance at which conspicuous objects can be seen and identified.

warp — To change the form of a wing by twisting it. Warping was formerly used to perform the function now performed by ailerons.

wash — The disturbance in the air produced by the passage of an airfoil. Also called the " wake " in the general case for any solid body.

washin — A warp of an airplane wing giving an increase of the angle of attack toward the tip.

washout — A warp of an airplane wing giving a decrease of the angle of attack toward the tip.

weight:

empty weight — The structure, power plant, and fixed equipment of an aircraft. Included in this fixed equipment are the water in the radiator and cooling system, all essential instruments and furnishings, fixed electric wiring for lighting, heating, etc. In the case of an aerostat, it also includes the amount of ballast that must be carried to assist in making a safe landing.

fixed power plant weight for a given airplane weight — The weight of the power plant and its accessories, exclusive of fuel and oil and their tanks.

gross weight (airplane) — The total weight of an airplane when fully loaded (*cf.* load, full).

net weight (stress analysis) — The gross weight, less some specific partial weight. Very often the partial weight is the dead weight of the wings, but it may be the useful load. The partial weight in question should always be clearly indicated by the context.

wheel, tail — A wheel used to support the tail of an airplane when on the ground. It may be steerable or non-steerable, fixed or swiveling.

wind, relative — The velocity of the air with reference to a body in it. It is usually determined from measurements made at such a distance from the body that the disturbing effect of the body upon the air is negligible.

window, inspection — A small transparent window fitted in the envelope of a balloon or airship, or in the wing or fuselage of an airplane, to allow inspection of the interior.

wind tunnel — An apparatus producing an artificial wind or air stream, in which objects are placed for investigating the air flow about them and the aerodynamic forces exerted on them.

wing — A general term applied to the airfoil, or one of the airfoils, designed to develop a major part of the lift of a heavier-than-air craft.

equivalent wing (stress analysis) — A wing of the same span as the actual wing, but with the chord at each section reduced in proportion to the ratio of the average beam load at that section to the average beam load at the section taken as the standard.

wingheavy, right or left — The condition of an airplane whose right or left wing tends to sink when the lateral control is released in any given attitude of normal flight.

wing-over — A maneuver in which the airplane is put into a climbing turn until nearly stalled, at which point the nose is allowed to fall while continuing the turn, then returned to normal flight from the ensuing dive or glide in a direction approximately 180° from that at the start of the evolution.

wing rib — A chord-wise member of the wing structure of an airplane, used to give the wing section its form and to transmit the load from the fabric to the spars.

compression wing rib — A heavy rib designed to perform the function of an ordinary wing rib and also to act as a strut opposing the pull of the wires in the internal drag truss.

former (or false) wing rib — An incomplete rib, frequently consisting only of a strip of wood extending from the leading edge to the front spar, which is used to assist in maintaining the form of the wing where the curvature of the airfoil section is sharpest.

wing section — A cross-section of a wing parallel to the plane of symmetry or to a specified reference plane.

wing skid — A skid placed near the wing tip to protect the wing from contact with the ground.

wing spar — A principal span-wise member of the wing structure of an airplane.

wing-tip rake — A term referring to the shape of the tip of the wing when the tip edge is sensibly straight in plan but is not parallel to the plane of symmetry. The amount of rake is measured by the acute angle between the straight portion of the wing tip and the plane of symmetry. The rake is positive when the trailing edge is longer than the leading edge.

wire (airplane):

antidrag wire — A wire intended primarily to resist the forces acting forward in the chord direction. It is generally enclosed in the wing.

drag wire — A wire intended primarily to resist the forces acting backward in the chord direction. It is generally enclosed in the wing.

landing wire — A wire or cable which braces the wing against the forces opposite to the normal direction of the lift.

lift wire — A wire or cable which braces the wings against the lift force; sometimes called " flying wire."

stagger wire — A wire connecting the upper and lower wings of an airplane and lying in a plane substantially parallel to the plane of symmetry; also called " incidence wire."

wire (airship):

antiflutter wire — A wire in the plane of the outer cover for local reinforcement and for reducing flutter due to variations in air pressure or propeller wash.

chord wire — A wire joining the vertices of a main transverse frame.

yaw — An angular displacement about an axis parallel to the normal axis of an aircraft.

zoom — To climb for a short time at an angle greater than the normal climbing angle, the airplane being carried upward at the expense of kinetic energy.

APPENDIX B

ANSWERS TO PROBLEMS

Chap. I, page 3

(**1**) 0.00175 slug per cu. ft.
(**2**) 0.00166 slug per cu. ft.
(**3**) 0.0659 lb. per cu. ft.

Chap. I, page 6

(**1**) 0.001225
(**2**) 0.000992
(**3**) 0.000810

Chap. I, page 7

(**1**) 0.000634
(**2**) 0.000472
(**3**) 0.000375

Chap. II, page 15

(**1**) 4190 lb.
(**2**) 3.8 hp.
(**3**) 2.4 hp.
(**4**) 1.8 hp.

Chap. II, page 17

(**1**) *a.* 79.8 lb.
b. 15.2 lb.
c. 81.4 lb.
d. 81.4 lb.
(**2**) 1.2 lb.
(**3**) *a.* 7.7 lb.
b. 1.1 lb.
c. 87° 28′
(**4**) *a.* 125.5 lb.
b. 1180 lb.
(**5**) 840 lb.

Chap. II, page 21

(**1**) 11.9 lb.
(**2**) 47.6 lb.
(**3**) 6.1 lb.

Chap. III, page 27

(**1**) 4,480,000
(**2**) 5,900,000
(**3**) 3,400,000
(**4**) 1,200 mi. per hr.
(**5**) 26.6 atmos.

Chap. IV, page 37

(**1**) 4,570 lb.
(**2**) 3,260 lb.
(**3**) 1.9°
(**4**) 4.6°
(**5**) 8.4°
(**6**) 606 sq. ft.
(**7**) 6,420 lb.
(**8**) 4,760 lb.
(**9**) 56.0 mi. per hr.
(**10**) 65.0 mi. per hr.
(**11**) 45 lb. per sq. ft.
(**12**) 33 lb. per sq. ft.

Chap. IV, page 38

(1) 56.9 mi. per hr.
(2) 55.0 mi. per hr.
(3) 418 sq. ft.
(4) 4.9 lb. per sq. ft.
(5) 2,700 lb.
(6) 69.0 mi. per hr.
(7) 7.4 lb. per sq. ft.
(8) *a.* 44.8 mi. per hr.
b. 52.0 mi. per hr.

Chap. IV, page 40

(1) 308 lb.
(2) 664 lb.
(3) 237 hp.
(4) 15.4 hp.
(5) *a.* 126 lb.
b. 21.7 hp.

Chap. IV, page 42

(1) *a.* 2,260 lb.
b. 116 lb.
(2) 866 lb.
(3) 2,320 lb.
(4) 55.8 mi. per hr.
(5) 65.1 mi. per hr.
(6) 3,250 lb.
(7) 2,380 lb.

Chap. IV, page 51

(1) *a.* 176 lb., 70.5 hp.
b. 150 lb., 60.3 hp.
c. 148 lb., 59.6 hp.
(2) *a.* 299 lb., 100 hp.
b. 400 lb., 133 hp.
c. 548 lb., 183 hp.
(3) *a.* 259 lb., 73.0 hp.
b. 259 lb., 92 hp.
(4) *a.* 96.5 hp.
b. 77.4 hp.
(5) *a.* $-1.5°$
b. 1.7°

Chap. IV, page 54

(1) *b.* 189 lb.
c. 250 lb.
d. 236 lb.
(2) *b.* 87 lb.
c. 5°
(3) *b.* 83 lb.
c. 3°
d. 107 lb.

Chap. IV, page 58

(1) 40.3 hp.
(2) 350 sq. ft.
(3) 194 lb.
(4) 5,760 lb.
(5) 244 lb.
(6) 63.5 hp.
(7) 34.9 hp.
(8) 18.8
(9) *a.* 11,900 lb.
b. 909 lb.
(10) 118 ft. per sec. or
80.4 mi. per hr.

Chap. IV, page 63

(1) 1,480 lb.
(2) 52 mi. per hr.
(3) 4°
(4) *a.* 4.5° *b.* 7.8°
(5) *a.* 110.9 lb.
b. 88.4 lb.
(6) *a.* 20.45 lb. per sq. ft.
b. 15.1 lb. per sq. ft.
(7) *a.* 90.9 mi. per hr.
b. 105.3 mi. per hr.
(8) 46.5 mi. per hr.
(9) *a.* 131 mi. per hr.
b. 152 mi. per hr.
(10) 98.6 sq. ft.

Chap. IV, page 64

(1) 66.5 hp.
(2) 102 mi. per hr.
(3) 286 sq. ft.
(4) *a.* 14.5 hp.
b. 18.4 hp.
(5) *a.* 24.25 hp.
b. 30.6 hp.

Chap. IV, page 66

(1) 98.7 hp.
(2) 36.8 hp.
(3) 76.5 hp.
(4) 23.1 hp.

Chap. IV, page 69

(2) 7,890 ft-lb. (3) 13,400 ft-lb. (4) 3,430 ft-lb. (5) 2,220 lb.

Chap. IV, page 73

(1) 41 per cent from L.E.
(2) *a.* −5,440 ft-lb.
b. −.114
c. −.0326
(3) *a.* 37 per cent from L.E.
b. 36.9 per cent from L.E.
c. −.078
(4) −.008
(5) .4
(6) −.05

Chap. V, page 86

(1) 1.4°, 0.016
(2) 2.42°, 0.0304
(3) 125 lb.
(4) 35.4 hp.
(5) 3.8 hp.
(6) 82 lb.
(7) 72 lb.
(8) 9.7 hp.
(9) *a.* 88 lb.
b. 119 lb.
c. 172 lb.
(10) *a.* 268 lb. *b.* 67 lb.

Chap. V, page 91

(1) *a.* 0.027
b. 25.9
(2) *a.* 0.0295
b. 23.7
(3) *a.* 0.036
b. 19.4
(4) *a.* −0.4°
b. 0.015
(5) *a.* 8.7°
b. 0.079
(6) *a.* 7.3°
b. 0.036
(7) *a.* −0.6°
b. 0.028
(8) *a.* 1.9°
b. 0.040
(9) *a.* 7.3°
b. 0.000142

Chap. V, page 93

(1) 141.6 lb.
(2) 166 lb.
(3) 48 lb.
(4) 145 lb.
(5) 549 lb.
(6) 258 lb.
(7) 276 lb.
(8) 671.3 lb.
(9) 295 lb.
(10) 388 lb.

Chap. V, page 95

(1) 0.0452
(2) 0.0372
(3) 0.0308
(4) 0.0324
(5) 0.0375

Chap. V, page 99

(1) 0.0657
(2) 0.0974
(3) 0.56
(4) 0.492
(5) 0.408

Chap. VI, page 108

(1) 4.47
(2) 4.6
(3) 3.9
(4) 3.7
(5) 2.93

Chap. VI, page 111

(1) 0.384, 0.48
(2) 0.416, 0.493
(3) *a.* 0.154
b. 0.257
(4) 0.25
(5) 0.293

Chap. VI, page 115

(1) 3.58
(2) 2.88
(3) 3.98
(4) 4.25
(5) 3.7
(6) 4.36
(7) 4.50
(8) 5.15
(9) 6.60
(10) 6.29

Chap. VI, page 116

(1) 29.4 ft.
(2) 36.0 ft.
(3) 38.7 ft.
(4) 41.5 ft.
(5) 99.0 ft.
(6) 23.1 ft.
(7) 44.5 ft.
(8) 34.9 ft.
(9) 34.0 ft.
(10) 38.9 ft.

Chap. VI, page 119

(1) 112 lb.
(2) 114 lb.
(3) 113 lb.
(4) 111 lb.
(5) 112 lb.
(6) 106 lb.
(7) 4.5 ft.
2.19 ft.
7.13 ft.
111 lb.
(8) 57.4 lb.
(9) 166 lb.
(10) 299 lb.

Chap. VI, page 123

(1) 254 lb.
(2) 255 lb.
(3) 183 lb.

Chap. VII, page 132

(1) 60.6 lb.
(2) 42.0 lb.
(3) 3.145 lb.
(4) 20.82 lb.
(5) 156 lb.

Chap. IX, page 164

(1) 1.07
(2) 1.29
(3) 0.87
(4) 0.90
(5) 1.46
(6) 1.85
(7) 1.55
(8) 1.37
(9) 1.22
(10) 1.85

Chap. IX, page 170

(1) 1.32, 8.7 ft., 19°, 79 per cent.
(2) 1.54, 8.05 ft., 23°, 82.5 per cent.
(3) 1.99, 7.73 ft., 28°, 86 per cent.
(4) 1.13, 10.1 ft., 15°, 76 per cent.
(5) 1.00, 9.35 ft., 13°, 73 per cent.

Chap. IX, page 171

(1) 27°
(2) 24°
(3) 24°
(4) 18°
(5) 21°

Chap. IX, page 174

(1) 16°
(2) 21°
(3) 23°
(4) 22°
(5) 26°

Chap. X, page 200

(1) 53.7 mi. per hr.
(2) 67.5 mi. per hr.
(3) 80.7 mi. per hr.
(4) 61.7 mi. per hr.
(5) 76.3 mi. per hr.

Chap. X, page 203

(1) 5042 miles.
(2) 231 mi.
(3) 3307 mi.
(4) 1341 mi.
(5) 1023 mi.

Chap. X, page 214

(1) 1280 ft.
(2) 1373 ft.
(3) 664 ft.
(4) 172 ft.
(5) 587 ft.

Chap. X, page 219

(2) 83.8 mi. per hr.
(3) 71,600 ft.
(4) 91.8 mi. per hr.
(5) 418 mi. per hr.
(6) 496 mi. per hr.

Chap. X, page 220

(2) 338.6 mi. per hr.
(3) 313 mi. per hr.
(4) 287 mi. per hr.
(5) 6.47 sq. ft.

Chap. X, page 223

(1) 47.6 mi. per hr.
(2) 46.6 mi. per hr.
(3) 82.7 mi. per hr.
(4) 50.2 mi. per hr.
(5) 53.3 mi. per hr.

Chap. X, page 225

(1) 104 sq. ft.
(2) 74.8 mi. per hr.
(3) 94.1 mi. per hr.
(4) 53.0 mi. per hr.
(5) 43.4 mi. per hr.

Chap. X, page 227

(1) 122 mi. per hr.
(2) 161.5 mi. per hr.
(3) 270 hp.
(4) 190 mi. per hr.
(5) 169.5 mi. per hr.

Chap. XI, page 238

(1) 14,400 ft.
(2) 15,650 ft.
(3) 7.6 hp.
(4) 473 ft. per min.
(5) 500 ft. per min.

Chap. XI, page 239

(1) *a.* 7.4 min.
b. 10.0 min.
(2) 47.8 min.
(3) 15.0 min.
(4) 23.0 min.
(5) 10.8 min.

Chap. XI, page 240

(1) 19,781 ft.
(2) 20,250 ft.
(3) 10,200 ft.
(4) 6,202 ft.
(5) 8,000 ft.

Chap. XII, page 254

(1) *a.* 2,000 lb.
b. 2,830 lb.
c. 672 ft.
(2) 100 mi. per hr.
(3) *a.* 64.6 hp.
b. 70.5 hp.
(4) 89.3°, 140,000 lb.
(5) *a.* 92 hp.
b. 121 hp.

Chap. XII, page 259

(1) 1.5
(2) 4.9
(3) 6.9
(4) 12.2
(5) 17.9

Chap. XII, page 261

(1) 226 ft., 8.6
(2) 554 ft., 8.8
(3) 768 ft., 14.6
(4) 96 ft., 4.4
(5) 202 ft., 6.5

Chap. XII, page 263

(1) 12.7
(2) 15.3
(3) 11.1
(4) 17.7
(5) 14.4

Chap. XIV, page 279

(1) *a.* 7.9 ft.
b. 0.9 ft.
c. 0.9 ft.
(2) *a.* 7.9 ft.
b. 0.4 ft.
c. 0.4 ft.
(3) *a.* 9.2 ft.
b. 0.9 ft.
c. 0.6 ft.
(4) *a.* 8.6 ft.
b. 0.5 ft.
c. 0.3 ft.
(5) *a.* 8.8 ft.
b. 0.8 ft.
c. 0.6 ft.
(6) *a.* 8.3 ft.
b. 0.4 ft.
c. 0.3 ft.
(7) *a.* 7.3 ft.
b. 0.9 ft.
(8) *a.* 7.4 ft.
b. 0.4 ft.
(9) *a.* 8.4 ft.
b. 0.8 ft.
(10) *a.* 7.9 ft.
b. 0.4 ft.

Chap. XIV, page 283

(1) *a.* −10.25°
b. 9.75°
(2) *a.* 0.50°
b. −9.50°
(3) 1.25°
(4) −6.00°
(5) −9.50°

Chap. XIV, page 285

(1) 5.8°
(2) −10.0°
(3) −5.6°
(4) 7.5°
(5) −1.5°

Chap. XIV, page 286

(1) 7,000 ft-lb.
(2) 1,890 ft-lb.
(3) 16,020 ft-lb.

Chap. XVIII, page 340

(1) 55°
(2) 200°
(3) 270°
(4) 328°
(5) 17°
(6) 345°
(7) 155°
(8) 27°
(9) 279°
(10) 119°

Chap. XX, page 371

(1) 4,445 mi.
(2) 2,387 mi.
(3) 5,305 mi.
(4) 2,453 mi.
(5) 1,885 mi.

Chap. XX, page 373

(1) 106 mi. per hr., 101°, 11° right.
(2) 172 mi. per hr., 26°, 4° right.
(3) 133 mi. per hr., 332°, 8° left.
(4) 85 mi. per hr., 245°, 20° left.
(5) 99 mi. per hr., 131°, 11° right.

Chap. XX, page 374

(1) 15°, 137 mi. per hr.
(2) 35°, 155 mi. per hr.
(3) 343°, 76 mi. per hr.
(4) 143°, 120 mi. per hr.
(5) 214°, 88 mi. per hr.

Chap. XX, page 377

(1) *a.* 34 mi. per hr.
b. 60°
c. 14°
d. 105 mi. per hr.

(2) *a.* 56 mi. per hr.
b. 155°
c. 64°
d. 170 mi. per hr.

(3) *a.* 48 mi. per hr.
b. 260°
c. 94°
d. 187 mi. per hr.

Chap. XXI, page 389

(1) 1,803 lb.
(2) 588 lb.
(3) 1,169 lb.
(4) 1,083 lb.
(5) 1,829 lb.

Chap. XXI, page 393

(1) *a.* 498 lb.
b. 498 lb.

(2) *a.* 990 lb.
b. 990 lb.
c. 990 lb.

(3) *a.* 60 lb.
b. 188 lb.

(4) *a.* 198 lb.
b. 3,560 ft.
c. 25 lb.

Chap. XXI, page 394

(1) *a.* 1,423 lb.
b. 1,350 lb.
c. 7 lb.
d. 25,600 cu. ft.
e. 1,343 lb.

(2) *a.* 1,319 lb.
b. 186 lb.
c. 344 lb.
d. 965 lb.
e. 954 lb.

Chap. XXI, page 397

(1) *a.* 730 lb.
b. 734 lb.

(2) *a.* 1,935 lb.
b. 1,957 lb.

(3) *a.* 488 lb.
b. 524 lb.

(4) *a.* 625 lb.
b. 702 lb.

(5) *a.* 742 lb.
b. 688 lb.

Chap. XXI, page 401

(1) 6,099 ft.

(2) *a.* 12,910 lb.
b. 13,463 lb.

(3) *a.* 12,435 lb.
b. 905 lb.

INDEX

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S